

Evolution of Stars by Kinetic Theory and Quantum Physics on the Basis Generalized Special Relativity

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Abstract:

The conditions of star equilibrium are discussed on the basis of the relation between pressure and gravity forces. The pressure expression was found first by using Gibbs and quantum laws. This leads to an equilibrium radius that depends on particle and mass density. The star explode on requires the energy to positive in this case thermal energy exceeds gravity potential. When the generalized special relativistic energy is negative contraction takes place when gravity energy exceeds the thermal one. Star equilibrium requires the radius to have critical value typical to that of black hole and the critical mass to be less than a certain critical temperature dependent mass.

Keywords: star, equilibrium, critical radius, critical mass, generalized special relativity.

Introduction:

The theory of general relativity is a model of gravity, the prediction of which leads to a remarkable change in the concepts of nature. it is now understood that in spite of the successes of this theory, it suffers from main defects reflected in its singular behavior at strong field limit .they are flawed in two ways: firstly, abnormal behavior in the strong field makes it conducive to the demolition of the same law that the adoption of it, where predict a so called gravitational collapse, and the attendant emergence of black holes. Secondly, which causes its being isolated from other physical laws, and forbid its being neither quantization nor unified with the rest of physics. We employed Gibbs's distribution relation and quantum laws to deal with the translational degrees of freedom of the constituent particles, and quantum mechanics to deal with the non-translational degrees of freedom. It turns out that this approach is necessary to deal with either low temperature or high density gases. At stellar densities which greatly exceed white dwarf densities; the extreme pressures cause electrons to combine with protons to form neutrons. Thus, any star which collapses to such an extent that its radius becomes significantly less than that characteristic of a white dwarf is effectively

transformed into a gas of neutrons. Eventually, the mean separation between the neutrons becomes comparable with their wavelength. At this point, it is possible for the degeneracy pressure of the neutrons to halt the collapse of the star [1, 2, 3]. A star which is maintained against gravity in this manner is called a neutron star. It is found that there is a critical mass and critical radius equivalent to the radius of Schwarzschild. Above which a neutron star cannot be maintained against gravity. And also cannot be maintained against gravity by degeneracy pressure, and must ultimately collapse to form a black hole [4, 5, 6]. One will discuss in this work the evolution of stars by kinetic theory and quantum physics on the basis of generalized special relativity in section two. Sections three and four are concerned with discussion and conclusion respectively.

Differential Equations for Stellar Structure:

Let us now discuss ideal gases from a purely quantum mechanical standpoint. It turns out that this approach is necessary to deal with either low temperature or high density gases. Furthermore, it also allows us to investigate completely non-classical “gases”, such as photons. From the kinetic theory and quantum physics; we can get an equation of star evolution by the pressure force and the force of gravity. For stars one has two forces, pressure force which counter balances the gravity force, thus

$$P = \frac{1}{3}nmv^2, \quad mv^2 = 3KT \quad (1)$$

The number density can be assumed to satisfy Maxwell’s distribution

$$n = n_0 e^{-\beta E} \quad (2)$$

We first consider an ideal gas consisting of a single type of non-relativistic particles. The ideal-gas law for the gas contained in a volume V is commonly written as

$$P = \frac{1}{3} \frac{N}{V} (3KT) = nKT \quad (3)$$

Where: $n = N/V$ (is the number of particles per unit volume).

Thus the pressure force is given by

$$F_p = PA = nKT(4\pi r^2) = 4\pi nKT r^2 = c_1 r^2 \quad (4)$$

The gravity force is given by

$$F_g = \int \left(\frac{4\pi}{3} \rho r^3 \right) (4\pi r^2 \rho) dr$$

For constant density

$$F_g = \frac{(4\pi)^2}{3} \rho^2 \int_0^r r^5 dr = \frac{1}{6} \frac{(4\pi)^2}{3} \rho^2 r^6$$

Thus

$$F_g = \frac{8}{9} \pi^2 \rho^2 r^6 = c_2 r^6 \quad (5)$$

Equation of hydrostatic equilibrium requires

$$F_p = F_g \quad (6)$$

Thus from equation (4), (5) and (6) one gets

$$c_1 r^2 = c_2 r^6 \quad \Rightarrow \quad r = \left(\frac{c_1}{c_2}\right)^{1/4}$$

The critical radius is thus given by

$$r_c = \left(\frac{9nKT}{2\pi\rho^2}\right)^{1/4} \quad (7)$$

Expansion takes place

$$F_p > F_g \quad (8)$$

While contraction is observed when

$$F_p < F_g \quad (9)$$

But according to the laws of quantum mechanics for particle in box the energy is given by

$$E = c_0 V^{-2/3} \quad (10)$$

At $T = 0$ all quantum states whose energy is less than the Fermi energy E_F are filled. The Fermi energy corresponds to a Fermi momentum $p_F = \hbar k_F$ is thus given by

$$E_F = \frac{p_F^2}{2m} = \frac{\hbar^2 k_F^2}{2m} \quad (11)$$

The above expression can be rearranged to give

$$k_F = (3\pi^2 n)^{1/3} = \frac{\Lambda}{\hbar} \left(\frac{N}{V}\right)^{1/3}$$

Where

$$\Lambda = (3\pi^2)^{1/3} \hbar$$

Hence

$$\lambda_F = \frac{2\pi}{k_F} = \frac{2\pi}{(3\pi^2 n)^{1/3}} = \frac{2\pi\hbar}{\Lambda} \left(\frac{V}{N}\right)^{1/3}$$

Which implies that the De-Broglie wavelength λ_F corresponding to the Fermi energy is of order the mean separation between particles $(V/N)^{1/3}$. All quantum states with De-Broglie wavelengths $\lambda > \lambda_F$ are occupied at $T = 0$, whereas all those with $\lambda < \lambda_F$ are empty.

According to equation (11), the Fermi energy at $T = 0$ takes the form

$$E_F = \frac{\hbar^2}{2m} (3\pi^2 n)^{2/3} = \frac{\Lambda^2}{2m} \left(\frac{N}{V}\right)^{2/3} = c_0 V^{-2/3} \quad (12)$$

$$\frac{dE_F}{dV} = -\frac{2}{3} c_0 V^{-5/3} \quad (13)$$

But for spherical body

$$V = \frac{4\pi}{3} r^3$$

Thus

$$\frac{dE_F}{dV} = -\frac{2}{3} c_0 \left(\frac{4\pi}{3} r^3\right)^{-5/3} = -\frac{2}{3} \left(\frac{4\pi}{3}\right)^{-5/3} c_0 r^{-5} \quad (14)$$

But according to canonical Gibbs' distribution

$$P = n \frac{dE_F}{dV} \quad (15)$$

Hence the pressure takes the form

$$P = -\frac{2}{3} \left(\frac{4\pi}{3}\right)^{-5/3} n c_0 r^{-5} = c_1 r^{-5} \quad (16)$$

Thus the pressure force is given by

$$\begin{aligned} F_P &= P(4\pi r^2) \\ F_P &= -\frac{2}{3} \left(\frac{4\pi}{3}\right)^{-5/3} n c_0 r^{-5} (4\pi r^2) = 4\pi c_1 r^{-3} \\ F_P &= c_2 r^{-3} \quad (17) \end{aligned}$$

But gravity force is given by

$$F_g = \frac{GmM}{r^2}$$

Where we assume that the density is constant within the star. The mass at distance r from the star center is

$$\begin{aligned} M(r) &= \frac{4\pi}{3} \rho r^3 \\ F_g &= \frac{4\pi r^3 \rho Gm}{3r^2} \end{aligned}$$

Thus

$$F_g = \frac{4}{3} \pi \rho Gm r = c_3 r \quad (18)$$

Equation of hydrostatic equilibrium requires

$$F_P = F_g \quad (19)$$

i.e.

$$c_2 r^{-3} = c_3 r$$

The critical radius r_c is thus given by

$$\begin{aligned} r_c^4 &= \frac{c_2}{c_3} \\ r_c &= \left(\frac{c_2}{c_3}\right)^{1/4} = \left(\frac{-(4\pi)^{-5/3} n c_0 (4\pi)}{2\pi \rho m G (3)^{-5/3}}\right)^{1/4} \quad (20) \end{aligned}$$

Expansion takes place

$$F_P > F_g \quad (21)$$

While contraction happens when

$$F_P < F_g \quad (22)$$

The conditions of star evolution can be started by adopting classical limit, of generalized special relativity (GSR) energy relation where

$$E = m_0 c^2 \left(1 + \frac{2\phi}{c^2}\right) \left(1 + \frac{2\phi}{c^2} - \frac{v^2}{c^2}\right)^{-1/2} \quad (23)$$

Considering Newtonian potential and thermal motion

$$\phi = -\frac{GM}{R}, \quad \frac{1}{2}mv^2 = \frac{3}{2}KT \Rightarrow v^2 = \frac{3KT}{m_0} \quad (24)$$

$$E = mc^2 = m_0c^2 \left(1 - \frac{2GM}{Rc^2}\right) \left(1 - \frac{2GM}{Rc^2} - \frac{3KT}{m_0c^2}\right)^{-1/2} \quad (25)$$

If the gravitational potential and thermal energy are everywhere small, so

$$\frac{2GM}{Rc^2} \ll 1, \quad \frac{3KT}{m_0c^2} \ll 1 \quad (26)$$

Thus (25) reduces to

$$E = m_0c^2 \left(1 - \frac{2GM}{Rc^2}\right) \left(1 + \frac{GM}{Rc^2} + \frac{3KT}{2m_0c^2}\right) \quad (27)$$

Neglecting higher order terms, yields

$$E = m_0c^2 \left(1 + \frac{GM}{Rc^2} + \frac{3KT}{2m_0c^2} - \frac{2GM}{Rc^2} - \frac{2G^2M^2}{R^2c^4} - \frac{3GMKT}{Rm_0c^4}\right)$$

Thus the energy E become

$$E = m_0c^2 + \frac{3}{2}KT - \frac{GMm_0}{R} \quad (28)$$

Assuming the kinetic energy is due to thermal motion

$$K.E = \frac{3}{2}KT \quad (29)$$

Assuming also the potential energy of mass m_0 to be

$$V = -\frac{GMm_0}{R} \quad (30)$$

Thus equation (28) gives

$$E = m_0c^2 + K.E + V$$

Thus the expression of energy includes the total kinetic energy of the degenerate electrons (the kinetic energy of the ion is negligible), the rest energy m_0c^2 and the gravitational potential energy V . Let us assume, for the sake of simplicity, that the density of the star is its uniform. The total energy of a star is its gravitational potential energy, its internal energy and its kinetic energy (due to bulk motions of gas inside the star, not the thermal motions of the gas particles).

Using the hypothesis of universe expansion, the star explodes and expands when the energy E is positive

$$E = m_0c^2 + \frac{3}{2}KT - \frac{Gm_0M}{R} > 0 \quad (31)$$

i.e.

$$m_0c^2 + \frac{3}{2}KT > \frac{Gm_0M}{R} \quad (32)$$

This is quite obvious from the point of view of common sense because this equation indicates that expansion happen when thermal and rest mass energies exceeds attractive gravity energy. However it collapse and contract when the energy E is negative, this requires

$$m_0c^2 + \frac{3}{2}KT < \frac{Gm_0M}{R} \quad (33)$$

Thus collapse takes place when gravity energy exceeds thermal one.

Can be obtained the critical radius, using the following energy for generalized special relativity

$$E = m_0c^2 \left(1 - \frac{2GM}{rc^2}\right) \left(1 - \frac{2GM}{rc^2} - \frac{3KT}{m_0c^2}\right)^{-1/2}$$

$$E = m_0c^2 \left(1 + \frac{2c_1}{rc^2}\right) \left(1 + \frac{2c_1}{rc^2} - \frac{3KT}{m_0c^2}\right)^{-1/2}$$

Where

$$c_1 = -GM$$

$$E = m_0c^2(1 + c_2r^{-1}) \left(1 + c_2r^{-1} - \frac{3KT}{m_0c^2}\right)^{-1/2} \quad (34)$$

Where

$$c_2 = \frac{2c_1}{c^2}$$

The critical radius of the star requires minimizing the total energy E and can be found by using the conditions for minimum value, i.e.

$$\frac{dE}{dr} = \frac{-m_0c^2c_2r^{-2}}{\left(1 + c_2r^{-1} - \frac{3KT}{m_0c^2}\right)^{1/2}} + \frac{\frac{1}{2}m_0(1 + c_2r^{-1})(c^2c_2r^{-2})}{\left(1 + c_2r^{-1} - \frac{3KT}{m_0c^2}\right)^{3/2}}$$

$$\frac{dE}{dr} = \frac{-m_0c^2c_2r^{-2} \left(1 + c_2r^{-1} - \frac{3KT}{m_0c^2}\right) + \frac{1}{2}m_0(1 + c_2r^{-1})(c^2c_2r^{-2})}{\left(1 + c_2r^{-1} - \frac{3KT}{m_0c^2}\right)^{3/2}} = 0$$

$$-m_0c^2c_2r^{-2} \left(1 + c_2r^{-1} - \frac{3KT}{m_0c^2}\right) + \frac{1}{2}m_0(1 + c_2r^{-1})(c^2c_2r^{-2}) = 0$$

$$m_0(1 + c_2r^{-1})(c^2c_2r^{-2}) = 2m_0c^2c_2r^{-2} \left(1 + c_2r^{-1} - \frac{3KT}{m_0c^2}\right)$$

$$1 + c_2r^{-1} = 2 + 2c_2r^{-1} - \frac{6KT}{m_0c^2}$$

$$2c_2r^{-1} - c_2r^{-1} = \frac{6KT}{m_0c^2} - 1$$

$$c_2r^{-1} = \frac{6KT - m_0c^2}{m_0c^2}$$

When temperature is neglected, i.e. when

$$T = 0$$

One gets

$$c_2r^{-1} = -1$$

$$r = -c_2 = -\frac{2c_1}{c^2} = \frac{2MG}{c^2}$$

The critical radius is thus given by

$$r_c = \frac{2GM}{c^2} \quad (35)$$

(This is the black hole radius)

Using the generalized special relativity energy relation

$$E = m_0 c^2 \left(1 - \frac{2GM}{rc^2}\right) \left(1 - \frac{2GM}{rc^2} - \frac{3KT}{m_0 c^2}\right)^{-1/2} \quad (36)$$

For star having spherical shape:

$$-GM = -G \left(\frac{4\pi}{3} \rho r^3\right) = -\frac{4\pi}{3} G \rho r^3 = c_3 r^3 \quad (37)$$

$$E = m_0 c^2 \left(1 + \frac{2c_3 r^2}{c^2}\right) \left(1 + \frac{2c_3 r^2}{c^2} - \frac{3KT}{m_0 c^2}\right)^{-1/2}$$

$$E = m_0 c^2 (1 + c_4 r^2) \left(1 + c_4 r^2 - \frac{3KT}{m_0 c^2}\right)^{-1/2} \quad (38)$$

Where

$$c_4 = \frac{2c_3}{c^2}$$

The radius of the star r that dimension which reduces the total energy E and his can be found by using the minimum energy condition that has to be less energy as soon as possible, i.e.

$$\frac{dE}{dr} = 0 \quad (39)$$

$$\frac{dE}{dr} = \frac{m_0 c^2 (2c_4 r)}{\left(1 + c_4 r^2 - \frac{3KT}{m_0 c^2}\right)^{1/2}} - \frac{m_0 c^2 (1 + c_4 r^2) \left(\frac{1}{2}\right) (2c_4 r)}{\left(1 + c_4 r^2 - \frac{3KT}{m_0 c^2}\right)^{3/2}} = 0$$

$$\frac{2m_0 c^2 c_4 r \left(1 + c_4 r^2 - \frac{3KT}{m_0 c^2}\right) - m_0 c^2 c_4 r (1 + c_4 r^2)}{\left(1 + c_4 r^2 - \frac{3KT}{m_0 c^2}\right)^{3/2}} = 0$$

$$2m_0 c^2 c_4 r \left(1 + c_4 r^2 - \frac{3KT}{m_0 c^2}\right) - m_0 c^2 c_4 r (1 + c_4 r^2) = 0$$

$$2m_0 c^2 c_4 r \left(1 + c_4 r^2 - \frac{3KT}{m_0 c^2}\right) = m_0 c^2 c_4 r (1 + c_4 r^2)$$

$$2 \left(1 + c_4 r^2 - \frac{3KT}{m_0 c^2}\right) = 1 + c_4 r^2$$

$$2 + 2c_4 r^2 - \frac{6KT}{m_0 c^2} = 1 + c_4 r^2$$

$$c_4 r^2 = \frac{6KT}{m_0 c^2} - 1$$

$$r^2 = \frac{6KT - m_0 c^2}{m_0 c^2 c_4} = \frac{6KT - m_0 c^2}{2m_0 c_3}$$

The minimum radius

$$r = \left(\frac{6KT - m_0c^2}{2m_0c_3} \right)^{1/2}$$

For r to be real

$$6KT > m_0c^2 \quad (40)$$

$$m_0 < \frac{6KT}{c^2}$$

Thus the critical mass is given by:

$$m_{0c} = \frac{6KT}{c^2}$$

Hence for equilibrium

$$m_0 < m_{0c}$$

Using equation (37)

$$c_3 = \frac{4\pi}{3} G\rho$$

The critical radius is thus given by

$$r_c = \left(\frac{6KT - m_0c^2}{\frac{8\pi}{3} m_0 G\rho} \right)^{1/2} \quad (41)$$

When

$$\frac{d^2E}{dr^2} = \frac{2m_0c^2c_4}{\left(1 + c_4r^2 - \frac{3KT}{m_0c^2}\right)^{1/2}} - \frac{2m_0c^2c_4^2r^2}{\left(1 + c_4r^2 - \frac{3KT}{m_0c^2}\right)^{3/2}} - \frac{m_0c^2c_4(1 + 3c_4r^2)}{\left(1 + c_4r^2 - \frac{3KT}{m_0c^2}\right)^{3/2}}$$

$$+ \frac{3m_0c^2c_4^2r^2(1 + c_4r^2)}{\left(1 + c_4r^2 - \frac{3KT}{m_0c^2}\right)^{5/2}}$$

$$\frac{d^2E}{dr^2} = \frac{2m_0c^2c_4\left(1 + c_4r^2 - \frac{3KT}{m_0c^2}\right)}{\left(1 + c_4r^2 - \frac{3KT}{m_0c^2}\right)^{3/2}} - \frac{[2m_0c^2c_4^2r^2 + m_0c^2c_4(1 + 3c_4r^2)]}{\left(1 + c_4r^2 - \frac{3KT}{m_0c^2}\right)^{3/2}}$$

$$+ \frac{3m_0c^2c_4^2r^2(1 + c_4r^2)\left(1 + c_4r^2 - \frac{3KT}{m_0c^2}\right)^{-1}}{\left(1 + c_4r^2 - \frac{3KT}{m_0c^2}\right)^{3/2}}$$

$$= \frac{m_0c^2c_4 - 6c_4KT - 3m_0c^2c_4^2r^2 + 3m_0c^2c_4^2r^2(1 + c_4r^2)\left(1 + c_4r^2 - \frac{3KT}{m_0c^2}\right)^{-1}}{\left(1 + c_4r^2 - \frac{3KT}{m_0c^2}\right)^{3/2}}$$

For maximum values

$$\frac{d^2E}{dr^2} < 0 \quad (42)$$

$$\frac{m_0 c^2 c_4 - 6c_4 KT - 3m_0 c^2 c_4^2 r^2 + 3m_0 c^2 c_4^2 r^2 (1 + c_4 r^2) \left(1 + c_4 r^2 - \frac{3KT}{m_0 c^2}\right)^{-1}}{\left(1 + c_4 r^2 - \frac{3KT}{m_0 c^2}\right)^{3/2}} < 0$$

$$\begin{aligned} & m_0 c^2 c_4 - 6c_4 KT - 3m_0 c^2 c_4^2 r^2 + 3m_0 c^2 c_4^2 r^2 (1 + c_4 r^2) \left(1 + c_4 r^2 - \frac{3KT}{m_0 c^2}\right)^{-1} < 0 \\ & \left[3m_0 c^2 c_4^2 r^2 (1 + c_4 r^2) \left(1 + c_4 r^2 - \frac{3KT}{m_0 c^2}\right)^{-1} \right] < [6c_4 KT + 3m_0 c^2 c_4^2 r^2 - m_0 c^2 c_4] \\ & \left[3m_0 c^2 c_4^2 r^2 (1 + c_4 r^2) \left(1 + c_4 r^2 - \frac{3KT}{m_0 c^2}\right)^{-1} \right] < \left[m_0 c^2 c_4 \left(\frac{6KT}{m_0 c^2} + 3c_4 r^2 - 1 \right) \right] \\ & \left[c_4 r^2 (1 + c_4 r^2) \left(1 + c_4 r^2 - \frac{3KT}{m_0 c^2}\right)^{-1} \right] < \left(\frac{2KT}{m_0 c^2} + c_4 r^2 - \frac{1}{3} \right) \\ & c_4 r^2 (1 + c_4 r^2) < \left(\frac{2KT}{m_0 c^2} + c_4 r^2 - \frac{1}{3} \right) \left(1 + c_4 r^2 - \frac{3KT}{m_0 c^2}\right) \end{aligned}$$

When temperature is neglected, i.e. when

$$T = 0$$

$$c_4 r^2 (1 + c_4 r^2) < \left(c_4 r^2 - \frac{1}{3} \right) (1 + c_4 r^2)$$

$$c_4 r^2 + c_4^2 r^4 < c_4 r^2 + c_4^2 r^4 - \frac{1}{3} - \frac{1}{3} c_4 r^2$$

$$\frac{1}{3} c_4 r^2 + \frac{1}{3} < 0$$

$$c_4 r^2 + 1 < 0$$

$$r^2 < -\frac{1}{c_4}$$

$$r < \left(\frac{1}{c_4} \right)^{1/2}$$

Where

$$c_4 = \frac{2c_3}{c^2}, \quad c_3 = \frac{4\pi G\rho}{3}$$

$$c_4 = \frac{8\pi G\rho}{3c^2} \quad (43)$$

$$r < c \left(\frac{3}{8\pi G\rho} \right)^{1/2}$$

While contraction takes place when

$$r < \sqrt{3} c (8\pi G\rho)^{-1/2} \quad (44)$$

For minimum values

$$\frac{d^2E}{dr^2} > 0 \quad (45)$$

Thus explosion is expected when

$$r > \sqrt{3} c(8\pi G\rho)^{-1/2} \quad (46)$$

Thus the critical radius is given by

$$r_c = \sqrt{3} c(8\pi G\rho)^{-1/2} \quad (47)$$

3. Discussion:

The equilibrium radius can be found by using ordinary expression for thermal pressure (see equation (3)) and the ordinary Newtonian force relation for star having constant density. Assuming pressure and attractive gravity force to balance each other, one can find temperature and density dependent critical radius r_c , where r_c increases as temp. Increase and decreases as density increase. This conforms to the fact that thermal pressure force causes contraction. The same result can be found by using quantum mechanical relation for pressure (see equations (11)-(17)), but here the increase of density increases r_c as shown by equation (20). Using generalized special relativity energy relation the condition of expansion requires the thermal energy to exceed gravity energy, while contraction requires gravity energy to be more than thermal energy which agrees with previous models. By assuming gravity Newtonian potential relation and thermal energy for generalized special relativity energy relation (see equation (34)). The minimum energy requires the existence of critical temperature depend Ent mass similar to Chandrasekhar mass. Where the star mass should be less than this mass to attain minimum energy.

4. Conclusion:

Gibbs's distribution relations, quantum laws beside generalized special relativity GSR energy relations are used to study star equilibrium conditions. It was shown that equilibrium conditions require certain critical value for the radius typical to that of the black hole. The critical mass is shown to depend on temperature. The equilibrium also requires rest and thermal energy to be equal to potential energy. The fact that the rest mass energy is joined with thermal energy comes from the fact that rest mass energy can be converted to thermal energy.

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