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Dumb-Bell Plots of Cumulative Discovery and Production of Nigerian Petroleum Resources Using Composite Underground Reservoir

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ABSTRACT

Process control concepts were introduced upon developed models from material balance of Nigerian petroleum around a composite reserve, to obtain several transfer functions, so that as input functions are varied, new models are obtained, to note which input have the best impact. The well-known Hubbert concept of oil depletion was then employed for the determination of the peak and dumbbell intersection. Nigerian Petroleum Data were obtained from the Department of Petroleum Resources (DPR) of the Ministry of Petroleum and Minerals Resources, which was used as the experimental data, for 57 years giving 57 data points. MatLab Package 7.9 version was subsequently used in the mathematical computations and curve-fitting analysis. The research has shown that the Nigerian oil reserve will finish in the year 2682AD and the gas will follow sooth in the year 3151AD. However, this result is accurate to an R^2 of 0.9955 - 0.9963 for oil and 0.9979 - 0.9983 for gas, as shown in model 5. These findings can be useful in governmental planning, economy diversification, and OPEC bargaining and positioning.

Key Words: Nigerian petroleum resources, composite underground reservoir, cumulative discovery and production, dumbbell profiles, intersections.

1.0 INTRODUCTION

Today, most of the countries in the world are importers of energy. The fossil fuels accumulated over aeons of geological activity are irreversibly consumed at a rate more than million times faster than they were formed. This has left us in a precarious position, especially for petroleum and its products. The hike in price of petroleum and its products, both in national and international scenes, is frequent for two simple reasons: the mounting demands and fast depletion of reserves. The importance of petroleum in present day civilization is ever increasing due to its unmatched contribution to our energy requirements in lubrication and in petrochemical field. Thus, its competence to serve mankind is unquestionable and unique too.

In 1973, the Middle-East war broke out and aided Nigeria to make more money from her petroleum production. With the nation’s desire for rapid development, coupled with our rising bills for imported food and industrial machinery, Nigeria became more and more dependent on oil. The question as of then was not how to get money but how and on what to spend it on. No wonder the then Head of State was quoted as saying: that the problem of Nigeria was not money but how to spend it, either to spend it on people-oriented projects or on gigantic white elephant schemes (Imoukhuede, 1996).

The reserves held by a country constitute a strong determinant in the assessment of the economic potentials of the country and rate of investment and long term economic planning. Reserve is the much oil known to be producible, within a known time with known techniques at known costs and in known fields. It is not just the much oil in the rock in the earth; this is because the oil is divided into conventional and non-conventional oil.

But new discoveries both onshore and offshore do not mean new petroleum formation in the earth. It simply means either that another small 'old' reserve has been detected or that the 'former' large reserve has another place where it is closest to the surface of the earth (Kamalu, 2010). However, Nigerians may not know that their petroleum recoverable reserves is seriously depleting; and any serious-minded patriot knows that if a pool is constantly being scooped away without replenishment, depletion and eventual exhaustion inevitably follows. The amount of oil and gas we get from the ground is about to reach the point of diminishing returns i.e getting its peak and starts coming down forever, no matter the ingenuity or determination to make it rise. (Adenikinju, 1996; Kamalu, 2010).

Modeling future production or depletion is straight forward in countries not constrained by political policy, a situation that does not exist so far within OPEC and FSU. Several methods can be identified on modeling future production or depletion. The creaming curve plotting cumulative means discoveries versus the cumulative number of new field. Another method is to plot the percentage of annual to cumulative production (or depletion) versus cumulative production (or depletion), and extrapolate the trend. The future production (or depletion) can also be modeled with one or more bell-shaped curves. The bell- shaped curve was originated by K.M. Hubbert, a geophysicist, in 1956 to predict that U.S oil production would peak in about 1970 and decline thereafter. He was proved remarkably accurate because in 1971 the production of the 48 US lower states peaked and has remained on the down ward trend ever since. In other regions where severe exploration and production (B and P) episodes, prevail one has to use several such curves, each one related to its discovery pattern (Laherrere, 2002). Such record for Nigeria is non-existent inspite of the strategic role of oil to the economy Oil production (or depletion) in the world outside OPEC and the FSU can be readily modeled with such a bell-shaped curve, despite the recent surge of deepwater production (Laherrere, 2002). Modeling the PSU petroleum has to recognize the overproduction of the late 1980s and the under-production during the 1990s following the collapse of the Soviet Union. What has been under-produced in the 90s or under-developed in the Soviet era is now creating a new cycle. The main contributing regions will be East Siberia and Sakhalin as well as new discoveries in the Caspian together giving a peak of about 10Mb/d in 2010. Modeling OPEC production (or depletion) is more questionable, but it could peak in 2020 at between 40 and 45 Mb/d (Laherrere, 2003).

Against this background, the problem is to find when Nigerian Petroleum will peak or had peaked in the past years, so that we can monitor the downward bumps of plateau to exhaustion. Since oil formation gives rise to gas formation or vice versa the exhaustion of one leads to the eventual exhaustion of the other with time. And of this, Nigerian petroleum depletion profiles, peaking and exhaustion dates, this study is poised to model and forecast.

This work covers only the Nigerian conventional petroleum reserves both for onshore and offshore reserves. It takes no account of non-conventional oil since its production is yet to begin in Nigeria. But the Nigerian petroleum ultimate reserves are dynamic since more explorations and discoveries are always adding to the known ultimately recoverable oil pool. This is one of the handicaps of the study. Also, solid fossil fuels (coal and wood) are precluded in the study but allusions could still be extended to them. They are part of the fossilification processes of these fuels.

2.0 NATURAL RESOURCES DEPLETION MODEL DEVELOPMENT

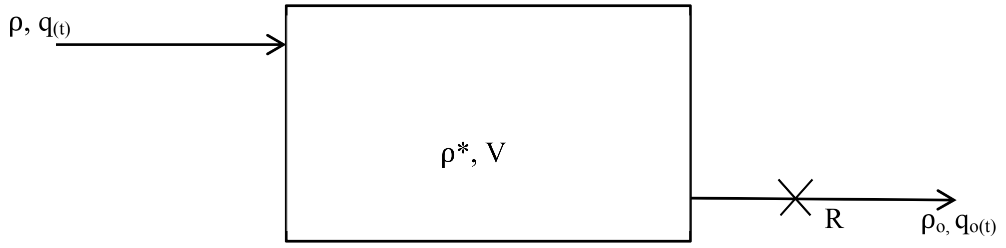
2.1 Background

Generally, the trend for most natural resources depletion is first increasing return, followed by a constant he resource is finally return and finally diminishing return. If there is no corresponding replenishment of the produced (consumed) resources, the diminishing return will continue until the resource is finally exhausted or flatten. The law of diminishing return is hence applicable to natural resource depletion with general model given by the equation below;

$$P = C_1 t - C_2 t^2 \qquad (2.1)$$

2.2 Assumptions and Model Development

1. One composite oil well replacing all oil wells drilled by all the oil companies in Nigeria.
2. The data collected from (DPR) is the sum total of all the production, discovery and reserves from all the oil companies in Nigeria.
3. The composite oil well have one input and one output.
4. The material balance around the well will result in first order ODE.
5. The density of the crude entering the reserve, in the reserve and out from the reserve is the same.
6. At the beginning there is initial volume of reserve, V_O.
7. The nation's crude will finish when the rate of change of the reserve is zero (Hubbert concept, ie $\frac{dQ_R}{dt} = 0$)



Mass Balance

Mass flow in –Mass flow out = rate of accumulation of mass in the reservoir.
Mathematically,

$$\rho q - \rho_o q_o = \frac{d(\rho * V)}{dt} \qquad (2.2)$$

Assumptions:

- At the t = 0, volume is V_O
- Uniform density, i.e $\rho_o = \rho = \rho*$
- Laminar flow, $q_o = \frac{V}{R}$

$$\text{Eqn (1) becomes } q - \frac{V}{R} = \frac{dV}{dt} \qquad (2.3)$$

Taking Laplace of eqn (2.3), $Q_{(S)} - \frac{V(s)}{R} = sV(_S) - V_O$

$$V(_S) = \frac{Vo}{[S + \frac{1}{R}]} + \frac{Q(s)}{[S + \frac{1}{R}]} \qquad (2.4)$$

*If the input function is $Q_{(S)} = \frac{1}{S[S+\frac{1}{R}]}$, then eqn (2.4) is

$V(s) = \frac{Vo}{[S+\frac{1}{R}]} + \frac{1}{S[S+\frac{1}{R}]^2}$ (2.5)

Taking the inverse Laplace of eqn (2.5) and simplifying gives;

$V_{(t)} = F(R^2(1-e^{-\frac{1}{R}t}) + (Vo- \ Rt)\ e^{-\frac{1}{R}t})$ (2.6a)

On differentiation with respect to time, eqn (2.6a) which is a cumulative production-time history yields an annual production-history, egn (2.6b).

$P_{(t)} = F(t - \frac{Vo}{R})e^{-\frac{1}{R}t}$ (2.6b)

At peak, $\frac{d^2V}{dt^2} = \frac{dP}{dt} = 0$; $Rt - R^2 - V_o = 0$

$t_{pk} = R + \frac{Vo}{R}$ (2.6c)

*If the input function is $Q(s) = \frac{1}{S[S+\frac{1}{R}]^2}$, then eqn (2.4) is

$V(s) = \frac{Vo}{[S+\frac{1}{R}]} + \frac{1}{S[S+\frac{1}{R}]^3}$ (2.7)

Taking inverse Laplace of eqn (2.7) and simplifying gives

$V_{(t)} = F(R^3(1-e^{-\frac{1}{R}t}) + (Vo- \ R^2t - \frac{Rt2}{2})\ e^{-\frac{1}{R}t})$ (2.8a)

On differentiation with respect to time, eqn (2.8a) which is a cumulative production-time history yields an annual production-time history, eqn (2.8b)

$P_{(t)} = F(\frac{t2}{2} - \frac{Vo}{R})e^{-\frac{1}{R}t}$ (2.8b)

At peak, $\frac{d^2V}{dt^2} = \frac{dP}{dt} = 0$; $Rt^2 - 2R^2t - 2Vo = 0$

$t_{pk} = R \pm \frac{\sqrt{(R4 + 2RVo)}}{R}$ (2.8c)

The derivations continue as shown in the table below

Table 2.1: An Array of developed models by altering the input functions

Model No.	Input function	Cumulative Production $V_{(t)}$	Annual Production $P_{(t)}$	Peak Time (t_{pk})
1.	$\frac{1}{s}$	$R + (R-Vo)\, e^{-t/R}$	$\left(\frac{V_0}{R}-1\right) e^{-\frac{t}{R}}$	at t_{pk}: $V_0 = R$
2.	$\frac{1}{S^2}$	$(t-R)\, R + (V_0 + R^2)e^{-t/R}$	$R-\frac{1}{R}(V_0 + R^2)\, e^{-t/R}$	at t_{pk}: $V_0 =-R^2$
3.	$\frac{1}{s+\frac{1}{R}}$	$(V_0 + t)\, e^{-t/R}$	$-\frac{1}{R}\,(V_0+t)\, e^{-t/R}$	$-V_0$
4.	$\frac{1}{s\left(s+\frac{1}{R}\right)}$	$R^2\,(1-e^{-t/R}) + (V_0-Rt)\, e^{-t/R}$	$\left(\frac{V_0}{R}-t\right) e^{-t/R}$	$Rt - R^2 - V_0 = 0$
5.	$\frac{1}{s\left(s+\frac{1}{R}\right)^2}$	$R^3\,(1-e^{-t/R}) + (V_0-R^2t - R\frac{t^2}{2})\, e^{-t/R}$	$\left(\frac{V_0}{R}-\frac{t^2}{2}\right) e^{-t/R}$	$Rt^2 - 2R^2t-2V_0=0$
6.	$\frac{1}{s\left(s+\frac{1}{R}\right)^3}$	$R^4\,(1-e^{-t/R}) + (V_0-R^3t + \frac{R^2t^2}{2} - \frac{Rt^3}{6})\, e^{-t/R}$	$\left(\frac{V_0}{R}-\frac{t^3}{6}\right) e^{-t/R}$	$Rt^3 - 3R^2t^2-6V_0=0$
7.	$\frac{1}{s\left(s+\frac{1}{R}\right)^4}$	$R^5\,(1-e^{-t/R}) + (V_0-R^4t - R^3\frac{R^3t^2}{2} - \frac{R^2t^3}{6} - \frac{Rt^4}{24})e^{-t/R}$	$\left(\frac{V_0}{R}-\frac{t^4}{24}\right) e^{-t/R}$	$Rt^4 - 2R^2t^3-24V_0=0$
8.	$\frac{1}{s\left(s+\frac{1}{R}\right)^n}$	$R^{n+1}\,(1-e^{-t/R}) + \left(V_0-R^nt - R^{n-1}\frac{t^2}{2!} - \ldots - R^{n-(n-2)}\frac{t^{n-1}}{(n-1)!} - \frac{R^{(n-1)}t^n}{n!}\right)e^{-\frac{t}{R}}$	$\left(\frac{V_0}{R}-\frac{t^n}{n!}\right) e^{-t/R}$	$Rt^n - nRt^{2n-1}-(n!)V_0$

9.	$\frac{1}{s\left(s+\frac{1}{C}\right)}$	RC + $V_0e^{-t/R}$+ $\frac{RC}{R-C}$ $(Ce^{-t/C}-Re^{-t/R})$	$\frac{RC}{R\ -C}(e^{-t/R}-e^{t/c})$ - $\frac{V_0}{R}$ e t/c	$\frac{RC}{R-C}\ In\left[\frac{RC-V_0(R-C)}{R^2}\right]$

2.3 Hubbert Concept

If we represent the cumulative production by the symbol Q_P, the cumulative proved discoveries by Q_D (producible discovery: not all discovery are producible), and the proved reserves by Q_R, then for each year,

$$Q_D = Q_P + Q_R \text{ or } Q_R = Q_D - Q_P \qquad (2.9)$$

The relation between rates of change of these quantities with time is obtained by taking the derivative with respect to time of equation (2.9), giving

$$\frac{dQ_D}{dt} = \frac{dQ_P}{dt} + \frac{dQ_R}{dt} \text{ or } \frac{dQ_R}{dt} = \frac{dQ_D}{dt} - \frac{dQ_P}{dt} \qquad (2.10)$$

in which, $\frac{dQ_D}{dt}$ is the rate of discovery,

$\frac{dQ_P}{dt}$ is the rate of production, and

$\frac{dQ_R}{dt}$ is the rate of increase of the proved reserves (Hubbert, 1956).

The manner in which the three quantities Q_D, Q_P, and Q_R must vary with time during the entire history of petroleum production from start to finish must be approximately as follows:

The cumulative production Q_P, when plotted as a function of time, will increase slowly during the early stages of petroleum production, increase more and more rapidly with time to about the halfway point, and then continue its ascent by rising slowly, and finally leveling off as the ultimate figure Q_∞, as production ceases (Hubbert, 1956).

The curve of proved reserves Q_R will start at zero, rise gradually until a maximum is reached at about the halfway point and then gradually decline to zero.

As petroleum must be found before it can be produced, the curve of cumulative proved discoveries Q_D must closely resemble that of cumulative production, except that it must plot ahead of the production curve by some time interval Δt, which on itself may vary during the cycle (Hubbert, 1956).

A plot of the family of the three curves Q_D, Q_P and Q_R is shown in Fig 2.1 as they may be expected to appear in the case of cumulative production, of petroleum in Nigeria.

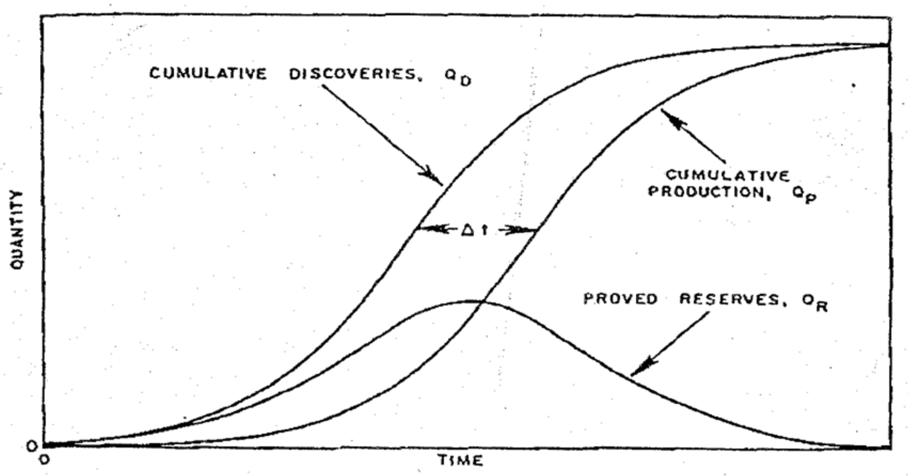


Fig 2.1: Cumulative discoveries and production, and proved reserves

Because of the close similarity between the curve of cumulative proved discoveries and that of cumulative production, it follows that the study of the discovery curve must give one a preview of what production will do at a time of approximately Δt in the future (Hubbert, 1956).

Taking the time derivatives of the three curves shown in Fig 2.1 gives us the rate of discovery, rate of production, and rate of increase of proved reserves, which are plotted as a function of time in Fig 2.2. It will be noted that the rate of discovery will reach a peak at about mid-range, and thereafter, gradually decline to zero.

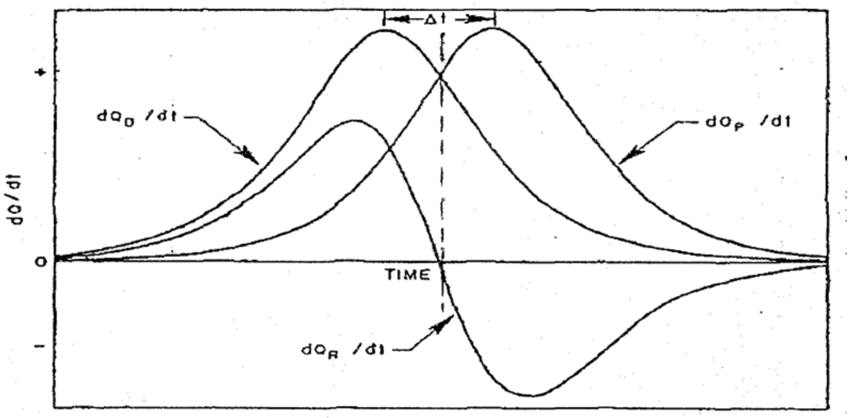


Fig 2.2: Rates of discovery, production and change of proved reserves

The rate of production will reach a peak at a time about Δt after that of discovery, and the increase of proved reserves will change from positive to negative about halfway between the discovery and production peaks. The reserves themselves, Q_R, will reach a maximum at the same time.

The relations between the three curves at this midpoint can be seen by noting that when reserves reach their maximum value, their derivative with time becomes zero.

$$\frac{dQ_R}{dt} = 0 \tag{2.11}$$

Which when inserted into equation (2.10) gives

$$\frac{dQ_D}{dt} = \frac{dQ_P}{dt} \tag{2.12}$$

This tells us that when reserves reach their maximum value, the curve of discovery rate and production rate will cross, production going up and discovery going down. This is shown in Figure 2.2.

Then combining sections 2.2 and 2.3, we see that; $y = Q$ (2.13)
Which is a cumulative (sigmoidal) discovery or production profile with t as time (an independent variable in years) and y (or Q) is dependent variable as petroleum volume in (mmB or bscf).

When model (2.13) is differentiated with time, we obtain; $\frac{dy}{dt} = \frac{dQ}{dt}$ (2.14)
The model (2.14) which is an annual production or discovery model has a dumb-bell profile. Recall that although, $y = Q_D = Q_P$, the numerical values of the characteristic constants of the model for production and discovery are not necessarily going to be the same, so that their coefficients of correlation (R^2) are not the same.

It is like solving two non-linear equations simultaneously or graphically, at their intersection, $\frac{dQ_P}{dt} = \frac{dQ_D}{dt}$ (the rates are equal at the intersection) and the time when this occurs is when the nation's petroleum reserve is finished (see figure 2.2).

2.4 The sigmoidal and Dumb-bell plots (Rate plots) using MATLAB 7.9 version

The data obtained from Department of Petroleum Resources, Ministry of Petroleum and Minerals, Nigeria, used in plotting scatter diagrams, and the models $y = Q_D = Q_P$ were superimposed to obtain sigmoidal profiles. Then, the analysis section of the MATLAB would be asked to produce the differentiated data with time from the sigmoidal data already typed in. This differentiated data (scatter diagrams) which are dumbbell profiles will be plotted and the differentiated models $\frac{dy}{dt} = \frac{dQ_P}{dt} = \frac{dQ_D}{dt}$ superimposed on them and the intersections read off.

Because of the difference between apparent and real interception points, the rate change plots (Dumb-bell) are done in duplicates: the 1st showing the apparent intersection while the 2nd shows enlarged real intersection points. The intersection point shows the point at which the reserve of a country has finished so that the rate of discovery will be equal to rate of production and so a very important point for the petroleum industry of a country.

3.0 RESULT PRESENTATION AND DISCUSSION

3.1 Result presentation

The rate of change of discovery and production with time are also outlaid in figures 3.4 – 3.6 (DO) and (DOI), and (DG) and (DGI). Also presented are figures 3.7 (DOI) and (DGI), and figures 3.9 (DOI) and (DGI).

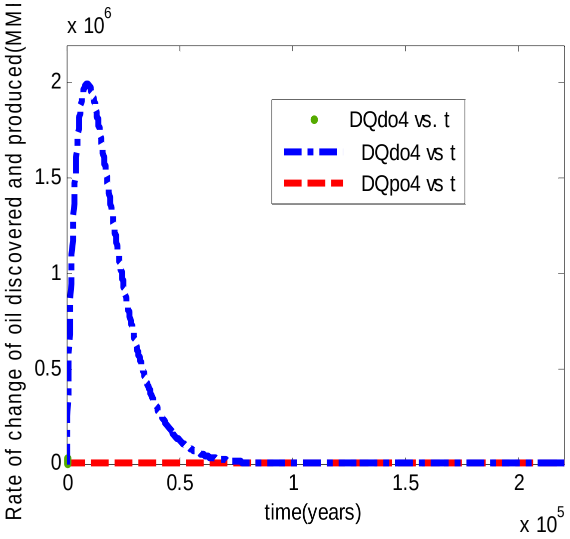


Fig 3.4(DO*): Rate of change of oil discovered and produced versus time

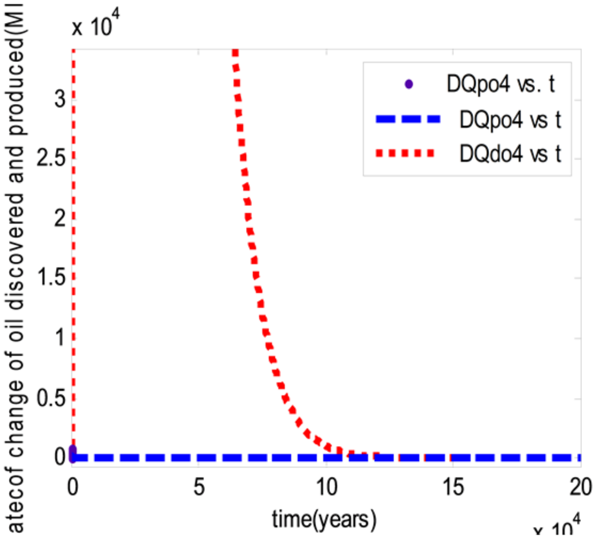


Fig 3.4(DOI): Rate of change of oil discovered and produced versus time

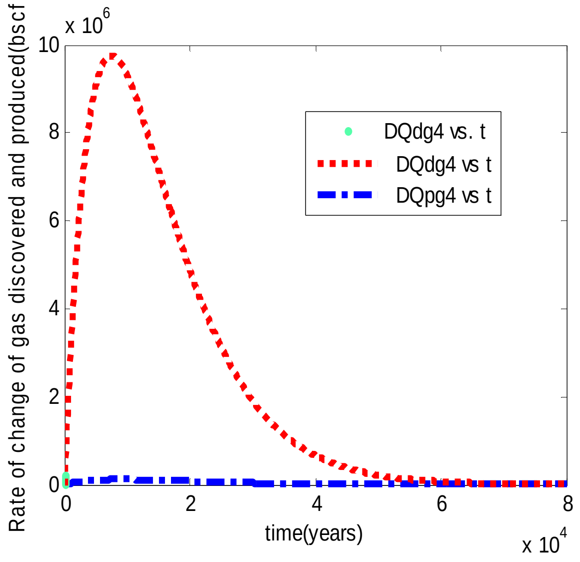


Fig 3.4(DG)*: Rate of change of gas discovered and produced versus time

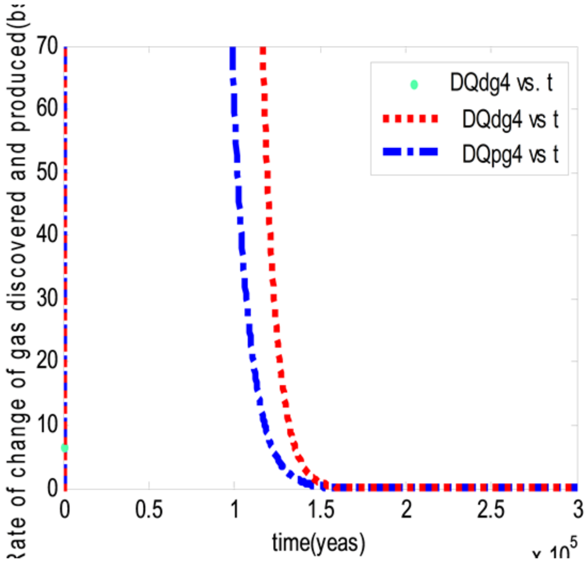


Fig 3.4(DGI): Rate of change of gas discovered and produced versus time

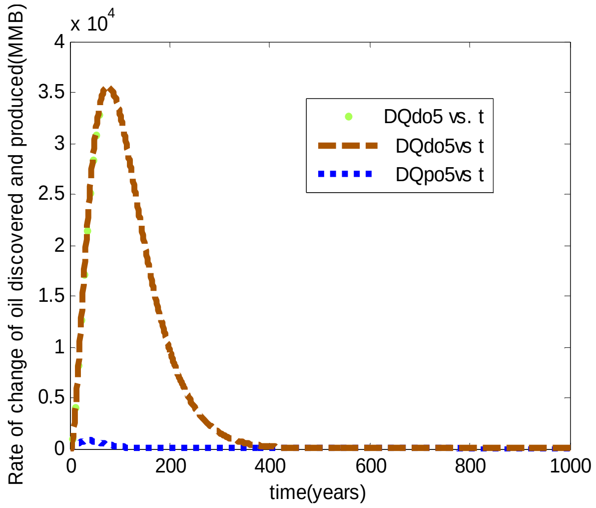


Fig 3.5(DO*): Rate of change of oil discovered and produced versus time

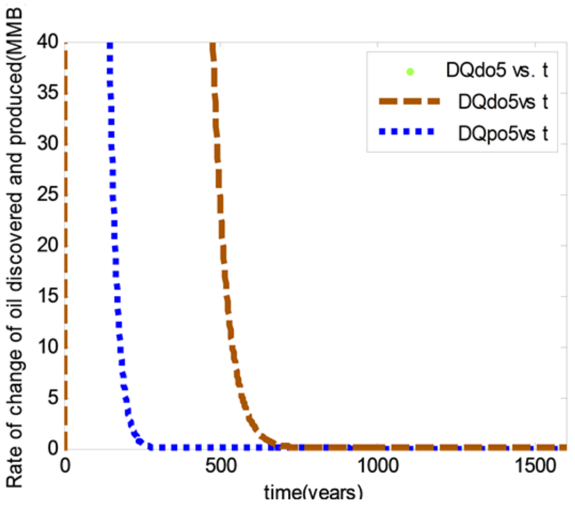


Fig 3.5(DOI): Rate of change of oil discovered and produced versus time

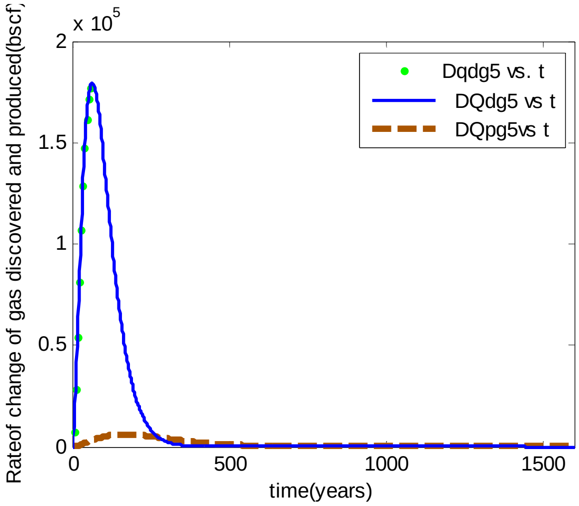


Fig 3.5(DG): Rate of change of gas discovered and produced versus time

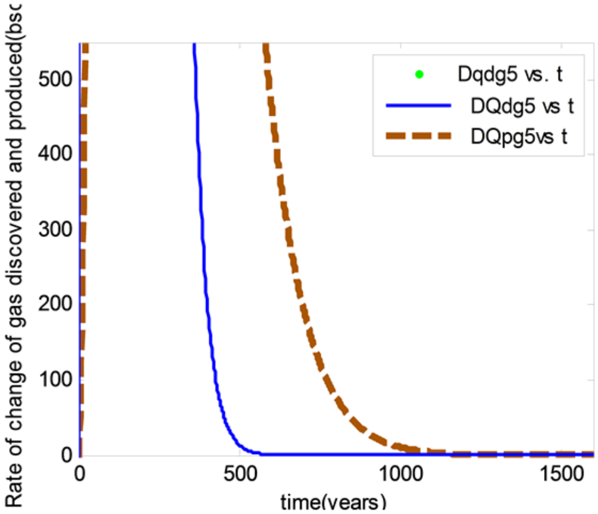


Fig 3.5(DGI): Rate of change of gas discovered and produced versus time

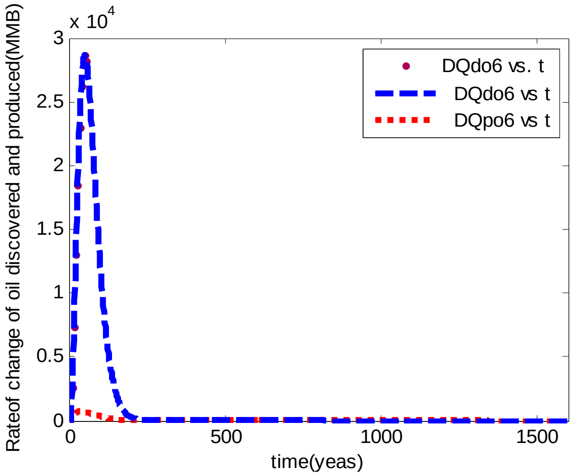


Fig 3.6(DO): Rate of change of oil discovered and produced versus time

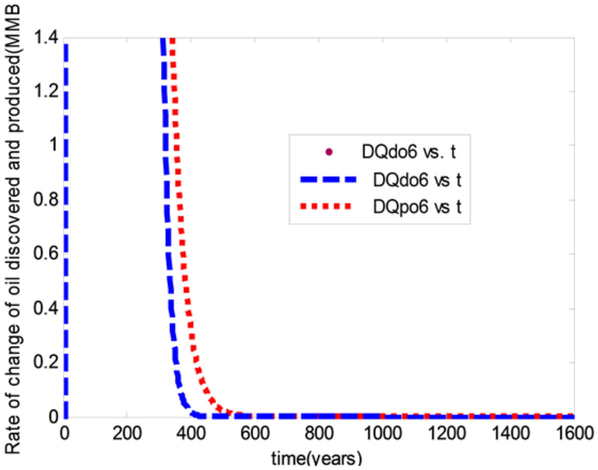


Fig 3.6(DOI): Rate of change of oil discovered and produced versus time

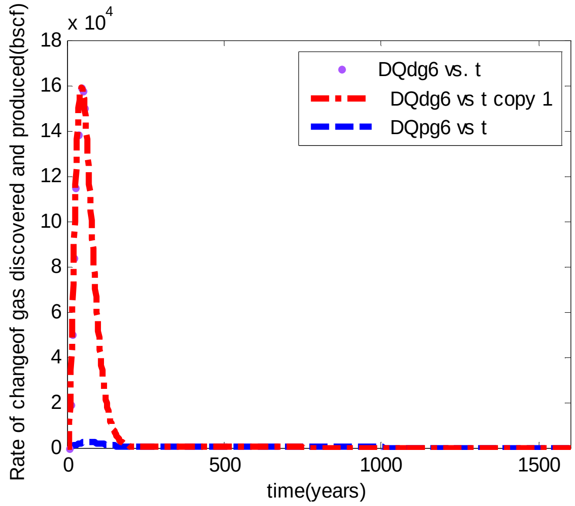


Fig 3.6(DG): Rate of change of gas discovered and produced versus time

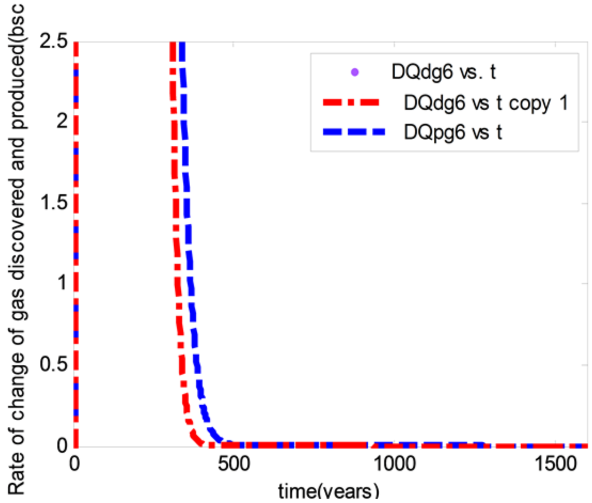


Fig 3.6(DGI): Rate of change of gas discovered and produced versus time

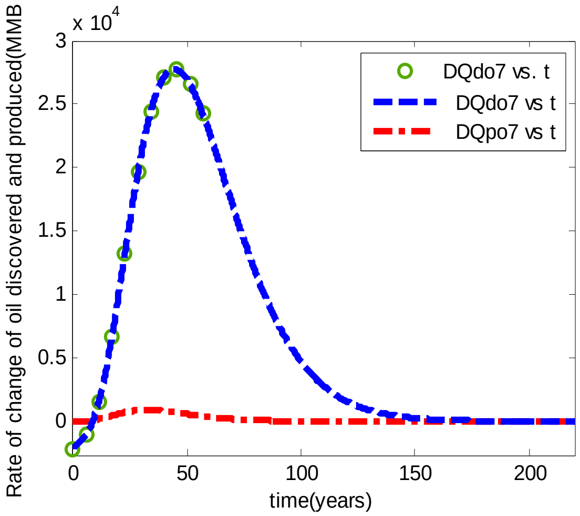


Fig 3.7(DOI)*: Rate of change of oil discovered and produced versus time

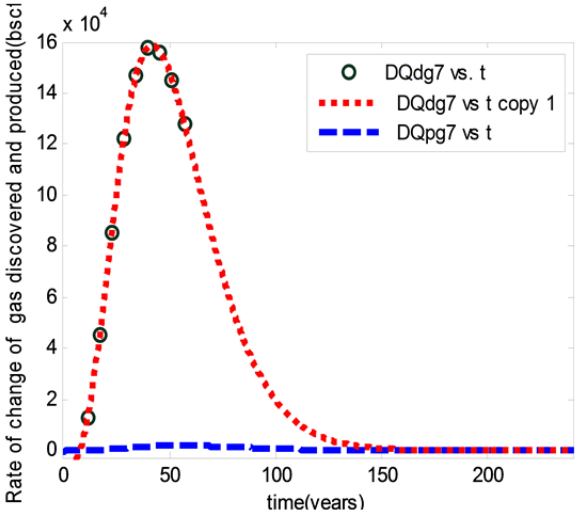


Fig 3.7(DGI)*: Rate of change of gas discovered and produced versus time

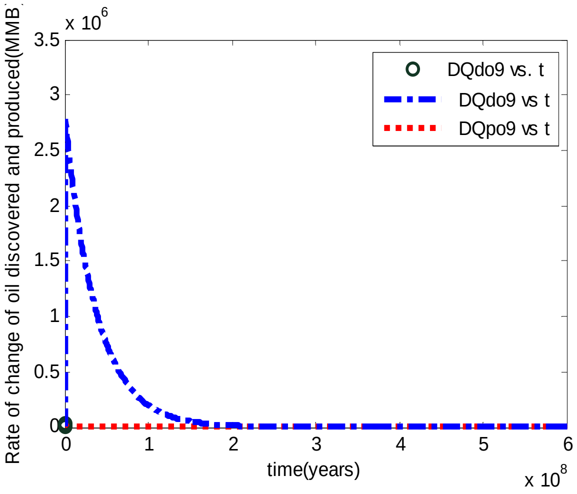


Fig 3.9(DOI)*: Rate of change of gas discovered and produced versus time

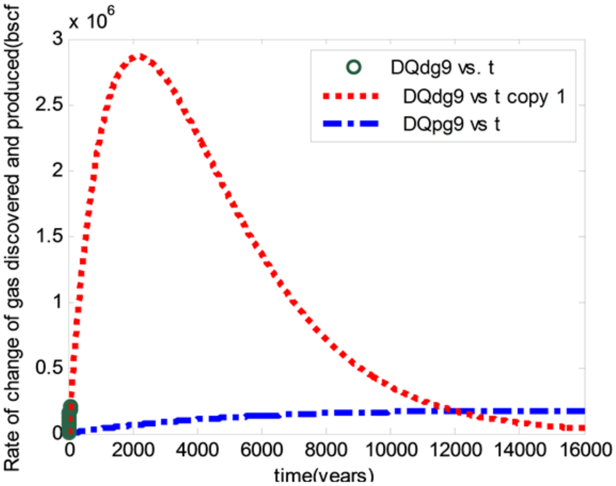


Fig 3.9(DGI)*: Rate of change of gas discovered and produced versus time

3.2 Discussion of Result

Models 1 – 3 had one problem or the other. For instance, a model may not fit the plot and will be discarded. Other times, a model may plot but will not predict because it does not tapper out asymptotically parallel to x-axis, so that an ultimate value of y can read off (i.e sigmoidal profile). In this case, except a model is sigmoidal (i.e S-shaped), it cannot predict (Hubbert, 1956). Hence, for a model to be used, it must both fit and predict.

Figure 3.4 (DO) show an apparent intersection of discovery and production of oil so that the enlarged intersection is shown in fig 3.4 (DOI). It is shown that from the enlarged intersection point, the years shifted up as the two graphs gum together and move as one.

This intersection is $f(119{,}000) = 0$, which means the reserve will finish in 119,000 years from 1957. In the case of the gas, i.e figs 3.4 (DG) and figs 3.4 (DGI), the intersection occurs at $f(151{,}000) = 0.895302$, i.e the gas reserve will finish in 151,000 years from 1957.

This is a good result except that in the model 4, gas have a negative initial reserve (V_O). In model 5, figs 3.5 (DO) and (DOI) show that the intersection occurred at $f(725) = 0.136817$, giving us 725 years from 1957, just as the intersection of the gas section of the same model figs 3.5 (DG) and (DGI) is $f(1194) = 1.62447$, i.e 1,194 years from 1957. This model is a good model because it has both positive initial reserve (V_O) as well as comparative high R^2.

In model 6, figs 3.6 (DO) and (DOI) show that the intersection occurs at $f(523) = 0.0133776$, i.e 523 years from 1957. The gas part of the model, figs (DG and DGI), intersect at $f(483) = 0.000149588$, i.e 483 years from 1957. Again, this model is a good model for it has given us a positive initial reserve (V_O) even as its R^2 is not high as model 5.

In model 7, there is no duplicate plot since the two plots can be contained In one graph. Fig 3.7 (DOI) showed an intersection of $f(180) = 36.6086$, i.e 180 years from 1957. The gas counterpart of the model 7, fig 3.7 (DGI) intersected at $f(224) = 3.45806$, i.e 224 years from 1957. This model also gives positive initial reserve (V_O) but its R^2 is comparatively lower than that of model 5.

There is no model 8 result since it is a generalized model.

In model 9, which does not also have duplicate plot, fig 3.9 (DOI) gives an intersection of $f(5.87e^{+8}) = 0.485818$. This big value merely means that it is for a very long time while the intersection of the gas counterpart of model 9, i.e fig 3.9 (DGI) at $f(11880) = 178310$, i.e 11880 years from 1957. This model is not a good model. Like model 4, it has negative initial reserve (V_O) even as R^2 is promising to be a good one.

It is said that, in oil industry, oil is to be discovered before production. But in this case, not only that there is no oil is outside the reservoirs (in the rock), i.e the meaning of negative initial reserve.

In models 6 and 7, though naturally okay with positive initial reserve, like model 5 but their R^2 are not the best. In model 5, it meets the natural reality of positive initial reserve as well as best R^2 in the remaining models.

Again, it is in order because oil will likely finish before gas 725 years against 1194 year, i.e if this is put in the present dispensation, the oil will finish in the year 2682AD and gas 3151AD. These are pretty long times. Nigeria can go to sleep!

The qualified input function for the selected model 5 is $Q(s) = \frac{1}{S[S+\frac{1}{R}]^2}$. This input function is made up of two parts i.e a product of $\frac{1}{S}$ (unit step) and $\frac{1}{[S+\frac{1}{R}]^2}$ (an exponential function which in time domain is $te^{-\frac{1}{R}}$).

$Q(s) = \frac{1}{S}$ has single pole at $s = 0$

$Q(s) = \frac{1}{[S+\frac{1}{R}]^2}$ has one pole at $s = -\frac{1}{R}$

Hence, the assumption of input flow rate into the one composite reservoir is justified.

4.0 CONCLUSSION

Process control concepts were introduced upon developed models from material balance of Nigerian petroleum around a composite reserve, to obtain several transfer functions, so that as input functions are varied, new models are obtained, to note which input have the best impact. The well-known Hubbert concept of oil depletion was then employed for the determination of the peak and dumbbell intersection. Nigerian Petroleum Data were obtained from the Department of Petroleum Resources (DPR) of the Ministry of Petroleum and Minerals Resources, which was used as the experimental data, for 57 years giving 57 data points. MatLab Package 7.9 version was subsequently used in the mathematical computations and curve-fitting analysis. The research has shown that the Nigerian oil reserve will finish in the year 2682AD and the gas will follow sooth in the year 3151AD. However, this result is accurate to an R^2 of 0.9955 - 0.9963 for oil and 0.9979 - 0.9983 for gas, as shown in model 5. These findings can be useful in governmental planning, economy diversification, and OPEC bargaining and positioning.

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APPENDIX : Dumb-bell Data for Models 4, 5, 6 and 7.

Year	**DQ_{po4}**	**DQ_{do4}**	**DQ_{pg4}**	**DQ_{dg4}**	**DQ_{po5}**	**DQ_{do5}**	**DQ_{pg5}**	**DQ_{dg5}**
0	10.5804	0.9830	0.1356	6.4968	-2.333	-349.0060	-11.6674	-1.1404 e^{+3}
5.7	204.5960	3.3782e^{+3}	205.7230	2.0209 e^{+4}	96.0005	959.7870	37.6476	7.7540 e^{+3}
11.4	363.6080	6.7529 e^{+4}	411.1580	4.0391 e^{+4}	288.3760	4.0851 e^{+3}	171.3880	2.8892 e^{+4}
17.1	491.6080	1.0124 e^{+4}	616.4410	6.0553 e^{+4}	481.9600	8.1985 e^{+3}	372.4530	5.4367 e^{+4}
22.8	593.1830	1.3495 e^{+4}	821.5680	8.0694 e^{+4}	635.4210	1.2708 e^{+4}	625.9920	8.1425 e^{+4}
28.5	672.0940	1.6862 e^{+4}	1.0265 e^{+3}	100814	735.9590	17201	919.1620	106872
34.2	731.6300	2.0227 e^{+4}	1.2314 e^{+3}	120913	785.4330	2.1404 e^{+4}	1.2409 e^{+3}	129128
39.9	774.6840	2.3589e^{+4}	1.4360 e^{+3}	140993	792.2420	2.5146 e^{+4}	1.5818 e^{+3}	147393
45.6	803.7600	2.6949 e^{+4}	1.6405 e^{+3}	161051	766.7920	2.8331 e^{+4}	1.9336 e^{+3}	161393
51.5	821.0570	3.0306 e^{+4}	1.8449 e^{+3}	181089	719.1290	3.0919 e^{+4}	2.2897 e^{+3}	171226
57	828.4830	3.3661 e^{+4}	2.0491 e^{+3}	201107	657.8680	3.2907 e^{+4}	2.6443 e^{+3}	177177

Year	**DQ_{po6}**	**DQ_{do6}**	**DQ_{pg6}**	**DQ_{dg6}**	**DQ_{po7}**	**DQ_{do7}**	**DQ_{pg7}**	**DQ_{dg7}**
0	-34.1950	-1.3000 e^{+3}	-60.7660	-5.5687 e^{+3}	0	0	0	0
5.7	27.5630	-363.1960	-26.2570	-79.0480	1.88	186	1.61	2262
11.4	220.7560	2.5880 e^{+3}	95.8730	19293	5.98	632	6.55	5217
17.1	468.3080	7.3915 e^{+3}	322.0700	4.9732 e^{+4}	13.35	1791	57.50	8538
22.8	677.2740	1.3020 e^{+4}	628.1750	8.3767 e^{+4}	29.15	3455	68.44	13098
28.5	802.2420	1.8449 e^{+4}	976.4260	114644	53.77	5233	85.62	17496
34.2	839.1940	2.2965 e^{+4}	1.3296 e^{+3}	138230	81.68	6692	107.73	22340
39.9	806.1330	2.6197 e^{+4}	1.6576 e^{+3}	152912	125.68	8947	144.06	28746
45.6	727.6900	2.8056 e^{+4}	1.9592 e^{+3}	-152912	225.04	11336	223.50	39756
51.5	626.4650	2.8642 e^{+4}	2.1621 e^{+3}	157421	377.47	14119	326.11	52888
57	519.5450	2.8162 e^{+4}	2.3213 e^{+3}	150231	494.03	17493	419.20	66941