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SEAV and SEMT Labeling for PVB-Tree

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***Abstract*—The PVB-tree T is a tree consists of b branches at each vertex of all the p copies of the path V on v vertices and these paths are joined to the vertices of a path P on p vertices. This $PVB-tree$ contains $pvb+pv+p$ vertices and $pvb+pv+p-1$ edges. Super edge antimagic vertex labeling is a bijective function $\lambda : V(T) \rightarrow \{1, 2, ..., pvb+pv+p\}$ with the property that the weights $\{w(xy) = \lambda(x)+\lambda(y); xy \in E(T)\}$ form an arithmetic progression$\{a, a+d, ..., a+(e-1)d\}$. The function $\lambda : V(T) \cup E(T) \rightarrow \{1, 2, ..., 2(pvb+pv+p)-1\}$ such that there is a minimal integer k satisfying $\lambda(x) + \lambda(y) + \lambda(xy) = k, \forall xy \in E(T)$ is called super edge magic total labeling. A graph that possess a super edge antimagic vertex (SEAV) labeling and/or possess super edge magic total (SEMT) labeling is called SEAV and/or SEMT graph. This paper establishes that the PVB-tree is both SEAV and SEMT graph.**

***Index Terms*—PVB-tree, super edge antimagic vertex labeling, super edge magic total labeling**

I. Introduction

Let G be a graph whose vertex set is $V(G)$ and Edge set is $E(G)$ with $|V(G)| = v$ and $|E(G)| = e$. If positive integers are assigned to vertices and/or edges of a graph, the graph is known as a labeled graph.

Consider a path P(main path) on p vertices, denoted by $x_1, x_2, ..., x_p$. Take p copies of another path V(subpath)on v vertices denoted by $x_{ij}, i = 1, 2, ..., p, j = 1, 2, ..., v$. At each vertex x_{ij} of these subpaths attach b pendant edges denoting these pendant vertices by $x_{ij}^k, i = 1, 2, ..., p, j = 1, 2, ..., v, k = 1, 2, ..., b$. The sub paths V are joined one at a vertex of the main path. In this Paper, it is established that the PVB-tree possesses both SEAV and SEMT labeling.

II. Basic definitions

***Definition** 1:* An (a, d)- super edge antimagic labeling of a graph G is a bijective function $\lambda : V(G) \rightarrow \{1, 2, ..., v\}$ such that the set of weights of the edges in G, $\{w(xy) = \lambda(x) + \lambda(y) : xy \in E(G)\}$, forms an antimagic arithmetic progression $\{a, a+d, a+2d, ..., a+(e-1)d\}$, where $a > 0$ and $d \geq 0$ are two fixed integers.

***Definition** 2:* An (a, d)- super edge magic total labeling of a graph G is a bijective map $\lambda : V(G) \cup E(G) \rightarrow \{1, 2, ..., v+e\}$, where $v = |v(G)|$ and $e = |E(G)|$, with the property that, given any edge $xy \in E(G)$,

$$\lambda(x) + \lambda(xy) + \lambda(y) = k$$

for some positive integer k. In other words,$w(xy) = k$ for any choice of edge xy. Such a minimal k is called the magic sum of G.

***Definition** 3:* Consider a path P (main path P) on p vertices. Take p copies of another path V (sub path) on v vertices. At each vertex of each copy of these sub paths attach b branches. These paths are joined to the main path P one at a vertex of the main path. The vertices of P are denoted by $x_1, x_2, ..., x_p$. The vertices of the sub paths V are denoted by x_{ij},$i = 1, 2, ..., p$,$j = 1, 2, ..., v$. The pendant vertices of the branches at x_{ij} are denoted by x_{ij}^k, $i = 1, 2, ..., p$,$j = 1, 2, ..., v$ and $k = 1, 2, ..., b$. The tree thus obtained is the $PVB-tree$.

The number of vertices of the $PVB-tree$ is $pvb+pv+p$, the number of edges is $pvb+pv+p-1$.

***Example** 1:*

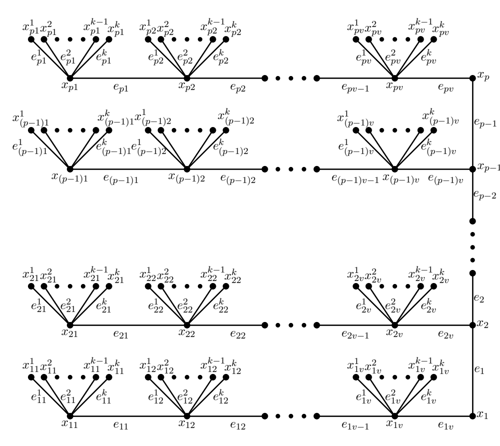

Figure 1.PVB-tree

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III. Main results

***Theorem** 3.1:* The PVB- tree T with odd or even vertices on P, odd vertices on V and b branches at each vertex of V admits a super edge antimagic vertex labeling.

Proof: Let $p \geq 3$ is odd or even, $v \geq 1$ is odd and $b \geq 0$. Define the vertex labeling $\lambda : V(T) \rightarrow \{1, 2, ..., pvb + pv + p\}$ as follows

For $i = 1, 3, ..., p$

$$\lambda(x_{ij}) = \frac{b(v(i-1)+j)+i(v+1)+j-v-b}{2} \quad \text{if } j = 1, 3, ..., v$$

$$\lambda(x_{ij}^k) = \frac{(b+1)(v(i-1)+j-1)-b+i+2k}{2} \quad \text{if } j = 2, 4, ..., v-1 \text{ and } k = 1, 2, ..., b$$

For $i = 2, 4, ..., p$

$$\lambda(x_i) = \frac{(b+1)(v(i-1))-b+i+1}{2}$$

$$\lambda(x_{ij}^k) = \frac{(b+1)(vi-j)+b+i-2k+3}{2} \quad \text{if } j = 1, 3, ..., v \text{ and } k = 1, 2, ..., b$$

$$\lambda(x_{ij}) = \frac{(b+1)(v(i-1))+(v-j)(b+1)}{2} + \frac{i+2}{2} \quad \text{if } j = 2, 4, ..., v-1$$

For $i = 1, 3, ..., p$, $j = 1, 3, ..., v$ and $k = 1, 2, ..., b$

$$\lambda(x_{ij}^k) = \begin{cases} \dfrac{(p+i-1)(v(b+1)+1)}{2} + \dfrac{(j-1)(b+1)+2k-b}{2} & \text{if } p \text{ is odd} \\ \dfrac{pvb+p(v+1)}{2} + \dfrac{(b+1)(v(i-1)+j-1)+i+2k-1}{2} & \text{if } p \text{ is even} \end{cases}$$

For $i = 1, 3, ..., p$ and $j = 2, 4, ..., v$

$$\lambda(x_{ij}) = \begin{cases} \dfrac{(p+i-1)(v(b+1)+b(j-1)+j+1}{2} & \text{if } p \text{ is odd} \\ \dfrac{(p+i-1)(v(b+1)+1)+j(b+1)}{2} & \text{if } p \text{ is even} \end{cases}$$

For $i = 1, 3, ..., p$

$$\lambda(x_i) = \begin{cases} \dfrac{(p+i)(v(b+1)+1)}{2} & \text{if } p \text{ is odd} \\ \dfrac{(p+i)(v(b+1)+1)+b}{2} & \text{if } p \text{ is even} \end{cases}$$

For $i = 2, 4, ..., p$ and $j = 1, 3, ..., v$

$$\lambda(x_{ij}) = \begin{cases} \dfrac{(p+i-1)(v(b+1)+1)}{2} + \dfrac{(v-j)(b+1)+2}{2} & \text{if } p \text{ is odd} \\ \dfrac{(p+i-1)(v(b+1)+1)}{2} + \dfrac{(v-j)(b+1)+b+2}{2} & \text{if } p \text{ is even} \end{cases}$$

For $i = 2, 4, ..., p$, $j = 2, 4, ..., v$ and $k = 1, 2, ..., b$

$$\lambda(x_{ij}^k) = \begin{cases} \dfrac{(p+i-1)(v(b+1)+1)}{2} + \dfrac{(v-j-1)(b+1)+2(b-k)+4}{2} & \text{if } p \text{ is odd} \\ \dfrac{(p+i-1)(v(b+1)+1)}{2} + \dfrac{(v-j)(b+1)+2(b-k)+3}{2} & \text{if } p \text{ is even} \end{cases}$$

It is clear that the edges receive the weights from $\dfrac{p(bv+v+1)+4-b}{2}$ or $\dfrac{p(bv+v+1)+4}{2}$ to $\dfrac{3p(bv+v+1)-b}{2}$ or $\dfrac{3p(bv+v+1)}{2}$ according as p is odd or even. So this PVB-tree admits super edge antimagic vertex labeling with arithmetic progression $\{a, a+d, ..., a+(e-1)d\}$ where $d = 1$ and $2a = p(bv+v+1)+4-b$ or $2a = p(bv+v+1)+4$ according as p is odd or even. ■

***Example** 2:*
Figure 2 gives the SEAV labeling of PVB-tree with $p = 5, v = 7, b = 3$, the edges receive the antimagic labels from 73 to 216, satisfying the conclusion of theorem 3.1. Also Figure 3 gives the SEAV labeling of PVB-tree with $p = 6, v = 9, b = 3$, the edges receive the antimagic labels from 113 to 333

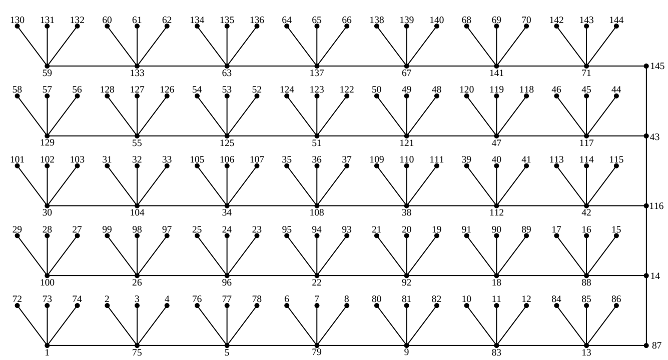

Figure 2. SEAV labeling of PVB-tree with $p = 5, v = 7$ and $b = 3$

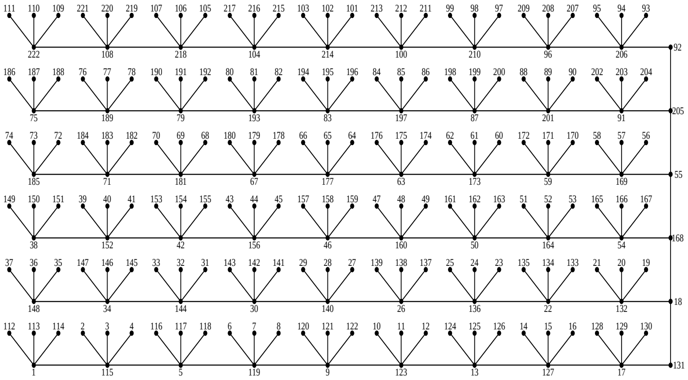

Figure 3. SEAV labeling of PVB-tree with $p = 6$, $v = 9$ and $b = 3$

***Theorem** 3.2:* The PVB- tree T with odd or even vertices on P, even vertices on V and b branches at each vertex of V admits super edge antimagic vertex labeling.

Proof: Let $p \geq 3$ is odd or even, $v \geq 2$ is even. Define the vertex labeling $\lambda : V(G) \rightarrow \{1, 2, ..., pvb + pv + p\}$ as follows

For $i = 1, 3, ..., p$

$$\lambda(x_{ij}^k) = \frac{(b+1)(v(i-1)+j-1)+(i-1)+2k}{2} \quad \text{if } j = 1, 3, ..., v \text{ and } k = 1, 2, ..., b.$$

$$\lambda(x_{ij}) = \frac{(b+1)(v(i-1)+j)+(i-1)}{2} \quad \text{if } j = 2, 4, ..., v$$

For $i = 2, 4, ..., p$

$$\lambda(x_i) = \frac{v(b+1)(i-1)+i}{2}$$

$$\lambda(x_{ij}^k) = \frac{v(b+1)(i-1)+i}{2} + \frac{(b+1)(v-j)+2(b-k+1)}{2} \quad \text{if } j = 2, 4, ..., v \text{ and } k = 1, 2, ..., b.$$

$$\lambda(x_{ij}) = \frac{(b+1)(vi-j+1)+i}{2} \quad \text{if } j = 1, 3, ..., v-1$$

For $i = 1, 3, ..., p$ and $j = 1, 3, ..., v-1$

$$\lambda(x_{ij}) = \begin{cases} \dfrac{(b+1)(v(p+i-1)+p+i}{2} & \text{if } p \text{ is odd} \\ \dfrac{(b+1)(v(p+i-1)+p+i+1}{2} & \text{if } p \text{ is even} \end{cases}$$

For $i = 1, 3, ..., p$, $j = 1, 3, ..., v-1$ and $k = 1, 2, ..., b$

$$\lambda(x_{ij}^k) = \begin{cases} \dfrac{(b+1)(v(p+i-1)+j-2)}{2} + \dfrac{p+i+2k}{2} & \text{if } p \text{ is odd} \\ \dfrac{(b+1)(v(p+i-1)+j-2)}{2} + \dfrac{p+i+2k+1}{2} & \text{if } p \text{ is even} \end{cases}$$

For $i = 1, 3, ..., p$

$$\lambda(x_i) = \begin{cases} \dfrac{(p+i)(v(b+1)+1)}{2} & \text{if } p \text{ is odd} \\ \dfrac{(p+i)(v(b+1)+1)+1}{2} & \text{if } p \text{ is even} \end{cases}$$

For $i = 2, 4, ..., p$ and $j = 2, 4, ..., v$

$$\lambda(x_{ij}) = \begin{cases} \dfrac{(p+i-1)(v(b+1)+1)}{2} + \dfrac{(v-j)(b+1)+2}{2} & \text{if } p \text{ is odd} \\ \dfrac{(p+i-1)(v(b+1)+1)}{2} + \dfrac{(v-j)(b+1)+3}{2} & \text{if } p \text{ is even} \end{cases}$$

For $i = 2, 4, ..., p$, $j = 1, 3, ..., v-1$ and $k = 1, 2, ..., b$

$$\lambda(x_{ij}^k) = \begin{cases} \dfrac{(p+i-1)(v(b+1)+1)}{2} + \dfrac{(v-j-1)(b+1)+2(b-k)+4}{2} & \text{if } p \text{ is odd} \\ \dfrac{(p+i-1)(v(b+1)+1)}{2} + \dfrac{(v-j-1)(b+1)+2(b-k)+5}{2} & \text{if } p \text{ is even} \end{cases}$$

It is clear that the edges receive the weights from $\dfrac{p(bv+v+1)+3}{2}$ or $\dfrac{p(bv+v+1)+4}{2}$ to $\dfrac{3p(bv+v+1)-1}{2}$ or $\dfrac{3p(bv+v+1)}{2}$ according as p is odd or even. So this PVB-tree admits super edge antimagic vertex labeling with arithmetic progression $\{a, a+d, ..., a+(e-1)d\}$ where $d = 1$ and $2a = p(bv+v+1)+3$ or $2a = p(bv+v+1)+4$ according as p is odd or even. ∎

***Example** 3:*

Figure 4 gives the SEAV labeling of PVB-tree with $p = 5, v = 6, b = 3$, where the edges receive the antimagic labels from 64 to 187, satisfying the conclusion of theorem 3.3. Also Figure 5 gives the SEAV labeling of PVB-tree with $p = 6, v = 8, b = 3$, the edges receive the antimagic labels from 101 to 297.

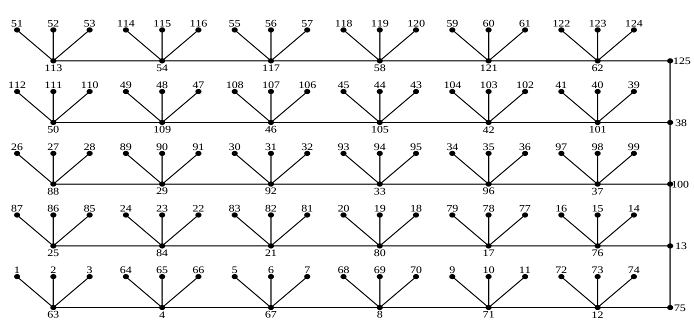

Figure 4. SEAV labeling of PVB-tree with $p = 5$, $v = 6$ and $b = 3$

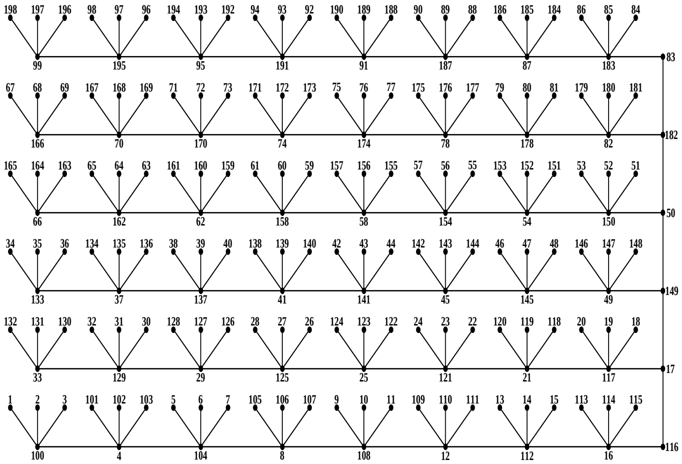

Figure 5. SEAV labeling of PVB-tree with $p = 6, v = 8$ and $b = 3$

***Notation** 1:* The edges joining the vertices x_i and x_{i+1} are denoted by $e_i, i = 1, ..., p-1$ i.e, $(x_i, x_{i+1}) = e_i$. In the same way for $i = 1, 2, ..., p$, the edges are denoted as $(x_{ij}, x_i x_{j+1}) = e_{ij}, j = 1, 2, ..., v-1$ with $(x_{iv}, x_i) = e_{iv}$. Also $(x_{ij}^k, x_{ij}) = e_{ij}^k; i = 1, 2, ..., p, j = 1, 2, ..., v$ and $k = 1, 2, ..., b$.

***Theorem** 3.3:* Let T be a PVB-tree on p (odd) vertices on P, v odd or even vertices on V and b branches. Then T admits a super edge magic total labeling.

Proof: Let T be a PVb-tree with the labeling $\lambda : V(T) \cup E(T) \to \{1, 2, ..., v+e\}; e = pvb+pv+v-1$
The vertices of this tree T follows the labeling as given in Theorem 3.1 when v is odd and the labeling as given in Theorem 3.3 when v is even. The edge labels are given as follows

For $i = 1, 3, ..., p$,$j = 1, 2, ..., v$

$\lambda(e_{ij})$=$(2p-i)(v(b+1)+1)+(v-j)(b+1)+1$

For $i = 1, 3, ..., p$, $j = 1, 2, ..., v$ and $k = 1, 2, ..., b$

$\lambda(e_{ij}^k)$ = $(b+1)(v(2p-i+1)-j)+2(p+1)+b-k-i$

For $i = 1, 2, ..., p-1$

$\lambda(e_i)$=$(2p-i)(v(b+1)+1)$

For $i = 2, 4, ..., p-1$,$j = 1, 2, ..., v$ and $k = 1, 2, ..., b$

$\lambda(e_{ij}^k)$=$(b+1)(v(2p-i)+j-1)+2p-i+k$

For $i = 2, 4, ..., p-1$ and $j = 1, 2, ..., v$

$\lambda(e_{ij})$=$(b+1)(v(2p-i)+j)+2p-i$

It is clear that the edges receive the constant weights. So this PVB-tree admits super edge magic total lableing with magic constant $k = \dfrac{5p(bv+v+1)-b+2}{2}$ if v is odd and $k = \dfrac{5p(bv+v+1)+1}{2}$ if v is even and hence the PVB-tree is a SEMT tree. ■

***Example** 4:*

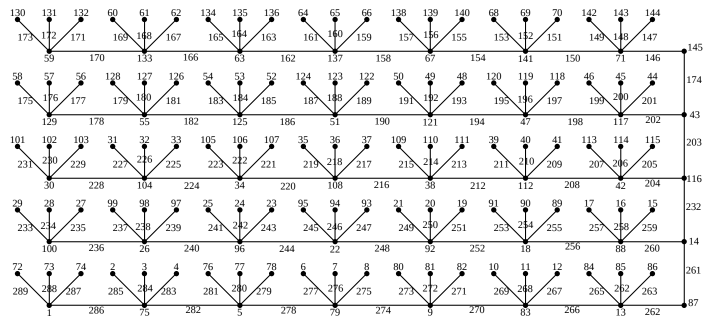

Figure 6.SEMT labeling of PVB-tree with $p = 5, v = 7, b = 3$ and $k = 362$

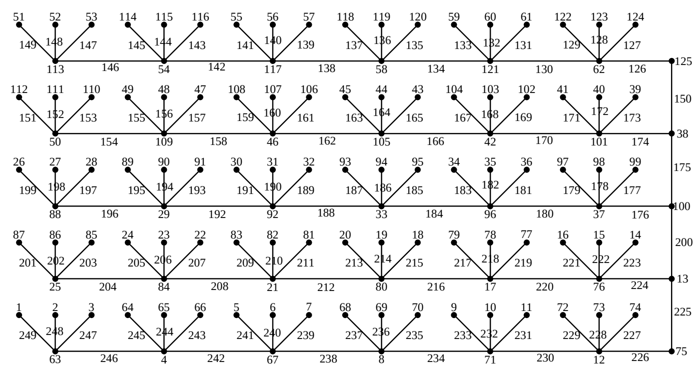

Figure 7. SEMT labeling of PVB-tree with $p = 5, v = 6, b = 3$ and $k = 313$

***Theorem** 3.4:* Let T be a PVB-tree on p (even) vertices on P, v odd or even vertices on V and b branches. Then T admits super edge magic total labeling.

Proof: Let T be a PVB-tree with the labeling $\lambda : V(T) \cup E(T) \to \{1, 2, ..., v+e\}$
The vertices of this tree T follows the labeling as given in Theorem 3.1 when v is odd and the labeling as given in theorem 3.3 when v is even . The edge labels are given as follows

For $i = 2, 4, ..., p$,$j = 1, 2, ..., v$ and $k = 1, 2, ..., b$

$\lambda(e_{ij}^k)$ = $(b+1)(v(2p-i)-j+1)+2(p+1)+b-k-i$

For $i = 2, 4, ..., p$ and$j = 1, 2, ..., v$

$\lambda(e_{ij})$ = $(b+1)(vp+j)+(p-i)(v(b+1)+1)+p$

For $i = 1, 2, ..., p-1$

$\lambda(e_i)$= $(2p-i)(v(b+1)+1)$

For $i=1,3,...,p-1$ and $j=1,2,...,v$

$\lambda(e_{ij})$=$(b+1)(v(2p-i+1)-j)+2p-i+1$

For $i = 1,3,...,p-1$, $j = 1,2,...,v$ and $k = 1,2,...,b$ $\lambda(e_{ij}^{k}) = (2p-i)(v(b+1)+1)+(v-j)(b+1)+b-k+2$ It is clear that the edges receive the constant weights. So this PVB-tree admits super edge magic total lableing with magic constant $k = \dfrac{5p(bv+v+1)+2}{2}$. Hence this PVB-tree is a SEMT tree. ■

***Example** 5:*

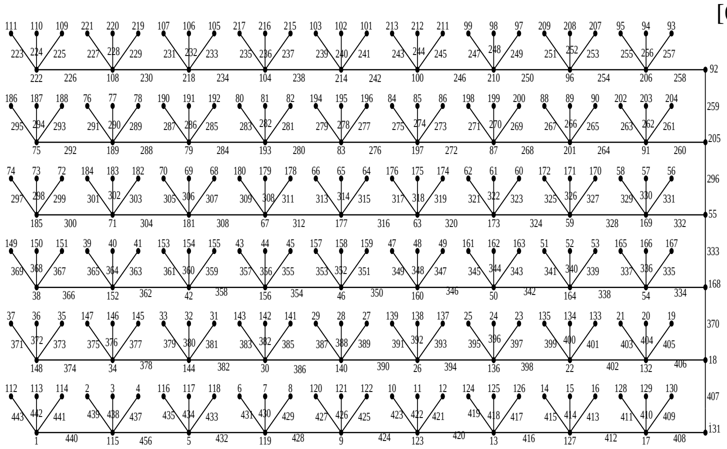

Figure 8. SEMT labeling of PVB-tree with $p=6, v=9, b=3$ and $k=556$

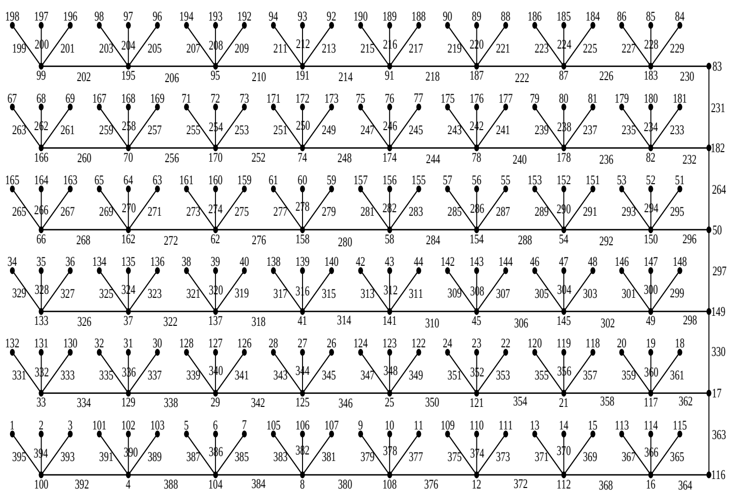

Figure 9. SEMT labeling of PVB-tree with $p=6, v=8, b=3$ and $k=496$

IV. CONCLUSION

Graphs are simboliesed for network communications, computational device etc. The focus of this paper is to present two kinds of labels to a particular type of tree. Further studies on this PVB- tree may be done by giving variations to v and b.

REFERENCES

[1] Z. Chen, On super edge magic graphs, *J. Combin. Math. Combin comput*.38,55-64
[2] A. Delman ,S. Koilraj, Radio Number for PVB-tree,*International Journal of Engineering Science, Advanced Computing & Bio-Technology*, Vol.7.No.4, October-December 2016,pp. 104-123
[3] H. Enomoto, K. Masuda and T. Nakamigawa, Induced graph theorem on magic valuations, *Ars combin. 56(2000)*,25-32
[4] J. Gallian, A dynamic survey of graphs labeling, *The Electronic Journal of Combinatorics*(2015)
[5] T. Nicholas, S. Somasundaram and V. Vilfred, On (a,d)-antimagic special trees, Unicylic graphs and complete bipartite graphs, *Ars Combin*70(2004),207-220
[6] K.A Sugeng, M. Miller and M. Baca, Super edge-antimagic total labelings,*Utilitas Math*.71(2006),131-141