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Understanding the Dynamics of Corruption Using Mathematical Modeling Approach

Felix Yakubu Eguda[1], Festus Oguntolu[2], Timothy Ashezua[3]

[1]. Department of Mathematics, Federal University, Dutse, Jigawa State.
[2]. Department of Mathematics and Statistics, Federal University of Technology, Minna, Niger State.
[3]. Department of Mathematics/Statistics/Computer Science, Federal University of Agriculture, Makurdi, Benue State.

Email: *. felyak_e@yahoo.co.uk

ABSTRACT

This paper develops and analyses a mathematical model for corruption dynamics. The basic reproduction number is obtained to monitor the dynamics of corruption and ascertain its level in order to suggest effective intervention strategies to reduce the spread of corruption in a population. The corruption free equilibrium was derived and it was proved that the corruption endemic equilibrium state exists if the basic reproduction number is greater than one. The results show that the disease free equilibrium is locally asymptotically stable if $R_0 < 1$. Numerical simulations of the model is presented to investigate the long term behaviour of the steady state solution of the corruption model in a population.
Keywords: *Corruption, basic reproduction number, stability, equilibrium*

Introduction:

Corruption is generally defined as the misuse of public authority and has been a subject of substantial amount of empirical research over the past three decades. In the words of transparency international, "Corruption is one of the greatest challenges of the contemporary world. It undermines good government, fundamentally distorts public policy, leads to the misallocation of resources, harms the private sector and particularly hurts the poor [1].

Corruption seriously harms the economy and society as a whole. Many countries around the world suffer from deep-rooted corruption that hampers economic development, undermines democracy, damages social justice and the rule of law. Corruption varies in nature and extent from one country to another, but it affects all member states. It is a complex phenomenon with economic, social, political and cultural dimensions which cannot easily be eliminated [2].

Corruption has contributed immensely to the poverty and misery of a large segment of the population. It is the misappropriation of government property or revenues made possible through government regulations. It is clearly an absence of accountability, law and order. Forms of corruption include – bribery, fraud, embezzlement, extortion, nepotism, favoritism. Some fundamental factors that engender corrupt practices in less developed nations are (i) the great inequality in the distribution of wealth (ii) political office as the primary means of gaining access to wealth (iii) conflict between changing moral codes (iv) the weakness of social and governmental enforcement mechanism (v) absence of a strong sense of national community. However, obsession with materialism, compulsion for a short-cut to affluence, glorification and approbation (of ill-gotten wealth) by the general public are among the reasons for the persistence of corruption in Nigeria [3]. Numerous authors have over the past two decades used mathematical models to evaluate the effects of corruption on national development [5].

2 Model Development

In order to formulate the model, the total population is categorized into five compartments namely Susceptible $(S(t))$, Immune $(V(t))$, Corrupt $(C(t))$, Prosecuted $(P(t))$ and Removed $(R(t))$ with the assumptions that;

(a) Susceptible individuals are those who have never been involved in any corrupt practices that will have harmful effects on a country's economic growth

(b) Immune individuals are individuals that can never involve themselves in corrupt practices irrespective of the circumstances around them

(c) Corrupt Individuals are individuals who are often involved in corrupt practices and are capable of influencing a susceptible individual to become corrupt

(d) Prosecuted individuals are those who have been convicted of corrupt practices and imprisoned for a specific period of time during which he cannot be involved in any corrupt acts and cannot influence others

(e) Removed Individuals are ex-convicts who have been reformed while serving their jail term and can still be influenced by corrupt individuals

The immune class is generated from daily recruitment of individuals born into homes with good moral standards and can never become corrupt at a rate $(1-\theta)\beta$ while those prone to become corrupt progress to susceptible class at a rate $\theta\beta$. Susceptible individuals acquire infection from corrupt individuals and become corrupt at a rate α while corrupt individuals are prosecuted at a rate δ. Corrupt and prosecuted individuals become reformed while serving their jail term at rates γ and ρ respectively. Individuals in the removed class become susceptible after a while at a rate ω while susceptible individuals prone to corruption become immune due to moral and religious beliefs as well as public enlightenment campaign. All classes are subjected to natural death at a rate μ

The assumptions result in the system of nonlinear ordinary differential equations;

$$\frac{dS}{dt} = \theta\beta - \frac{\alpha SC}{N} - (\upsilon + \mu)S + \omega R + \phi P$$

$$\frac{dV}{dt} = (1-\theta)\beta + \upsilon S - \mu V$$

$$\frac{dC}{dt} = \frac{\alpha SC}{N} - (\delta + \gamma + \mu)C \tag{1}$$

$$\frac{dP}{dt} = \delta C - (\rho + \phi + \mu)P$$

$$\frac{dR}{dt} = \gamma C + \rho P - (\omega + \mu)R$$

Let,

$$k_1 = \upsilon + \mu,\ k_2 = \delta + \gamma + \mu,\ k_3 = \rho + \phi + \mu,\ k_4 = \omega + \mu \tag{2}$$

$$\frac{dS}{dt} = \theta\beta - \frac{\alpha SC}{N} - k_1 S + \omega R + \phi P$$

$$\frac{dV}{dt} = (1-\theta)\beta + \upsilon S - \mu V$$

$$\frac{dC}{dt} = \frac{\alpha SC}{N} - k_2 C \tag{3}$$

$$\frac{dP}{dt} = \delta C - k_3 P$$

$$\frac{dR}{dt} = \gamma C + \rho P - k_4 R$$

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Table 1: Description of parameters of the model 1 (yr^{-1})

Parameter	Description	Values	References
θ	Proportion of individuals not born immune	0.00403	Estimated
β	Birth rate	0.042	Estimated
α	Effective corruption contact rate	(0,1)	Estimated
v	Rate at which susceptible individuals become immune to corruption	0.0004	Estimated
ω	Rate at which honest individuals become susceptible	0.0021	Estimated
ϕ	Rate at which prosecuted individuals are discharged from prison into the society	0.143	Abdulrahman (2014)
δ	Rate of prosecution and imprisonment of corrupt individuals	0.0001	Abdulrahman (2014)
γ	Rate at which corrupt individuals become honest due to public enlightenment	0.000001	Abdulrahman (2014)
ρ	Rate at which prosecuted individuals become honest	0.000001	Abdulrahman (2014)
μ	Natural death rate	0.0189	Estimated

3 Analysis of the Model

3.1 Feasible Region

Consider the region

$\Omega = \left\{ (S, V, C, P, R) \in \Re^5 : N \le \frac{\beta}{\mu} \right\}$. The dynamics of the system (1) can be studied in Ω and it can be shown that Ω is positively invariant and an attractor of the feasible solution set of the system (1). The rate of change of the total human population is given as $\frac{dN}{dt} = \beta - \mu N$ which gives

$$\frac{dN}{dt} + \mu N = \beta .$$

Applying the integrating factor method,

$N = \frac{\beta}{\mu} + ce^{-\mu t}$ at $t = 0,\ N(0) = N_0$,

$c = N_0 - \frac{\beta}{\mu}$ so that

$N(t) = \frac{\beta}{\mu} + \left(N_0 - \frac{\beta}{\mu} \right) e^{-\mu t}$. Using the standard comparison theorem, it can be shown that

$N(t) \le \frac{\beta}{\mu}$ if $N(0) \le \frac{\beta}{\mu}$ so that Ω is a positively invariant set. Thus, all solutions enter Ω and remain non-negative for non-negative initial conditions. In this region, the model is well posed mathematically and epidemiologically.

3.2 Corruption Free Equilibrium

At the corruption-free equilibrium, the population is free of corruption so that only the susceptible and immune individuals exist in the population. Setting the right-hand side of (3) to zero, the corruption-free equilibrium is obtained and is given by;

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$$\begin{pmatrix} S^0 \\ V^0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \dfrac{\theta\beta}{k_1} \\ \dfrac{[(1-\theta)\beta k_1 + \upsilon\theta\beta]}{\mu} \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad (4)$$

3.3 Basic Reproduction Number

The basic reproduction number is a useful non-dimensional quantity in epidemiology as it sets the threshold in the study of a disease both for predicting its outbreak and for evaluating its control strategies. Using the next generation matrix method as described in [4], the matrices F_i and V_i for the new infection terms and the remaining transfer terms are given and the basic reproduction number is obtained as follows;

$$F_i = \begin{pmatrix} \dfrac{\alpha SC}{N} \\ 0 \end{pmatrix} \qquad V_i = \begin{pmatrix} k_2 C \\ -\delta C + k_3 P \end{pmatrix} \qquad (5)$$

$$F = \begin{pmatrix} \dfrac{\alpha S}{N} & 0 \\ 0 & 0 \end{pmatrix} \qquad V = \begin{pmatrix} k_2 & 0 \\ -\delta_2 & k_3 \end{pmatrix} \qquad (6)$$

$$FV^{-1} = \begin{pmatrix} \dfrac{\alpha S}{Nk_2} & 0 \\ 0 & 0 \end{pmatrix} \qquad (7)$$

The characteristic equation $\left|FV^{-1} - \lambda I\right| = 0$ becomes

$$\begin{vmatrix} \dfrac{\alpha S}{Nk_2} - \lambda & 0 \\ 0 & -\lambda \end{vmatrix} = 0 \qquad (8)$$

The spectral radius given by $\rho(FV^{-1})$ is

$$R_0 = \frac{\alpha S^0}{Nk_2} = \frac{\alpha\theta\beta}{(\upsilon+\mu)(\delta+\gamma+\mu)} \qquad (9)$$

3.4 Existence of Local Stability of Corruption Free Equilibrium

Lemma 1: The corruption free equilibrium E^0 of the model is locally asymptotically stable if $R_0 < 1$

Proof: The Jacobian stability technique which requires that all eigen values be negative for stability to hold will be used to prove the existence of local stability of corruption-free equilibrium. Linearization of the model equations at equilibrium point gives the Jacobian matrix

$$J(E^0) = \begin{pmatrix} -k_1 & 0 & \dfrac{-\alpha S^0}{N^0} & \phi & \omega \\ \upsilon & -\mu & 0 & 0 & 0 \\ 0 & 0 & \dfrac{\alpha S^0}{N^0} - k_2 & 0 & 0 \\ 0 & 0 & \delta & -k_3 & 0 \\ 0 & 0 & \gamma & \rho & -k_4 \end{pmatrix} \qquad (10)$$

Thus the characteristic equation is given by

$$\begin{vmatrix} -k_1-\lambda & 0 & \frac{-\alpha S^0}{N^0} & \phi & \omega \\ \upsilon & -\mu-\lambda & 0 & 0 & 0 \\ 0 & 0 & \frac{\alpha S^0}{N^0}-k_2-\lambda & 0 & 0 \\ 0 & 0 & \delta & -k_3-\lambda & 0 \\ 0 & 0 & \gamma & \rho & -k_4-\lambda \end{vmatrix} = 0 \qquad (11)$$

Which gives

$$(-k_1-\lambda)(-\mu-\lambda)(-k_3-\lambda)(-k_4-\lambda)\left(\frac{\alpha S^0}{N^0}-k_2-\lambda\right)=0 \qquad (12)$$

Clearly, the eigenvalues are

$$\lambda_1 = -k_1$$
$$\lambda_2 = -\mu$$
$$\lambda_3 = -k_3$$
$$\lambda_4 = -k_4$$
$$\lambda_5 = \frac{\alpha S^0}{N^0} - k_2 < 0 \qquad (13)$$

If $\frac{\alpha S^0}{N^0} < k_2$

i.e. $\frac{\alpha S^0}{k_2 N^0} < 1$

$R_0 < 1$

All the eigen values are negative if $R_0 < 1$

Therefore, the corruption free endemic equilibrium is locally asymptotically stable if $R_0 < 1$

3.5 Existence of Corruption Endemic Equilibrium

The corruption endemic equilibrium is the steady state solution of the corruption transmission model where corruption persists in the population and all subpopulations are positive.

The corruption endemic equilibrium exist at the point

$$\begin{pmatrix} S \\ V \\ C \\ P \\ R \end{pmatrix} = \begin{pmatrix} \frac{Nk_2}{\alpha} \\ \frac{(\alpha(1-\theta)\beta+\upsilon Nk_2)}{\alpha\mu} \\ \frac{Nk_1k_2k_3k_4(R_0-1)}{\alpha((k_2k_3-\phi\delta)k_4-(\gamma k_3+\rho\delta)\omega)} \\ \frac{N\delta k_1k_2k_4(R_0-1)}{\alpha((k_2k_3-\phi\delta)k_4-(\gamma k_3+\rho\delta)\omega)} \\ \frac{Nk_1k_2(\gamma k_3+\rho\delta)(R_0-1)}{\alpha((k_2k_3-\phi\delta)k_4-(\gamma k_3+\rho\delta)\omega)} \end{pmatrix} \qquad (14)$$

Where $((k_2k_3-\phi\delta)k_4-(\gamma k_3+\rho\delta)\omega)>0$

3.6 Local Stability of Corruption Endemic Equilibrium

The Jacobian stability technique will be used to prove the local stability of the corruption endemic equilibrium. Linearization of the model equations at equilibrium point gives the Jacobian matrix;

$$J(E^0)=\begin{pmatrix} -\left(\frac{\alpha C}{N}+k_1\right) & 0 & \frac{-\alpha S}{N} & \phi & \omega \\ \upsilon & -\mu & 0 & 0 & 0 \\ \frac{\alpha C}{N} & 0 & \frac{\alpha S}{N}-k_2 & 0 & 0 \\ 0 & 0 & \delta & -k_3 & 0 \\ 0 & 0 & \gamma & \rho & -k_4 \end{pmatrix} \quad (15)$$

The characteristic equation of the upper triangular matrix is

$$\begin{vmatrix} -\left(\frac{\alpha C}{N}+k_1\right)-\lambda & 0 & \frac{-\alpha S}{N} & \phi & \omega \\ 0 & -\mu\left(\frac{\alpha C}{N}+k_1\right)-\lambda & \frac{-\upsilon\alpha S}{N} & \upsilon\phi & \upsilon\omega \\ 0 & 0 & A-\lambda & \frac{\alpha C\phi}{N} & \frac{\alpha C\omega}{N} \\ 0 & 0 & 0 & E-\lambda & \frac{\delta\alpha C\omega}{N} \\ 0 & 0 & 0 & 0 & \frac{\delta\alpha C\omega D}{N}+\delta k_4 E-\lambda \end{vmatrix}=0 \quad (16)$$

Where

$$A=\left(\frac{\alpha C}{N}+k_1\right)\left(\frac{\alpha S}{N}-k_2\right)-\frac{\alpha^2 SC}{N^2} \quad (17)$$

$$D=(\delta\rho+\gamma k_3)$$

$$E=\left(\frac{\delta\alpha C\phi}{N}+Ak_3\right)$$

Therefore, the eigenvalues are

$$\lambda_1=-\left(\frac{\alpha C}{N}+k_1\right)<0 \quad \text{since } C>0$$

$$\lambda_2=-\mu\left(\frac{\alpha C}{N}+k_1\right)<0$$

$$\lambda_3=A=\left(\frac{\alpha C}{N}+k_1\right)\left(\frac{\alpha S}{N}-k_2\right)-\frac{\alpha^2 SC}{N^2}=\frac{-\alpha Ck_2}{N}<0$$

$$\lambda_4=E=\left(\frac{\delta\alpha C\phi}{N}+Ak_3\right)=\frac{\delta\alpha C\phi}{N}\frac{-\alpha Ck_2k_3}{N}$$

$$=\frac{\alpha C}{N}(\delta\phi-k_2k_3)=\frac{\alpha C}{N}\big(\delta\phi-(\delta+\gamma+\mu)(\rho+\phi+\mu)\big)<0 \quad (18)$$

$$\lambda_5 = \frac{\delta\alpha C\omega D}{N} + \delta k_4 E = \frac{\delta\alpha C\omega}{N}(\delta\rho + \gamma k_3) + \delta k_4\left(\frac{\alpha C}{N}(\delta\phi - k_2 k_3)\right)$$

$$= -\left(\begin{array}{l} \frac{\delta^2\alpha C\omega\mu}{N} + \frac{\delta\alpha C\omega\rho\mu}{N} + \frac{\delta\alpha C\omega\mu\phi}{N} + \frac{\delta\alpha C\omega\mu^2}{N} + \frac{\delta^2\alpha C\mu\rho}{N} + \frac{\delta^2\alpha C\mu^2}{N} \\ + \frac{\delta\alpha C\mu\gamma\rho}{N} + \frac{\delta\alpha C\mu\gamma\phi}{N} + \frac{\delta\alpha C\mu\gamma\mu}{N} + \frac{\delta\alpha C\mu^2\rho}{N} + \frac{\delta\alpha C\mu^2\phi}{N} + \frac{\delta\alpha C\mu^3}{N} \end{array}\right) < 0$$

Since $C = \dfrac{Nk_1k_2k_3k_4(R_0 - 1)}{\alpha((k_2k_3 - \phi\delta)k_4 - (\gamma k_3 + \rho\delta)\omega)} > 0$

If $R_0 > 1$, we conclude that the corruption endemic equilibrium state is locally asymptotically stable if $R_0 > 1$

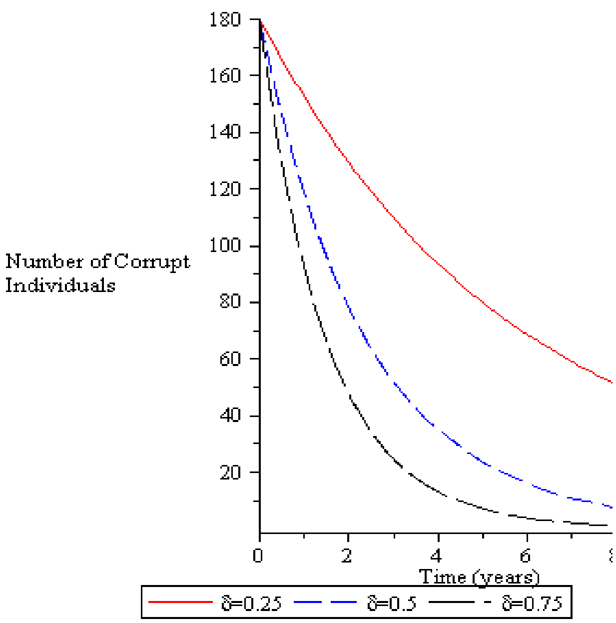

Figure 1: Total number of corrupt individuals with different corruption prosecution rate. The figure 1 shows that the higher the prosecution rate, the lower the population of corrupt individuals in the society

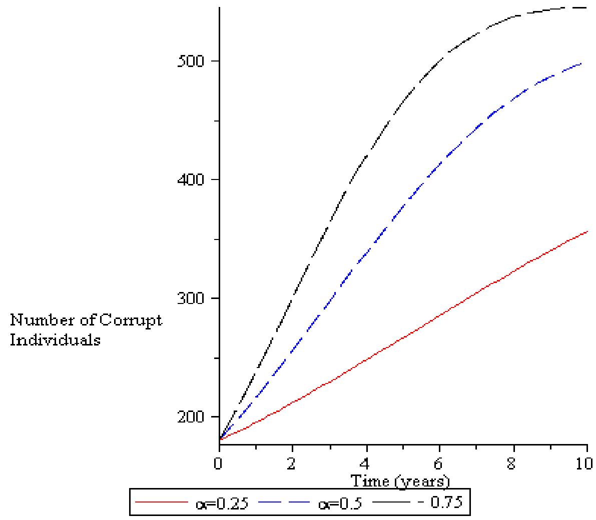

Figure 2: Total number of corrupt individuals with different corruption contact rate. The figure 2 shows that the higher the effective corruption contact rate, the higher the number of corrupt individuals.

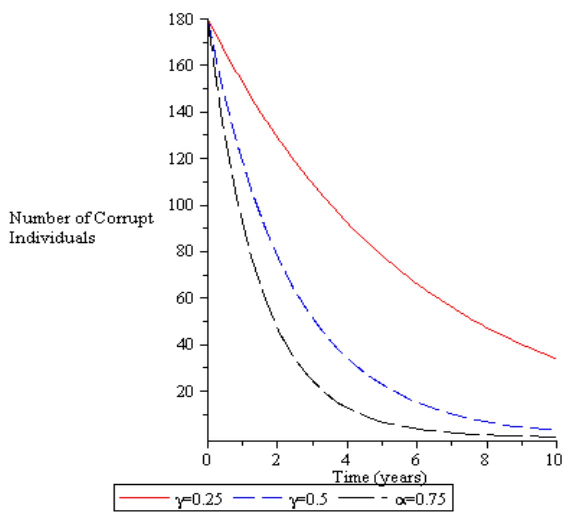

Figure 3: Total number of corrupt individuals with different public enlightenment rate. The figure 3

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shows that the higher the rate of public enlightenment against corruption, the lower the number of corrupt individuals.

4. Conclusion: In this paper, a mathematical model with standard incidence that investigates the dynamics of corruption as a disease was formulated and analysed. Local stability analysis of the corruption-free and corruption endemic equilibrium were conducted. We obtained the basic reproduction number using the next generation operator method. The analysis revealed that the corruption free equilibrium is locally asymptotically stable for $R_0 < 1$ while the corruption endemic equilibrium exists and is locally asymptotically stable for $R_0 > 1$. Numerical simulations are presented to explain the role of parameters in the long run path of the model .

Interestingly, the negative impact of corruption on economic growth is largely identified in public procurement as it affects life in many different ways. Therefore, renewed attention on the part of policy makers and government bodies should be targeted towards increasing minimum wage and the amount of tax revenues used to monitor corruption.

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