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Energy Calculation for Rotational Excited States Even-Even Nuclei in Lanthanide and Actinide Series

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Abstract

The measurement of energy of excited states in nuclei is one of the most active areas of nuclear structure physics. In this work the asymmetric rotor model of Davydov- Filippov (DF) has been employed to calculate the energies of rotational excited ground and gamma band states even-even nuclei. These energies raging up to 5^+ spin states have been studied for nuclei whose mass number ranges as 150<=A<=190 and A>=228, and for those the first excited state 2^+ and the second excited state $2^{+'}$ gamma band energies are available. In data calculations empirical method has been employed, and the best input parameters have been determined. These input parameters includes the energies of the first excited state 2^+ and the second excited state $2^{+'}$ gamma band energies, and the nuclear asymmetric parameter γ. The energies calculated have been compared with the most recent available experimental data, and shows an excellent agreement.

Keywords: energy, asymmetric rotor model, rotational excited states, asymmetric parameter.

1. Introduction

An interesting feature of nuclear structure is nuclear deformation. It has been known for a long time, that heavy nuclei with many valence particles of both kinds tend to take on a static prolate axially symmetric quadrupole deformation in their ground state. Regular rotational excitation bands are beautiful evidence of this fact. Excitations and behavior of deformed even-even nuclei can be very successfully described using nuclear collective models [1].

Further from closed shells, the accumulating p-n interaction strength leads to additional configuration mixing and deviations from spherical symmetry even in the ground state, and so we now turn to consider nuclei with stable and permanent deformations. The lowest applicable shape component is a quadrupole distortion. There can also be octupole and hexadecapole shapes in the nuclear deformation.

Nuclear shape is usually specified in terms of the two nuclear deformation parameters β and γ. The β parameter represents the extent of quadrupole deformation, while γ gives the degree of axial asymmetry. Nuclear triaxiality is associated with the breaking of axial symmetry of the quadrupole deformation [2]. In the frame work of the rigid tri-axial rotor model, deformation parameters β and γ are extracted from both level energies and E2 transition rates in even-even nuclei. The rigid tri-axial rotor model considers the nucleus as a rigid rotor with rigid tri-axial asymmetry as specified by β and γ [3].

The nuclear deformation parameters β and γ of the collective model are basic description of the nuclear equilibrium shape and structure, while values for these variables have been discussed for many nuclei [4]. It has been shown by the study of even-even nuclei in the regions A <=150 and A >=190 that the properties of their excited levels can be accounted for by considering the rotational motion of non-axial (non-axially symmetric) nuclei or nuclear vibrations.

A new collective theory of the behavior of nuclei has been developed by Davydov and Filippov (DF) taking into account possible violations of the axial symmetry of the nucleus. This violation affects the rotational spectrum of the axial even nucleus, and some new rotational states with total angular momenta of 2, 3, 4,… appear. A deformed nucleus has a rotational degree of freedom. For even-even nuclei, the 0^+ state is always the ground state. The next states with a rotational degree of freedom are $I^\pi = 2^+, 4^+, 6^+, 8^+$,… on symmetry grounds [5].

Even-even nuclei are known to have 0^+ ground states and several low-energy integer spin states. The transition strengths between these levels are sufficiently strong and well established to support the view that most nuclei are collective. Davydov and Filippov axially asymmetric model has been found quite suitable [6, 7, 8] in explaining the rotational levels of the deformed even-even nuclei, the large electric quadrupole moments and the transition probabilities.

In this study the systematic study of the properties of nuclear rotational excited levels in the deformed even-even nuclei will be considered in the framework of Asymmetric Rotor Model of Davydov and Filippov. The aim of the present work is to apply the Davydov-Fillipov model for calculating energies for the rotational excited ground and gamma band states even-even nuclei of rare earth and actinide series.

2. Energy States in Non-Axial Nuclei

Proceeding from the generalized nuclear model, consider the energy states of an even nucleus corresponding to rotation of the latter as a whole with no change of its internal state. The operator corresponding to the rotation energy of the nucleus has the form [7, 9, 10, 11, 12]

$$H = \frac{\hbar^2}{4B\beta^2}\sum_{\lambda=1}^{3}\frac{J_\lambda^2}{2\mathrm{Sin}^2(\gamma-\frac{2\pi}{3}\lambda)} \qquad (1)$$

Where B is the mass parameter, β is the nuclear quadrupole deformation parameter, γ varies between 0 and π/3 and determines the deviation of the shape of the nucleus from axial symmetry, and the J_λ are operators of the projections of the nuclear angular momenta on the axes of a coordinate system connected with the nucleus. The commutation rules for these projections differ from the rules for the projections in a space-fixed coordinate system by the signs in the right hand side.

According to Eq. (1), for γ≠0 or π/3 the nucleus should be regarded as an asymmetric top. In stationary states of the asymmetric top not one of the projections of the total angular momentum on axes 1,2,3 of the body-fixed coordinate system has a definite value and hence the energy levels cannot be specified by the values of $K= J_3$. Each value of the total angular momentum in the asymmetric top corresponds to 2J+ 1 different energy levels. These levels can be classified with respect to the irreducible representations of group D_2 with symmetry elements C_2^1, C_2^2, C_2^3 corresponding to rotation through π about the coordinate axes 1,2,3, because operator (2.1) and the commutation between the J_λ are invariant with respect to this transformation group. Thus the energy levels of an asymmetric top split up into four types of levels which correspond to the four irreducible representations of group D_2.

In virtue of the symmetry conditions imposed on the wave function in even nuclei of the 2J+1 different levels only those energy levels with a given J can exist which correspond to a completely symmetric representation of group D2. Rotation states of the required symmetry will not exist if J=1. Two such states will exist for J=2, one for J=3, three for J=4, two for J=5, four for j=6, etc.

If the energy is expressed in units of $A= \frac{\hbar^2}{4B\beta^2}$ the energy of two levels of the required symmetry are, for j=2, defined by the expression

$$\mathcal{E}_1(2) = \frac{9(1-\sqrt{1-\frac{8}{9}\mathrm{Sin}^2(3\gamma)}}{\mathrm{Sin}^2(3\gamma)}, \quad \mathcal{E}_2(2) = \frac{9(1+\sqrt{1-\frac{8}{9}\mathrm{Sin}^2(3\gamma)}}{\mathrm{Sin}^2(3\gamma)} \qquad (2)$$

The energy of a level with angular momentum j=3 is given by

$$\mathcal{E}(3) = \frac{18}{\mathrm{Sin}^2(3\gamma)} \qquad (3)$$

The three spin 4 energy levels are the roots of the third degree equation

$$\mathcal{E}^3 - \frac{90}{\mathrm{Sin}^2(3\gamma)}\mathcal{E}^2 + \frac{48}{\mathrm{Sin}^4(3\gamma)}[27+26\mathrm{Sin}^2(3\gamma)]\mathcal{E} - \frac{640}{\mathrm{Sin}^4(3\gamma)}[27+7\mathrm{Sin}^2(3\gamma)] = 0 \qquad (4)$$

The roots of this equation has been done by using the online wolfram alpha equation solver software [13] and the solution has not been presented in this study because it so lengthy.

The two spin 5 energy levels are given by the formula

$$\mathcal{E}_1(5) = \frac{45-9\sqrt{9-8\mathrm{Sin}^2(3\gamma)}}{\mathrm{Sin}^2(3\gamma)}, \quad \mathcal{E}_2(5) = \frac{45+9\sqrt{9-8\mathrm{Sin}^2(3\gamma)}}{\mathrm{Sin}^2(3\gamma)} \qquad (5)$$

The energy levels of states possessing an angular momentum equal to 6 and 8 are defined by a fourth and fifth degree equation respectively.

The following simple relation between the spin 2 and spin 3 energy levels follows from Eq. (2) and Eq. (3) as

$$\mathcal{E}_1(2) + \mathcal{E}_2(2) = \mathcal{E}(3) \qquad (6)$$

3. Methods

In the frame work of the rigid tri-axial rotor model [3], deformation parameters β and γ are extracted from both level energies and E2 transition rates in even-even nuclei. The rigid tri-axial rotor model considers the nucleus as a rigid rotor with rigid tri-axial asymmetry as specified by β and γ.

In this research we have employed the most widely used method to calculate the asymmetry parameter γ which is used in empirical calculation. The asymmetry parameter is evaluated from the ratio of two band head energies $E_2^{+'}/E_2^{+}$ [14, 15],

$$\frac{E2+'}{E2+} = \frac{\left[1+\sqrt{1-\frac{8\sin^2(3\gamma)}{9}}\right]}{\left[1-\sqrt{1-\frac{8\sin^2(3\gamma)}{9}}\right]} \qquad (7)$$

where $E_2^{+'} = \frac{9}{\sin^2(3\gamma)}\left[1+\sqrt{1-\frac{8\sin^2(3\gamma)}{9}}\right]\frac{\hbar^2}{4B\beta^2}$ and

$$E_2^{+} = \frac{9}{\sin^2(3\gamma)}\left[1-\sqrt{1-\frac{8\sin^2(3\gamma)}{9}}\right]\frac{\hbar^2}{4B\beta^2}$$

Further simplifying the ratio we can write the asymmetric parameter γ as

$$\gamma = \frac{1}{3}\sin^{-1}\left\{\frac{9}{8}\left[1-\frac{(m-1)^2}{(m+1)^2}\right]\right\}^{1/2} \qquad (8)$$

Where m is the energy ratio $E_2^{+'}/E_2^{+}$.

The energies calculated are considered in two different units. The first is in an energy unit of $A = \frac{\hbar^2}{4B\beta^2}$ discussed in section 2 above. The second is in a unit of KeV. It is possible to convert the energy calculated in a unit $\frac{\hbar^2}{4B\beta^2}$ to KeV unit by considering the following relation.

The hydrodynamic relation relates $E_2^{+'}$ with moment of inertia J_0 and asymmetric parameter γ for asymmetric nucleus (γ≠0) as [16]

$$E_2^{+'} = \frac{6\hbar^2}{2J_o}\,\frac{9-\sqrt{81-72\sin^2(3\gamma)}}{4\sin^2(3\gamma)} \qquad (9)$$

Where $J_o = 4B\beta^2$

Now we can rewrite equation (3.3) to fit to the energy unit we have considered in section 2 above as

$$\frac{\hbar^2}{4B\beta^2} = \frac{8}{6}\,\frac{E_2^{+'}}{\frac{9-\sqrt{81-72\sin^2(3\gamma)}}{\sin^2(3\gamma)}} \qquad (10)$$

And we have approximated this equation for this particular empirical energy calculation as

$$\frac{\hbar^2}{4B\beta^2} \approx \frac{E_2^{+'}}{\frac{9-\sqrt{81-72\sin^2(3\gamma)}}{\sin^2(3\gamma)}} \qquad (11)$$

4. Result

In this section we present the calculated [17] asymmetric parameter γ in Figure 1, and the calculated band energies in a unit of A in Figure 2.

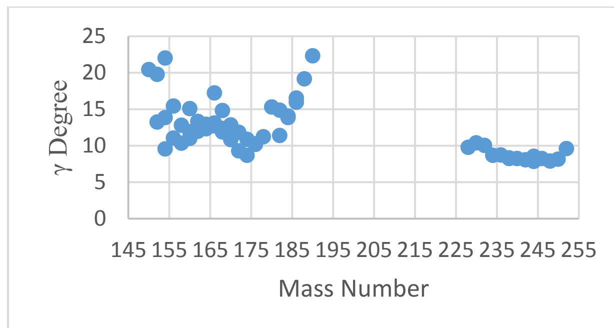

Figure 1: Mass Number versus Asymmetric Parameter γ.

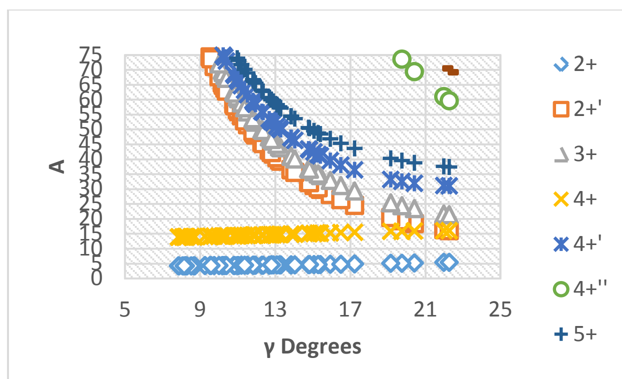

Figure 2: Energy (in a unit of A) versus Asymmetric Parameter γ.

The empirical energy calculations (in a unit of A) is tabulated in Table 1, and the energies converted to a unit KeV is tabulated in Table 2.

Table 1: Calculated Band Energies (in a unit of A) of Rare Earth and Actinide Series.

S/No	Nucleus	Spin							
		2^+	$2^{+'}$	3^+	4^+	$4^{+'}$	$4^{+''}$	5^+	$5^{+'}$
1.	$^{150}_{62}Sm$	5.12	18.29	23.41	15.86	31.90	69.31	38.77	78.29
2.	$^{152}_{62}Sm$	4.45	39.69	44.13	14.67	50.30	155.70	57.48	163.19
3.	$^{154}_{62}Sm$	4.23	74.22	78.45	14.05	84.26	293.95	91.14	301.12
4.	$^{152}_{64}Gd$	5.04	19.31	24.36	15.79	32.48	73.52	39.49	82.29
5.	$^{154}_{64}Gd$	4.49	36.37	40.86	14.79	47.14	142.37	54.35	149.97
6.	$^{156}_{64}Gd$	4.31	55.89	60.19	14.29	66.06	220.62	73.12	227.85
7.	$^{158}_{64}Gd$	4.27	63.75	68.01	14.16	74.12	251.38	80.82	259.25
8.	$^{160}_{64}Gd$	4.30	56.57	60.87	14.28	66.76	223.32	73.79	230.58
9.	$^{154}_{66}Dy$	5.30	16.29	21.59	15.99	31.05	60.91	37.49	70.44
10.	$^{156}_{66}Dy$	4.62	29.86	34.48	15.08	41.07	116.25	48.34	124.08
11.	$^{158}_{66}Dy$	4.42	42.30	46.72	14.60	52.75	166.23	59.97	173.61
12.	$^{160}_{66}Dy$	4.36	48.58	52.94	14.43	58.86	191.42	66.02	198.69
13.	$^{162}_{66}Dy$	4.36	48.04	52.40	14.45	58.35	189.21	65.49	196.52
14.	$^{164}_{66}Dy$	4.39	45.55	49.93	14.51	55.89	179.25	63.09	186.57
15.	$^{160}_{68}Er$	4.59	31.15	35.74	15.00	42.36	121.21	49.51	129.18
16.	$^{162}_{68}Er$	4.45	39.29	43.74	14.68	49.92	154.09	57.10	161.61
17.	$^{164}_{68}Er$	4.43	41.62	46.05	14.61	52.13	163.49	59.32	170.91
18.	$^{166}_{68}Er$	4.41	43.00	47.41	14.57	53.45	168.97	60.64	176.39
19.	$^{168}_{68}Er$	4.39	45.13	49.52	14.51	55.64	177.45	62.69	184.91
20.	$^{170}_{68}Er$	4.34	51.51	55.85	14.38	61.60	203.26	68.86	210.38
21.	$^{166}_{70}Yb$	4.44	40.49	44.93	14.65	51.04	158.94	58.24	166.39
22.	$^{168}_{70}Yb$	4.36	48.90	53.25	14.43	59.13	192.71	66.32	199.95
23.	$^{170}_{70}Yb$	4.29	58.40	62.70	14.25	68.50	230.73	75.58	237.90
24.	$^{172}_{70}Yb$	4.22	78.47	82.68	14.02	88.33	311.05	95.33	318.09
25.	$^{174}_{70}Yb$	4.19	89.61	93.80	13.93	99.66	355.17	106.36	362.63
26.	$^{176}_{70}Yb$	4.26	65.45	69.71	14.15	75.45	258.97	82.49	266.07
27.	$^{166}_{72}Hf$	4.78	24.45	29.24	15.41	36.32	94.45	43.58	102.59
28.	$^{168}_{72}Hf$	4.57	32.24	36.81	14.95	43.34	125.71	50.51	133.54
29.	$^{170}_{72}Hf$	4.42	42.18	46.60	14.60	52.61	165.77	59.85	173.13
30.	$^{172}_{72}Hf$	4.35	49.20	53.56	14.42	59.55	193.81	66.62	201.16
31.	$^{174}_{72}Hf$	4.30	57.88	62.18	14.26	68.01	228.61	75.07	235.81
32.	$^{176}_{72}Hf$	4.26	64.70	68.97	14.16	74.76	255.91	81.76	263.07
33.	$^{178}_{72}Hf$	4.32	54.47	58.79	14.32	64.61	214.97	71.74	222.19
34.	$^{182}_{74}W$	4.33	52.82	57.14	14.35	63.06	208.30	70.13	215.59
35.	$^{184}_{74}W$	4.49	36.47	40.97	14.79	47.20	142.84	54.44	150.38
36.	$^{186}_{74}W$	4.66	28.12	32.78	15.18	39.46	109.26	46.77	117.13
37.	$^{180}_{76}Os$	4.61	30.33	34.94	15.06	41.49	118.15	48.76	125.93
38.	$^{182}_{76}Os$	4.57	32.04	36.62	14.97	43.11	124.92	50.33	132.75
39.	$^{184}_{76}Os$	4.51	35.46	39.97	14.83	46.24	138.81	53.50	146.37
40.	$^{186}_{76}Os$	4.71	26.39	31.10	15.28	37.97	102.27	45.25	110.27
41.	$^{188}_{76}Os$	4.98	20.34	25.32	15.71	33.17	77.70	40.25	86.33
42.	$^{190}_{76}Os$	5.34	15.96	21.29	16.01	30.95	59.52	37.31	69.16
43.	$^{228}_{90}Th$	4.24	71.01	75.25	14.09	81.00	281.16	87.97	288.29
44.	$^{230}_{90}Th$	4.27	62.79	67.06	14.18	72.90	248.23	79.88	255.44
45.	$^{232}_{90}Th$	4.25	67.60	71.86	14.13	77.63	267.52	84.61	274.67
46.	$^{234}_{92}U$	4.19	89.22	93.41	13.93	99.02	354.05	105.97	361.07
47.	$^{236}_{92}U$	4.19	88.61	92.80	13.94	98.43	351.63	105.37	358.64
48.	$^{238}_{92}U$	4.17	98.51	102.68	13.88	108.33	391.20	115.19	398.22

49.	$^{238}_{94}Pu$	4.17	97.35	101.52	13.88	107.15	386.57	114.04	393.58
50.	$^{240}_{94}Pu$	4.17	98.98	103.15	13.87	108.80	393.09	115.66	400.10
51.	$^{242}_{94}Pu$	4.16	102.85	107.02	13.85	112.69	408.45	119.50	415.58
52.	$^{244}_{94}Pu$	4.18	92.28	96.46	13.91	102.11	366.29	109.01	373.31
53.	$^{244}_{96}Cm$	4.15	108.84	112.99	13.82	118.68	432.47	125.45	439.52
54.	$^{246}_{96}Cm$	4.17	98.98	103.15	13.87	108.80	393.09	115.66	400.10
55.	$^{248}_{96}Cm$	4.15	107.79	111.94	13.83	117.55	428.32	124.40	435.30
56.	$^{250}_{98}Cf$	4.17	100.67	104.83	13.87	110.41	399.93	117.33	406.84
57.	$^{252}_{98}Cf$	4.23	73.62	77.85	14.06	83.55	291.65	90.54	298.73

Table 2: Calculated Empirical Band Energies (in KeV) of Rare Earth and Actinide Series.

S/No.	Nucleus	Energy	Spin							
			2^+	$2^{+'}$	3^+	4^+	$4^{+'}$	$4^{+''}$	5^+	$5^{+'}$
1.	$^{150}_{62}Sm$	Expt.	333.96	1193.84	1504.57	773.37				
		Emp.	333.87	1193.04	1526.90	1034.23	2080.55	4519.80	2528.50	5106.02
2.	$^{152}_{62}Sm$	Expt.	121.78	1085.84	1233.86	366.48	1371.74		1559.62	
		Emp.	121.78	1086.47	1208.26	401.63	1377.08	4262.57	1573.63	4467.65
3.	$^{154}_{62}Sm$	Expt.	81.98	1440.04	1539.19	266.82	1664.82		1804.99	
		Emp.	81.98	1439.17	1521.15	272.48	1633.73	5699.55	1767.08	5838.67
4.	$^{152}_{64}Gd$	Expt.	344.28	1318.42	1692.41	755.40	1550.16		1861.58	
		Emp.	344.28	1317.93	1662.20	1077.40	2216.41	5017.20	2695.04	5615.99
5.	$^{154}_{64}Gd$	Expt.	123.07	996.26	1127.80	371.00	1263.78		1432.59	
		Emp.	123.07	995.91	1118.98	404.91	1291.01	3898.63	1488.20	4106.72
6.	$^{156}_{64}Gd$	Expt.	88.97	1154.15	1248.01	288.19	1355.42		1506.86	
		Emp.	88.97	1154.05	1243.02	295.10	1364.11	4555.89	1509.92	4705.17
7.	$^{158}_{64}Gd$	Expt.	79.51	1187.14	1265.52	261.46	1358.47		1481.42	
		Emp.	79.51	1187.61	1267.12	263.82	1380.96	4683.34	1505.65	4829.96
8.	$^{160}_{64}Gd$	Expt.	75.26	988.40	1057.54	248.52	1147.78		1261.07	
		Emp.	75.26	989.07	1064.33	249.64	1167.27	3904.70	1290.11	4031.55
9.	$^{154}_{66}Dy$	Expt.	334.34	1027.04	1334.19	746.78	1442.28		1739.60	
		Emp.	334.58	1027.63	1362.21	1008.67	1959.26	3843.14	2365.94	4445.08
10.	$^{156}_{66}Dy$	Expt.	137.77	890.50	1022.08	404.19	1168.47		1335.56	
		Emp.	137.83	891.21	1029.05	450.01	1225.58	3469.23	1442.54	3702.69
11.	$^{158}_{66}Dy$	Expt.	98.92	946.32	1044.60	317.14	1163.75		1314.78	
		Emp.	98.92	947.12	1046.04	326.80	1181.20	3722.10	1342.79	3887.39
12.	$^{160}_{66}Dy$	Expt.	86.79	966.17	1049.10	283.82	1155.84		1288.66	
		Emp.	86.79	967.30	1054.09	287.41	1171.89	3811.23	1314.45	3956.00
13.	$^{162}_{66}Dy$	Expt.	80.66	888.16	962.94	265.66	1060.99		1182.76	
		Emp.	80.66	888.04	968.70	267.05	1078.62	3497.74	1210.68	3632.81
14.	$^{164}_{66}Dy$	Expt.	73.39	761.82	828.19	242.23	916.00		1024.64	
		Emp.	73.39	762.28	835.67	242.81	935.44	3000.05	1055.85	3122.50
15.	$^{160}_{68}Er$	Expt.	125.80	854.40	987.30	389.90			1316.70	
		Emp.	125.80	853.83	979.63	411.17	1161.03	3322.64	1357.03	3541.11
16.	$^{162}_{68}Er$	Expt.	102.04	900.72	1002.06	329.62	1128.11		1286.22	
		Emp.	102.04	900.22	1002.26	336.44	1143.86	3530.73	1308.38	3702.93
17.	$^{164}_{68}Er$	Expt.	91.38	860.25	946.40	299.43	1058.48		1197.46	
		Emp.	91.40	859.67	951.07	301.81	1076.77	3376.82	1225.27	3530.09
18.	$^{166}_{68}Er$	Expt.	80.58	785.91	859.39	264.99	956.23		1075.28	
		Emp.	80.58	785.54	866.11	266.26	976.60	3087.17	1107.85	3222.73
19.	$^{168}_{68}Er$	Expt.	79.80	821.17	895.79	264.09	994.75		1117.57	
		Emp.	79.80	820.57	900.38	263.81	1011.63	3226.58	1139.79	3362.09

20.	$^{170}_{68}$Er	Expt.	78.60	934.03	1010.54	260.15	1103.33		1236.61	
		Emp.	78.59	933.50	1012.09	260.58	1116.29	3683.53	1247.87	3812.60
21.	$^{166}_{70}$Yb	Expt.	102.37	932.38	1039.14	330.48	1162.74		1327.85	
		Emp.	102.37	933.81	1036.18	337.83	1177.18	3665.88	1343.29	3837.59
22.	$^{168}_{70}$Yb	Expt.	87.73	984.00	1067.15	286.55	1171.36		1302.30	
		Emp.	87.73	984.73	1072.46	290.60	1190.79	3880.87	1335.65	4026.65
23.	$^{170}_{70}$Yb	Expt.	84.25	1145.72	1225.35	277.43	1329.31		1459.75	
		Emp.	84.25	1145.90	1230.16	279.62	1344.02	4527.15	1482.92	4667.86
24.	$^{172}_{70}$Yb	Expt.	78.74	1465.88	1549.15	260.27	1657.79		1778.86	
		Emp.	78.75	1465.95	1544.69	261.88	1650.20	5811.14	1780.92	5942.54
25.	$^{174}_{70}$Yb	Expt.	76.47	1633.97	1709.42	253.12	1805.40		1926.00	
		Emp.	76.47	1636.71	1713.18	254.37	1820.14	6486.93	1942.59	6623.31
26.	$^{176}_{70}$Yb	Expt.	82.14	1260.89	1336.38	271.85	1435.50		1558.34	
		Emp.	82.13	1261.79	1343.92	272.83	1454.53	4992.28	1590.31	5129.28
27.	$^{166}_{72}$Hf	Expt.	158.64	809.96	1007.16	470.46	1332.41		1418.90	
		Emp.	158.50	810.44	968.94	510.66	1203.81	3130.39	1444.44	3400.26
28.	$^{168}_{72}$Hf	Expt.	124.10	875.94	1030.93	385.92	1216.50		1386.38	
		Emp.	124.00	875.54	999.54	406.04	1176.90	3413.45	1371.54	3626.16
29.	$^{170}_{72}$Hf	Expt.	100.80	961.30	1087.59	321.99	1227.30			
		Emp.	100.80	962.05	1062.85	333.03	1200.05	3781.19	1365.25	3949.01
30.	$^{172}_{72}$Hf	Expt.	95.22	1075.29	1180.87	309.24	1304.66		1462.88	
		Emp.	95.22	1076.04	1171.26	315.29	1302.40	4238.67	1456.92	4399.37
31.	$^{174}_{72}$Hf	Expt.	90.99	1226.77	1336.48	297.38	1448.85		1658.41	
		Emp.	90.99	1225.55	1316.54	301.91	1440.01	4840.74	1589.49	4993.20
32.	$^{176}_{72}$Hf	Expt.	88.35	1341.31	1445.79	290.18	1540.30		1727.80	
		Emp.	88.35	1340.77	1429.12	293.44	1549.15	5303.05	1694.17	5451.42
33.	$^{178}_{72}$Hf	Expt.	93.18	1174.63	1268.54	306.62	1384.46		1533.15	
		Emp.	93.18	1175.66	1268.84	309.04	1394.48	4639.89	1548.38	4795.80
34.	$^{182}_{74}$W	Expt.	100.11	1221.40	1331.12	329.43	1442.84		1623.51	
		Emp.	100.06	1221.14	1321.21	331.67	1457.98	4816.06	1621.39	4984.63
35.	$^{184}_{74}$W	Expt.	111.22	903.31	1005.97	364.07	1133.85		1294.94	
		Emp.	111.21	902.80	1014.01	366.00	1168.28	3535.62	1347.63	3722.40
36.	$^{186}_{74}$W	Expt.	122.63	737.96	862.28	396.55	1006.73		1197.30	
		Emp.	122.33	737.58	859.91	398.20	1035.21	2866.00	1226.90	3072.64
37.	$^{180}_{76}$Os	Expt.	132.11	870.44	1022.85	408.63	1196.83		1405.55	
		Emp.	132.30	870.87	1003.17	432.28	1191.35	3392.53	1400.07	3615.79
38.	$^{182}_{76}$Os	Expt.	126.89	890.61	1039.04	400.29	1190.30		1399.47	
		Emp.	127.00	890.42	1017.42	415.90	1197.88	3471.21	1398.42	3688.67
39.	$^{184}_{76}$Os	Expt.	119.77	942.87	1080.97	383.68	1224.99		1428.15	
		Emp.	119.80	942.37	1062.17	393.95	1228.65	3688.35	1421.57	3889.29
40.	$^{186}_{76}$Os	Expt.	137.16	767.48	910.47	434.09	1070.48		1275.61	
		Emp.	137.16	767.73	904.89	444.52	1104.77	2975.15	1316.37	3208.09
41.	$^{188}_{76}$Os	Expt.	155.02	633.02	789.98	477.94	965.65		1180.86	
		Emp.	155.02	633.15	788.17	489.00	1032.74	2419.03	1253.23	2687.61
42.	$^{190}_{76}$Os	Expt.	186.72	557.98	756.02	547.85	955.37		1203.86	
		Emp.	186.72	558.10	744.82	559.82	1082.52	2081.71	1304.97	2419.11
43.	$^{228}_{90}$Th	Expt.	57.77	968.38	1022.54	186.84	1091.05		1174.52	
		Emp.	57.76	967.61	1025.37	191.96	1103.71	3831.11	1198.64	3928.20
44.	$^{230}_{90}$Th	Expt.	53.23	781.38	825.66	174.11	883.60		955.04	
		Emp.	53.20	781.93	835.14	176.64	907.86	3091.17	994.74	3180.94
45.	$^{232}_{90}$Th	Expt.	49.37	785.25	829.60	162.12	890.10		960.24	
		Emp.	49.37	785.03	834.40	164.02	901.45	3106.48	982.50	3189.47
46.	$^{234}_{92}$U	Expt.	43.50	926.72	968.43	143.35	1023.77		1090.89	

		Emp.	43.50	926.75	970.24	144.74	1028.48	3677.51	1100.74	3750.48
47.	$^{236}_{92}\mathrm{U}$	Expt.	45.24	957.90	1001.50	149.48	1058.80		1127.38	
		Emp.	45.24	957.02	1002.26	150.54	1063.03	3797.61	1137.99	3873.33
48.	$^{238}_{92}\mathrm{U}$	Expt.	44.92	1060.27	1105.71	148.38	1168.00		1232.00	
		Emp.	44.91	1061.13	1106.04	149.48	1166.86	4213.81	1240.77	4289.43
49.	$^{238}_{94}\mathrm{Pu}$	Expt.	44.07	1028.54	1069.93	145.94	1125.75			
		Emp.	44.08	1028.73	1072.81	146.71	1132.26	4084.95	1205.06	4159.02
50.	$^{240}_{94}\mathrm{Pu}$	Expt.	42.82	900.32	1030.55	141.69	1076.22			
		Emp.	42.82	1016.90	1059.72	142.54	1117.70	4038.37	1188.19	4110.41
51.	$^{242}_{94}\mathrm{Pu}$	Expt.	44.54	1102.00		147.30				
		Emp.	44.54	1100.74	1145.28	148.25	1206.01	4371.23	1278.90	4447.50
52.	$^{244}_{94}\mathrm{Pu}$	Expt.	44.20	1015.00		155.00				
		Emp.	46.00	1015.24	1061.24	153.07	1123.40	4029.78	1199.24	4106.96
53.	$^{244}_{96}\mathrm{Cm}$	Expt.	42.97	1020.76		142.35				
		Emp.	42.96	1126.13	1169.10	143.04	1227.89	4474.53	1297.99	4547.49
54.	$^{246}_{96}\mathrm{Cm}$	Expt.	42.85	1124.26	1165.48	141.99	1219.95			
		Emp.	42.85	1017.56	1060.41	142.63	1118.43	4041.00	1188.97	4113.09
55.	$^{248}_{96}\mathrm{Cm}$	Expt.	43.40	1131.00		143.80	1222.00			
		Emp.	43.38	1125.56	1168.94	144.42	1227.52	4472.72	1299.08	4545.61
56.	$^{250}_{98}\mathrm{Cf}$	Expt.	42.72	1031.85	1071.37	141.88	1123.00			
		Emp.	42.72	1032.48	1075.20	142.21	1132.35	4101.74	1203.37	4172.63
57.	$^{252}_{98}\mathrm{Cf}$	Expt.	45.72	804.80	845.70	151.74	900.30			
		Emp.	45.72	795.81	841.53	151.99	903.14	3152.49	978.69	3228.97

5. Discussion

In this particular work we have employed an updated experimental data of energies. The important determining parameter γ of the DF model has been calculated using Eq. (8), and shown in Figure 1. We have used Eq. (2) through Eq. (5) to calculate the theoretical energies in a unit of A, and the result has been shown in Figure 2 and tabulated in Table 1. These energies have been converted to a unit of KeV by utilizing Eq. (11), and the result has been tabulated in Table 2 for the 57 isotopes of rare earth and actinide series.

We have seen from Figure 1 that the values of the asymmetric parameter γ is somewhat scattered for the mass region 150≤A≤190 (rare earth), and almost similar values for mass region 228≤A≤252 (actinide series).

The Experimental data used in this study have been taken from [18, 19]. We couldn't find a sufficient available research work on this similar topic to compare with, and that we have compared, our results, only with the available experimental ones. For the $4^{+''}$ and $5^{+'}$ we couldn't find the appropriate experimental data to compare with, and so that these empirical results could be taken as the estimated values of the respective band energies.

This present study reveals by comparing the percentage difference between experimental data and empirical calculations obtained that the percentage difference is very small for almost all nuclei included in this study except for $^{150}_{62}\mathrm{Sm}$, $^{152}_{64}\mathrm{Gd}$ and $^{154}_{66}\mathrm{Dy}$. For these nuclei the percentage difference is greater than 30% for total angular momentum of 4 and greater, but for lower momentum the empirical calculations has perfectly match the experimental data available. This comparison show that the present empirical calculation is in an excellent agreement with the available experimental data.

6. Conclusion

The calculation of energy values for the ground and gamma band states of the even-even deformed nuclei in the rare earth and actinide regions have been considered. We have compared our empirical calculation of energies with the respective experimental values, and have found an excellent agreement. This investigation show that the DR predictions are very closer to the experimental values, thus giving support to the idea that the nuclear shape tends to become triaxial in the region of mass number in the rare earth and actinide nuclei considered.

So we conclude that the calculated empirical energies are observed to be in an excellent agreement especially at lower transitions for all isotopes considered, and that the asymmetric rotor model could be helpful in calculating the rotational excited even-even ground and gamma band energy states.

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