

Generation of Elementary Particles inside Black Holes at Planck Time

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Abstract

Using generalized special relativity together with Newton's laws of gravitation and treating particles as quantum strings a useful expression for self energy was found. The critical radius of a star when particles are created is that of a black hole. The critical radius and mass are dependent on the speed of light and gravitational constant. For mass formation the radius and mass should be small which agrees with the fact that elementary particles have very small mass and radius. The formation should also take place at Planck time which also conforms with that proposed by big bang model.

Keywords: critical radius, critical mass, generalized special relativity, black hole, Planck length, Planck time.

Introduction:

Our vision for the beginning of universe is on the basis of the fundamental forces unify at the beginning of time. Unification forces leads to the answer to the most important questions in the cosmology, how did the creation and how space and time begin. A proper unification of all interactions should include the fundamental constants, representing the four basic Interactions, gravitational, electromagnetic beside nuclear interactions. Appropriate combinations of the physical constant that determine interaction nature provide a proper description of the anticipated unified interaction. We have found that these fundamental constants describe completely our universe, at all stages. One will discuss in this work generation of elementary particles inside black holes in section two. Sections three and four are devoted for discussion and conclusion respectively.

Model Universe with A cosmology Constant:

Generalized special relativistic energy (GSR) expression, beside ordinary Newtonian gravity potential are given by

$$E = m_0 c^2 \left(1 + \frac{2\varphi}{c^2} \right) \left(1 + \frac{2\varphi}{c^2} - \frac{v^2}{c^2} \right)^{-1/2} \quad (1)$$

Where the Newtonian potential takes the form

$$\varphi = -\frac{MG}{R} \quad (2)$$

$$E = m_0 c^2 \left(1 - \frac{2MG}{Rc^2} \right) \left(1 - \frac{2MG}{Rc^2} - \frac{v^2}{c^2} \right)^{-1/2} \quad (3)$$

Minimizing E w.r.t M yields

$$\frac{dE}{dM} = m_0 c^2 \left[\frac{-\frac{2G}{Rc^2}}{\left(1 - \frac{2MG}{Rc^2} - \frac{v^2}{c^2} \right)^{1/2}} + \frac{\left(1 - \frac{2MG}{Rc^2} \right) \left(-\frac{1}{2} \right) \left(\frac{-2G}{Rc^2} \right)}{\left(1 - \frac{2MG}{Rc^2} - \frac{v^2}{c^2} \right)^{3/2}} \right] = 0$$

Thus

$$\frac{-\frac{2G}{Rc^2} \left(1 - \frac{2MG}{Rc^2} - \frac{v^2}{c^2} \right) + \frac{G}{Rc^2} \left(1 - \frac{2MG}{Rc^2} \right)}{\left(1 - \frac{2MG}{Rc^2} - \frac{v^2}{c^2} \right)^{3/2}} = 0$$

If one consider

$$\begin{aligned} & v^2 \ll c^2 \\ & -\frac{2G}{Rc^2} \left(1 - \frac{2MG}{Rc^2} - \frac{v^2}{c^2} \right) + \frac{G}{Rc^2} \left(1 - \frac{2MG}{Rc^2} \right) = 0 \\ & -\frac{G}{Rc^2} \left(1 - \frac{2MG}{Rc^2} \right) = 0 \end{aligned}$$

This requires

$$\begin{aligned} \frac{2MG}{Rc^2} &= 1 \\ 2MG &= Rc^2 \quad (4) \end{aligned}$$

Thus the mass which makes E minimum is

$$M = \frac{Rc^2}{2G} \quad (5)$$

Consider also the generalized special relativity energy E equilibrium condition by minimizing E with respect to radius r from equation (3), when the star particles speed are small compared to speed of light

$$\frac{v^2}{c^2} \ll 1$$

Thus

$$E = m_0 c^2 \left(1 - \frac{2MG}{rc^2} \right)^{1/2} \quad (6)$$

$$\frac{dE_r}{dr} = m_0 c^2 \left(\frac{2MG}{r^2 c^2} \right) \left(\frac{1}{2} \right) \left(1 - \frac{2MG}{rc^2} \right)^{-1/2}$$

$$\frac{dE_r}{dr} = \frac{m_0 c^2 \left(\frac{MG}{r^2 c^2} \right) \left(1 - \frac{2MG}{rc^2} \right)}{\left(1 - \frac{2MG}{rc^2} \right)^{3/2}} = 0$$

Thus the radius which makes E minimum is given by

$$1 - \frac{2MG}{rc^2} = 0$$

The critical radius is thus given by

$$r_c = \frac{2MG}{c^2} \tag{7}$$

(This is the black hole radius)

But the critical mass is given by equation (7), i.e.

$$M = m_c = \frac{c^2 r_c}{2G} \tag{8}$$

Hence from (8)

$$2m_c G = r_c c^2 \tag{9}$$

The condition governing the equilibrium of the universe, from (9) and (4) we get

$$\frac{m_c R}{Mr_c} = 1 \tag{10}$$

Where M and R are the mass and radius of the universe respectively. The mass of the universe ($M = 2.2 \times 10^{56} \text{g}$) and the radius ($R = 1.6 \times 10^{28} \text{cm}$).

According to generalized general relativity (GGR) there is a short range repulsive gravitational force beside long range attractive gravity force given by [1]:

$$\varphi_s = \frac{c_1}{r} e^{-\frac{r}{r_c}} \tag{11}$$

$$\varphi_L = -\frac{GM}{r} \tag{12}$$

$$\varphi = \varphi_s + \varphi_L = \frac{c_1}{r} e^{-\frac{r}{r_c}} - \frac{GM}{r}$$

$$\varphi = \frac{1}{r} \left[c_1 e^{-\frac{r}{r_c}} - GM \right] \tag{13}$$

For small radius r or strictly speaking small $\frac{r}{r_c}$:

$$e^{-\frac{r}{r_c}} = 1 - \frac{r}{r_c} \tag{14}$$

Hence

$$\varphi = \frac{1}{r} \left[c_1 - c_1 \frac{r}{r_c} - GM \right] \tag{15}$$

To secure finite self energy φ at small r , one requires

$$c_1 = GM \tag{16}$$

Thus the star self energy is given by

$$\varphi = -\frac{c_1}{r_c} = -\frac{GM}{r_c} \quad (17)$$

Since the star is a particle at rest thus the minimization of E requires (see equation (2), (4) and (17))

$$\varphi = -\frac{c_1}{r_c} = -\frac{c^2}{2} \quad (18)$$

For photon ($v = c$) thus one gets

$$\varphi = \frac{c^2}{2} \quad (19)$$

From equation (17) and (18)

$$\varphi = -\frac{GM}{r_c} = -\frac{c^2}{2} \quad (20)$$

Thus the critical radius is given by

$$r_c = \frac{2GM}{c^2} \quad (21)$$

(This is the black hole radius)

Since r_c should be small as shown by equation (14), thus requires

$$r_c < 1 \quad , \quad \frac{2GM}{c^2} < 1$$

$$M < \frac{c^2}{2G} \quad (22)$$

Thus there is a critical mass

$$M_c = \frac{c^2}{2G} \quad (23)$$

Above it the particle rest mass energy cannot be formed from potential.

We see from equation (4) that the present radius of the universe should be

$$R_0 = \frac{2GM_0}{c^2} \sim 10^{28} \text{ cm} \quad (24)$$

Which conforms to observations. Consider a star as consisting of photons gas, such that the critical radius is related to the wave number according to the relation

$$p = m_0c = \hbar k = \frac{\hbar}{r_c} \quad , \quad k = \frac{1}{r_c} \quad (25)$$

For oscillating string the energy takes the form

$$E_{r_c} = m_0c^2 = \frac{\hbar c}{r_c} \quad (26)$$

Hence

$$r_c = \frac{\hbar}{m_0c} \quad (27)$$

The photon which obeys quantum laws equations (19) and (1) gives

$$E = \frac{2m_0c^2}{\sqrt{2-1}} = 2m_0c^2 \quad (28)$$

This conforms with the fact that photons can produce particle pairs.
Newton's law of potential gives

$$E_{r_c} = U(r) = -G \frac{m_1 m_2}{r} \quad (29)$$

Gravity force is also given by

$$F = -G \frac{m_1 m_2}{r^2} \cdot \frac{r}{r} \quad (30)$$

If

$$m_1 = m_2 = m_c$$

Thus (26) and (29) given

$$E_{r_c} = \frac{G m_c^2}{r_c} = \frac{\hbar c}{r_c} \quad (31)$$

Therefore

$$\hbar c = G m_c^2 \quad (32)$$

Hence

$$m_c = \left(\frac{\hbar c}{G} \right)^{1/2} \quad (33)$$

Where

$$\hbar = 1.05 \times 10^{-27} \text{ erg} \cdot \text{s}, c = 3 \times 10^{10} \text{ cm} \cdot \text{s}^{-1}, G = 6.67 \times 10^{-8} \text{ erg} \cdot \text{cm} \cdot \text{g}^{-1}$$

$$m_c = \left(\frac{\hbar c}{G} \right)^{1/2} \sim 2.2 \times 10^{-5} \text{ g} \quad (34)$$

(Equivalent Planck's mass)

Which matches the proposed value. The same equation applies to Planck's length, namely

$$R_P = \frac{G_P M_P}{c^2} \sim 10^{-33} \text{ cm} \quad (35)$$

(Planck's length) At distances smaller than this scale the gravitational interaction should be stronger than the quantum effects [2]. Also the critical distance r_c is equal

$$r_c = \frac{\hbar}{m_c c} = \left(\frac{G \hbar}{c^3} \right)^{1/2} \sim 1.6 \times 10^{-33} \text{ cm} \quad (36)$$

(Equivalent Planck's length) One can calculate the critical density σ_c of the material when the particles are considered as a hollow sphere surrounded by thin layer or membrane. In this case the surface density is given by

$$\sigma = \frac{m_c}{A}, \quad m_c = \frac{\hbar}{r_c c}, \quad A = 4\pi r_c^2 \quad (37)$$

$$\sigma = \left(\frac{\hbar}{r_c c} \right) \left(\frac{1}{4\pi r_c^2} \right) = \frac{\hbar}{4\pi r_c^3 c} \quad (38)$$

Where

$$m_c = \frac{\hbar}{r_c c} \quad (39)$$

$$\sigma = \frac{m_c}{4\pi r_c^2} \sim 6.7 \times 10^{59} \text{ g. cm}^{-2} \quad (40)$$

Thus the critical density satisfies

$$\sigma_c = \frac{m_c}{r_c^2} = \left(\frac{c^7}{G^3 \hbar} \right)^{1/2}$$

Where

$$\sigma_c = 4\pi\sigma \sim 8.4 \times 10^{60} \text{ g. cm}^{-2} \quad (41)$$

According to this model the universe began at a time and specific place, at the critical point (r_c, t_c) , where all fundamental forces are unified into a single force. The Planck time is thus given by

$$t_c = \frac{r_c}{c} = \left(\frac{G\hbar}{c^3} \right)^{\frac{1}{2}} \left(\frac{1}{c} \right) = \left(\frac{G\hbar}{c^5} \right)^{1/2} \sim 5.4 \times 10^{-44} \text{ s} \quad (42)$$

(Equivalent Planck's time)

The value speed of light c at the critical point (r_c, t_c) .

$$c = \frac{r_c}{t_c} \sim 3 \times 10^{10} \text{ cm. s}^{-1} \quad (43)$$

Began creation of the universe at the critical point (r_c, t_c) , and show the fundamental constants such as (\hbar, c, G) known values, since that time and keep as it is without any change, the structure of the our universe is sensitive to precise degree to less change in these fundamental constants. The status of the universe at different stages is shown to be described in terms of the constants (\hbar, c, G) only. This masterly organization of the universe is the result for precise tuning arbitrator. The acceleration was great, which is equal to [3]:

$$a_c = R_c = \frac{c}{t_c} \quad (44)$$

Where R_c critical curvature (the maximal acceleration occurred at Planck's time).

From a purely dimensional argument one can constant a quantum acceleration from the set of fundamental constants (\hbar, c, G) to be valid at Planck's time, and according to our hypothesis, an analogous acceleration of the form

$$a_c = \frac{c}{t_c} = \frac{r_c}{t_c^2} = \left(\frac{c^7}{G\hbar} \right)^{1/2} \sim 5.7 \times 10^{53} \text{ cm. s}^{-2} \quad (45)$$

Getting limited value to a larger curvature or maximal acceleration in the relation (43) resolved the problem singular behavior. and the matching bending dimensions to pry acceleration are consistent with the principles of general relativity. Conform to the critical value of the acceleration a_c in this relation with the researches results [4]. This acceleration on unwavering c constants, and associated critical point (r_c, t_c) . The existence of this greatest acceleration confirms the occurrence of stretch accelerator of the universe at the beginning of time [5]. The acceleration declining at critical value a_c generates the force to attract at the beginning of time, when the universe takes its way to expansion, and this explains why the presence of the cosmic force of the overall attraction.

The critical force F_c as follows

$$F_c = m_c a_c = \frac{c^4}{G} \sim 1.25 \times 10^{49} \text{ dyne} \quad (46)$$

We can find critical energy E_c that unites all fields be the rank of ($\sim 10^{19} \text{ GeV}$):

$$E_c = m_c c^2 = \left(\frac{\hbar c^5}{G} \right)^{1/2} \sim 10^{19} \text{ GeV} \quad (47)$$

Discussion:

Generalized special relativity energy relation used to find the mass and radius at which the energy is minimum. The two conditions lead to relate critical mass and radius to the mass and universe radius. This relation is typical to that obtained by Ibrahim and others [6], (see equation (10)). It is also very interesting to note that according to equations (11) - (21) that the stars having short and long range gravity force have finite self energy that is formed when the radius is very small, provided that the mass should be less than a critical value. This means that only elementary particles having very small radius and very small mass can have self energy due to the transformation of potential field energy to rest mass energy, where equations (17), (19) and (20) gives:

$$V = m\varphi = \frac{mc^2}{2} = \frac{GMm}{r_c}$$

It is very interesting to note that the radius for self energy is that of black holes, which can be considered as a vent producing elementary particle. It is also very interesting note that, using quantum oscillator and relativistic energy expressions (25) and (26) beside Newtonian potential relation a useful expressions for Planck mass, length and time are obtained in equations (34), (36) and (42). The numerical values of these parameters agree with standard values.

Conclusion:

Using generalized special relativity, quantum mechanics and Newton's laws of gravitation it is shown that elementary particles are created inside black holes at Planck's time.

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