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«Generalized navier Condition With Regard To Influence Of Quantum-Mechanical Effects In Nono hydrodynamics»

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Abstract

Physico-mathematical dependence of the fluid velocity and fluid slippage conditions on the boundary with the vessel wall is proposed with regard to influence of quantum-mechanical effects in low-dimensional systems (nanosystems).

The Navier generalized formula is obtained. It is established that the velocity and slippage values of the fluid on the interface of fluid and empty wall space be composed of the sum of three velocities: the first one that arises due to the influence of structural inhomogeneity of the wall part of the fluid; the second one arising due to apparent length of fluid slippage on the boundary of the fluid and empty space; the third is the slippage velocity of the entire system of velocity diagrams over the depth of the fluid, which is due to apparent slippage length of the fluid proposed by Navier.

Keywords: *nanohydrodynamics with regard to quantum-mechanical effects, low-dimensional systems, nanotube, inhomogeneous fluid, quantum-mechanical effects, locally-ordered structure of fluid.*

I. Problem statement.

In nanohydrodynamics, one of the little researched problems is the form of the boundary condition on the fluid velocity at the vessel wall, and also the type of fluid slippage conditions along the vessel wall.

In 1823, in macrohydrodynamics, as a condition for fluid slippage along the vessel wall Navier first suggested the condition for the slippage of a viscous fluid along the vessel wall in the form **[7 - Navier C.L.M.N., 1823]:**

$$\upsilon = L_0 \cdot \frac{\partial \upsilon}{\partial n}$$

This relation was written under the following assumptions:

- the fluid is homogeneous,

- the profile of the fluid velocity distribution along the height of the vessel is represented by convexity in the direction of motion in the form of a parabola,

- fluid velocity on the boundary of contact of the fluid and vessel equals zero, i.e. the complete adherence condition takes place;

- According to Maxwell, the size of the apparent slippage length L_0 is equal to double length of free run of an atom in a varified gas,

- there are no side non-mechanical interaction forces on the boundary of contact of the fluid and vessel wall.

However, in low-dimensional systems $10^{-9}\,м \le d \le 10^{-4}\,м$, specific interaction forces arise in the form of physical fields on the basis of the influence of quantum-mechanical effects. The result of interaction of these forces is the formation of an empty space between the fluid and the vessel wall called the in physics “forbidden zone”.

Secondly, under the influence of the intensity of the physical field penetrating deep into the fluid, a homogeneous fluid is transformed into fluid. This phenomen leads to change in the properties of the fluid the density and viscosity **[2 – Aliyev G.G. and Aliyev A.G. 2016].**

Using the computer simulation method to experimental studies in flow of liquid particles in low-dimensional channels, the numerical value of an empty interlayer between the fluid and the tube with thickness of layers $R_f \le r \le R_0$ equal to $R_f / R_0 = 0{,}88$, was established **[10,14,17].**

The second phenomenon in low-dimensional systems is the variability of physico-mechanical properties of fluids, i.e. density and viscosity. Physicists-experimenters have established the strange behavior of fluid in low-dimensional channels. Placed in a low-dimensional tube (micro, nanotube), a usual homogeneous fluid turns into a structurally-inhomogeneous fluid. Such a local inhomogeneity of atomic and molecular structure of the fluid affects the nature of fluid flow in nanochannels. This strange phenomenon of locally-ordered structure of the fluid in low-dimensional channels was stated in the form of a hypothesis by Ya.I.

Frenkel in 1941 **[6].** Over the 15 years of the XXI century, full-scale and computer experiments which confirm the existence of ice-like structure of water in micro and nanotubes **[9,12,13,15,16,18]** were carried out.

However, we note that causality of changing the properties of fluid and their mathematical representation in the form of models is not available to date.

For the first time, the problem of causality of the phenomenon of transformation of a homogeneous fluid into inhomogeneous one in low-dimensional systems (in particular, in nano-dimensional systems) with regard to influence of quantum-mechanical effects was first studied in the works of Aliyev G.G. and Aliyev A.G. **[1-5].** The authors have proved that the causality of the phenomenon of transformation of a homogeneous fluid into an inhomogeneous one is related to the size of density of the influence of intensity of physical field $\tilde{E} = \frac{E(r)}{E_0}$ penetrating deep into fluid. The authors also proposed the following physico-mathematical model of dependence of mechanical characteristics of fluid (density $\rho(r)$ and viscosity $\mu(r)$) on intensity of physical field $\tilde{E} = \frac{E(r)}{E_0}$:

$$\rho = \rho_0 \cdot [1 - \tilde{E}(r)], \qquad \mu = \mu_0 \cdot [1 - \tilde{E}(r)]$$

Based on these models, in the work **[2]** Aliyev G.G. and Aliyev A.G. constructed determining equations of hydromechanics of ideal and viscous fluid in low-dimensional systems with regard to influence of quantum-mechanical effects.

The goal of the reseach.

In the paper we propose a mathematical formula of generalized boundary condition of fluid slippage at the interface of fluid and vessel wall with regard to influence of arising quantum-mechanical effects, in the form:

$$\upsilon(r) = a + b \cdot \frac{\partial \upsilon}{\partial r},$$

where the coefficients a and b are the expressions dependent on quantum-mechanical effects.

II. Construction of physico-mathematical model of generalized fluid slippage condition on the boundary.

Let us consider flow of viscous fluid in the tube of diameter $10^{-9}\,м \le d \le 10^{-4}\,м$. Under the action of quantum-mechanical effects taking place on the boundary with the vessel wall and also intensity of physical field penetrating deep into the fluid, there will arise the following phenomena in the fluid:

- formation of empty space between the vessel wall and fluid,

- along the height of the fluid close to the wall, homogeneous fluid will turn into structurally- inhomogeneous one,

- variability of mechanical characteristics of inhomogeneous part of fluid (density $\rho(r)$ and viscosity $\mu(r)$) along the depth depending on the influence intensity of physical flied, will be in the form:

$$\rho = \rho_0 \cdot [1 - \tilde{E}(r)], \qquad \mu = \mu_0 \cdot [1 - \tilde{E}(r)] \qquad (1)$$

Under these conditions, the diagram of velocity along the section of the tube, will not be parabolic, it will be in the form shown in fig. 1. On the other hand, by the symmetry of fluid flow with respect to the center, the flow velocity graph is represented in the form of the curve MA_1N.

Under the action of quantum-mechanical effects, at first, there will be formed an empty space between the vessel wall of radius R_0 and the boundary of fluid flow ($R_0 - \Delta$) of size $\Delta = 0{,}12 \cdot R_0$;

Secondly, by the structural-inhomogeneity of fluid along the depth in the zone $(R_0 - \Delta, r_0)$, the diagram of fluid velocity will be in the form of the curve A_1N.

We consider that the forms of functions of fluid velocities in domains $(0, r_0)$ and $(r_0, R_0 - \Delta)$ are determined from the solution of corresponding equations of motion of homogeneous and structurally-inhomogenous parts of fluids. Note that these equations of a new type first were constructed in the work of Aliyev G.G. and Aliyev A.G. **[2].**

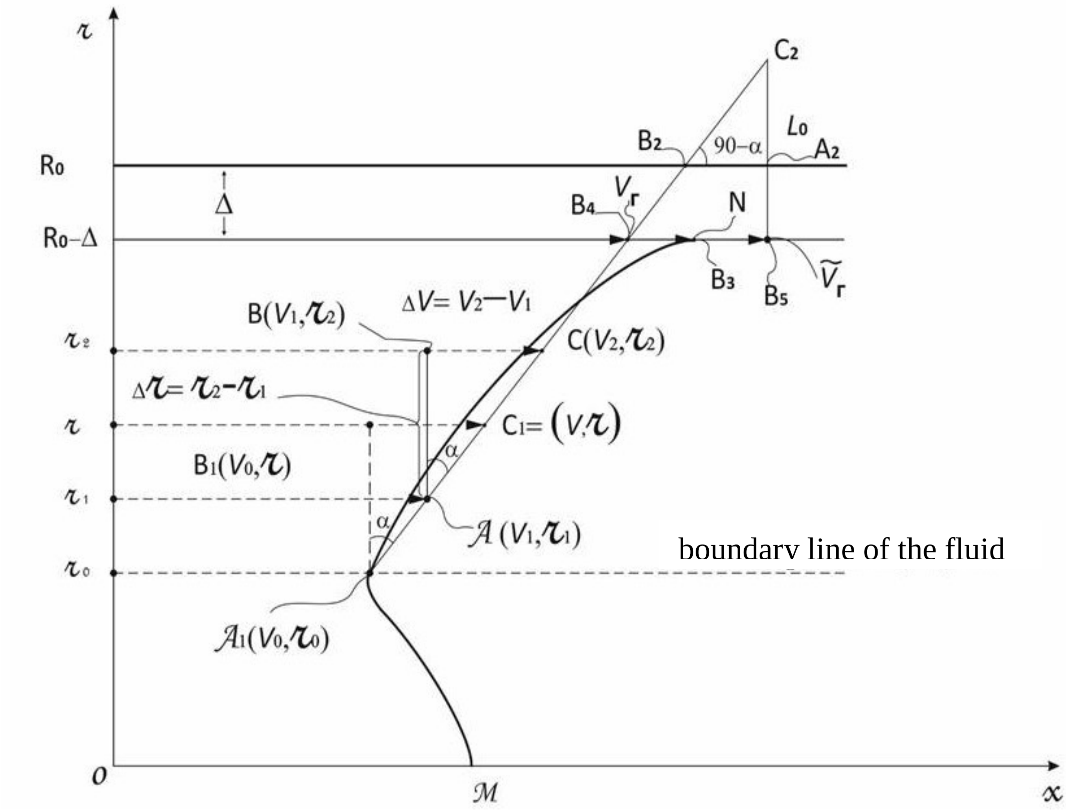

Fig.1. Nature of fluid flow velocity in the depth of the vessel with regard to quantum-mechanical effects.

The main goal of the paper is to propose a new boundary condition for velocity on the boundary $(R_0 - \Delta)$ and also to propose a new condition for inhomogeneous fluid slippage on this boundary with regard to influence of quantum-mechanical effects.

From ΔABC we have (fig.1):

1) $\Delta\upsilon = \upsilon_2 - \upsilon_1 > 0$, $\Delta r = r_2 - r_1 > 0$, $tg\alpha = \dfrac{d\upsilon}{dr} > 0$ for $d\upsilon > 0$

On the other hand, from $\Delta A_1 B_1 C_1$ we have:

$tg\alpha = \dfrac{\upsilon - \upsilon_0}{r - r_0} > 0$ for $\upsilon - \upsilon_0 > 0$, $r - r_0 > 0$

Whence:

$$\upsilon(r) = \upsilon_0 + (r - r_0) \cdot \frac{d\upsilon(r)}{dr} \qquad (2)$$

Here $\upsilon(r)$ is the velocity at arbitrary point of the curve A_1C, υ_0 is the fluid velocity at the intersection point of the curves at the point A_1. Hence, it is seen that

- for $r = r_0$ at the intersection point of curves, the velocity will equal $\upsilon = \upsilon_0$,

- for $r = R_0 - \Delta$ on the boundary with empty space, the velocity will be equal to :

$$\upsilon_\Gamma = \upsilon\big|_{r=R_0-\Delta} = \upsilon_0 + (R_0 - \Delta - r_0) \cdot \frac{d\upsilon}{dr} \qquad (3)$$

Note that formula (3) establishes relation between the velocity on the boundary between the fluid and empty space $y_\Gamma = y\big|_{r=R_0-\Delta} = R_0 - \Delta$ with the fluid velocity υ_0 at the intersection point of two curves r_0.

If follows from (3) that for $r_0 \neq 0$, $\upsilon_0 \neq 0$ and $\Delta \neq 0$ the velocity on the boundary between the wall and fluid is always non-zero, i.e. the slippage takes place due to influence of quantum mechanical effects. Hence, it follows that adhesion of fluid to the vessel wall in low-adhesional systems is always absent.

In the case $\upsilon_0 = 0$, $\Delta = 0$ and $r_0 = R_0$ the velocity on the interface of the wall and fluid will be equal to zero $\upsilon_\Gamma = 0$. This case will correspond to the condition of adhesion on the boundary, the form of the velocity curve will be in the form of a parabola. This special case will correspond to the classical case used in macro hydrodynamics.

Now let us show the slippage condition Navier type. In this case, along with the slippage quantity of the form (3) that arises due to quantum-mechanical effects, we take into account the form of fluid slippage at motion of the whole system (diagrams) of distribution of velocity in the depth of the vessel along the axis Ox.

From $\Delta B_4C_2B_5$ we have:

$$\frac{L_0}{B_4B_5} = tg(90 - \alpha), \quad \text{where } B_4B_5 = \tilde{\upsilon}_\Gamma - \upsilon_\Gamma$$

Whence:

$$\tilde{\upsilon}_{\Gamma} = \upsilon_{\Gamma} + (L_0 + \Delta) \cdot \frac{d\upsilon}{dr} \tag{4}$$

Having substituted (3) in (4), we find total value of slippage in the form:

$$\tilde{\upsilon}_{\Gamma} = \upsilon_0 + (R_0 - \Delta - r_0) \cdot \frac{d\upsilon}{dr} + \Delta \cdot \frac{d\upsilon}{dr} + L_0 \cdot \frac{d\upsilon}{dr}$$

Here L_0 is the apparent length of slippage of the whole system according to Navier; Δ is the apparent length of fluid slippage on the boundary of fluid and empty space and equals the size of the "forbidden zone" between the atoms, of size $\Delta = R_0 - R_{ж} = R_0 \cdot (1 - \frac{R_{ж}}{R_0}) = R_0 \cdot (1 - 0{,}88) = 0{,}12 \cdot R_0$ **(fig.1).**

In a compact form we can write it in the form:

$$\tilde{\upsilon}(r)\Big|_{\Gamma} = \tilde{\upsilon}(r)\Big|_{r = R_0 - \Delta} = a + b \cdot \frac{\partial \upsilon(r)}{\partial r}, \tag{5}$$

where $a = \upsilon_0$, $b = R_0 - r_0 + L_0$

Thus, the velocity of slippage on the interface between the fluid and empty space will be composed of the sum of three velocities:

The first velocity arising due to influence of structural in homogeneity of the wall part of the fluid, equal $\upsilon_1 = \upsilon_0 + (R_0 - \Delta - r_0) \cdot \frac{d\upsilon}{dr}$;

The second velocity arising due to apparent length of fluid slippage on the boundary of the fluid and empty space $\Delta = 0{,}12 \cdot R_0$, equal to the size of the "forbidden zone" between the atoms and is equal to $\upsilon_2 = \Delta \cdot \frac{d\upsilon}{dr}$, and the third velocity is the velocity of slippage of all the system of velocity diagrams in the depth of the fluid arising due to apparent length of fluid slippage L_0 of the size equal to $\upsilon_3 = L_0 \cdot \frac{d\upsilon}{dr}$, suggested by Navier.

III. Results

In the application term, this result has the following meaning. In the motion of the fluid, in low-dimensional tubes (nanotubes) on the boundary there will arise three forms of velocities:

$$\upsilon_1 = \upsilon_0 + (R_0 - \Delta - r_0)\cdot\frac{d\upsilon}{dr}, \qquad \upsilon_2 = \Delta\cdot\frac{d\upsilon}{dr}, \qquad \upsilon_3 = L_0\cdot\frac{d\upsilon}{dr}$$

Depending on the nature of structural inhomogeneity of the thin layer of the fluid and size of the empty space Δ the following forms of fluid flow velocities on the boundary are possible:

Case 1.

For $L_0 = 0$ there will be two forms of velocities on the boundary:

$$\upsilon_1 = \upsilon_0 + (0{,}88 - \frac{r_0}{R_0})\cdot\frac{d\upsilon}{dr}, \qquad \upsilon_2 = 0{,}12\cdot\frac{d\upsilon}{dr}, \qquad \upsilon_3 = 0$$

Here the functions υ_0 and $\frac{d\upsilon}{dr}$ should be determined from the solution of specifically stated problem that will depend on experimental data of $\frac{r_0}{R_0}$, Δ, L_0. Hence, it follows that $\upsilon_1 \neq 0$ and $\upsilon_2 \neq 0$. This means that the fluid slippage effect will always hold on the boundary due to structural inhomogeneity of the wall layer of the fluid, and also on the availability of inter layer of empty space between the fluid and the vessel wall of size $\Delta = \frac{R_{ж}}{R_0} = 0{,}12\cdot R_0$.

Case 2.

When $L_0 = 0$ and $\Delta = 0$ the velocity of fluid particles on the boundary (displacement) will be in the form:

$$\upsilon_1 = \upsilon_0 + R_0\cdot(1 - \frac{r_0}{R_0})\cdot\frac{d\upsilon}{dr}, \qquad \upsilon_2 = 0, \qquad \upsilon_3 = 0$$

In other words, on the boundary, the velocity of fluid particles will be nonzero, and the slippage effect of the system due to Δ and L_0 will be equal to zero.

Case 3.

In the case when $L_0 = 0$, $\Delta = 0$ and $r_0 = R_0$ all three velocities will be equal to zero:

$$\upsilon_1 = 0, \quad \upsilon_2 = 0, \quad \upsilon_3 = 0, \quad \upsilon_0 = 0$$

This will correspond to the case of absence of the influence of quantum-mechanical effects on the fluid flow and also to absence of fluid slippage on the vessel wall. In this case, the complete adhesion of the fluid to the vessel wall will take place.

Case 4.

In the case when $L_0 \neq 0$, $\Delta = 0$ and $r_0 = R_0$ will correspond to the slippage of homogeneous fluid in the vessel wall, corresponding to the Navier condition.

$$\upsilon_1 = 0, \quad \upsilon_2 = 0, \quad \upsilon_3 = L_0 \cdot \frac{d\upsilon}{dr}$$

Thus, in conclusion we can summarize that in the paper we propose physico-mathematical dependence of the velocity and slippage conditions of the fluid on the boundary with the vessel wall with regard to influence of quantum-mechanical effects in low-dimensional systems (nanosystems) which is the generalization of the Navier formula.

The sizes of the velocity of slippage of the fluid on the interface between the fluid and empty space will be composed of the sum of three velocities:

The first velocity arising due to influence of nonhomogeneity of the wall part of the fluid, and also the size of the empty space, and will be equal to $\upsilon_1 = \upsilon_0 + (R_0 - \Delta - r_0) \cdot \frac{d\upsilon}{dr}$;

The second velocity arising due to apparent length of fluid's slippage on the boundary of the fluid and empty space $\Delta = 0{,}12 \cdot R_0$ *,that equals the size of the "forbidden sone" between the atoms and equals* $\upsilon_2 = \Delta \cdot \frac{d\upsilon}{dr}$.

The third velocity of slippage of the system of velocity diagram in the depth of the fluid arising due to apparent length of fluid slippage L_0 *of the length equal to* $\upsilon_3 = L_0 \cdot \frac{d\upsilon}{dr}$*,suggested by Navier.*

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