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Application of Higer Order Shear Deformation Theory in the Analysis of Critical Thickness on a Thick Plate.

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ABSTRACT: In this paper, the refined plate theory (RPT) is examined for the bending analysis of clamped-simply supported (CCSS) isotropic rectangular thick plate. The axial displacement field uses parabolic function in terms of thickness coordinate to include the effect of transverse shear deformation. The transverse displacement consists of bending and shear components. In this work, it is assumed that the transverse shear strain is assumed to be constant across the thickness. The present theory satisfies the traction free conditions on the upper and lower surfaces of the plate without using problem dependent shear correction factors of Timoshenko. The main objective of this study is static bending analysis of an isotropic thick plate to determine the actual maximum critical thickness of the plate before deflection reaches the specified maximum limit. The aim is to avoid structural failure or consequently collapse by ensuring that deflection did not exceed allowable value. It can also be used as a check to achieve most possible economical design in a structure. Governing differential equations with its associated boundary conditions are obtained according to Ibearugbulem et al. To prove the credibility of the present theory, we also applied it to the bending analysis of plate with adjacent edge clamped and the other adjacent edge simply supported (CCSS) at varying side-thickness aspect ratio under general boundary conditions. The numerical results of non-dimensional displacements obtained by using the present theory are presented and compared with those of other refined theories available in the literature along with the elasticity solution. From the result of the analysis it was found that the higher shear deformation is free from the draw back suffered by the use of shear correction factors as in the case of lower order shear deformation theories.

Keywords: *Transverse shear deformation, shear correction factor, transverse shear stress, Critical lateral load.*

A. INTRODUCTION:

Thick plate as a structural components are increasingly being used in various engineering applications due to their attractive properties in strength, stiffness, and lightness. The effect of transverse shear deformation is more pronounced in thick plate because of excess vertical shear stress that exist due to heavy thickness.

The classical beam theory (CPT) does not predict the correct bending behaviour of thick plate. The classical plate theory is inconsistent in the sense that elements are assumed to remain perpendicular to the mid-plane, yet equilibrium requires that stress components τ_{xz} and τ_{zz} still arise (which would cause these elements to deform). The theory of thick plates is more consistent, but it still makes the assumption that $\sigma_{zz} = 0$ and $\varepsilon_{zz} = 0$. Note that *both* are approximations of the exact three-dimensional equations of elasticity. As with plate theory, it turns out that the solutions based on the classical theory agree well with the full elasticity solutions (away from the edges of the plate), provided the plate thickness is small relative to its other linear dimensions. When the plate is relatively thick, one is advised to use a more exact theory, for example one of the shear deformation theories. The first order shear deformation beam theory (FSDT) developed by Timoshenko (1921) includes the effect of transverse shear deformation but does not satisfy the zero shear stress conditions on the top and bottom surfaces of the beam, hence, it requires shear correction factor. Many higher order theories are available in the literature for the bending, buckling and free vibration analysis of plate which take into account the effect of transverse shear deformation and do not require shear correction factor. The third order theory of Reddy (1984) is the most commonly used higher order theory for beams as well as for plates. A recent review of higher order theories available for the analysis of laminated composite plate has been presented by Ghugal and Shimpi (2001).

Kadoli et al. (2008) applied the third order theory of Reddy for the static analysis of functionally graded plates.

Recently, Sayyad et al. (2015) developed a new trigonometric shear deformation theory for the bending analysis of laminated composite plates.

An important difference between the thin plate and thick plate theories is that in the former the moments are related to the curvatures through (using *x M* for illustration).

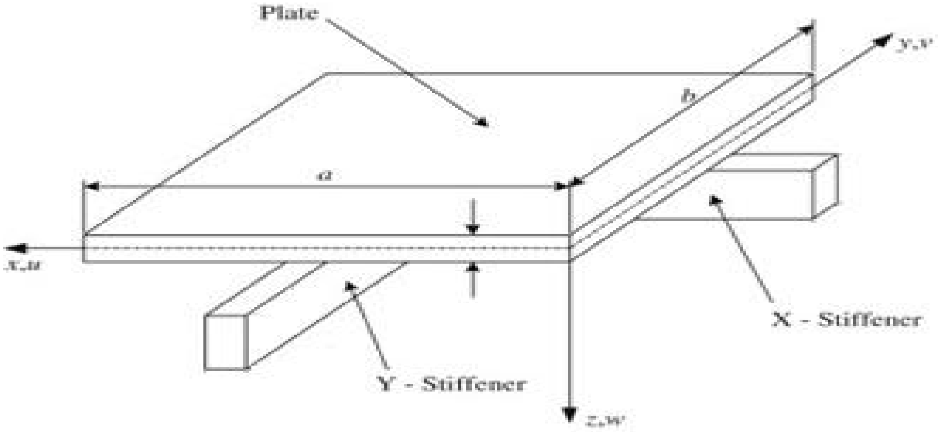
$$M_x = -D\left[\frac{\partial^2 w}{\partial x^2} + \mu\frac{\partial^2 w}{\partial y^2}\right] \qquad 1$$

This is only an approximate relation (although it turns out to be exact in the case of pure bending). The thick plate theory predicts that, in the case of a uniform lateral load q, the relationship is given by:

$$M_x = -D\left[\frac{\partial^2 w}{\partial x^2} + \mu\frac{\partial^2 w}{\partial y^2}\right] + h^2 q\left[\frac{8 + \mu + \mu^2}{20(1-\mu)}\right] \qquad 2$$

B. DISPLACEMENT FIELD

Figure **3.1** shows a bent elastic plate under lateral loading. Our intention is to obtain the displacement – strain relationships in terms of curvatures. From the assumptions, we have three displacement of thick place which includes the deflection, w(x,y) and the two inplane displacements, u(x,y,z), and v(x,y,z). Ibearugbulem (2015) in his lecture note defined deflection, w.



Where the plate thickness h = t

Fig. 1. Plate geometry and co-ordinate system

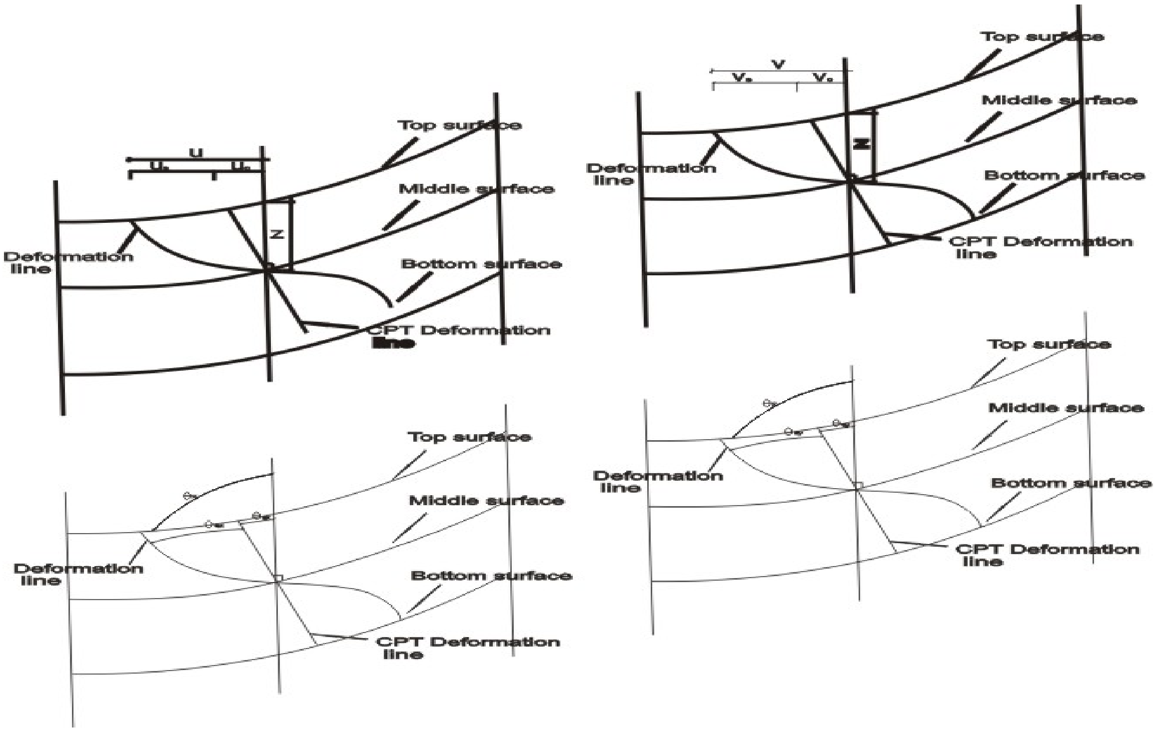


Fig. 2. Deformation of a section of a thick Plate

C. ENGINEERING STRAIN – DISPLACEMENT RELATIONS

The refined plate theory (RPT) displacements and strain as presented on figure1 are defined mathematically as:

$$w = A_1 h \qquad 2$$

From assumptions herein, the strain normal to z axis is zero. This left us with only five engineering strain components $\varepsilon_x, \varepsilon_y, \gamma_{xy}, \gamma_{xz}$ and γ_{yz}.

$$\varepsilon_x = \frac{du}{dy} \equiv \left(-\frac{zd^2w}{dx^2} + \frac{Fd\theta_{Sx}}{dx}\right) \qquad 3$$

That is:

$$\varepsilon_x = [-A_1 z + A_2 F(z)]\frac{d^2h}{dx^2} \qquad 4$$

$$\sigma_x = \frac{E}{1-\mu^2}\left[[-A_1 z + A_2 F(z)]\frac{d^2h}{dx^2} + \mu[-A_1 z + A_3 F(z)]\frac{d^2h}{dy^2}\right] \qquad 5$$

Similarly reasoning in y direction, we shall obtain:

$$\varepsilon_y = \frac{dv}{dy} \equiv \left(-\frac{zd^2w}{dy^2} + \frac{Fd\theta_{Sy}}{dy}\right) \qquad 6$$

That is:

$$\varepsilon_y = [-A_1 z + A_3 F(z)]\frac{d^2h}{dy^2} \qquad 7$$

$$\gamma_{xy} = 2\frac{z\partial^2 w}{\partial x\partial y} + F\left(\frac{d\theta_{Sx}}{dy} + \frac{Fd\theta_{Sy}}{dx}\right) \qquad 8$$

That is:

$$\gamma_{xy} = \frac{du}{dy} + \frac{dv}{dx} \equiv [-2A_1 z + A_2 F(z) + A_3 F(z)]\frac{d^2h}{\partial x\partial y} \qquad 9$$

$$\gamma_{xz} = \frac{dF}{dz}.\theta_{Sx} \qquad 10$$

That is:

$$\gamma_{xz} = A_2\frac{dF(z)}{dz}\frac{dh}{dx} \qquad 11$$

$$\gamma_{yz} = \frac{dF}{dz}.\theta_{Sy} \qquad 12$$

That is:

$$\gamma_{yz} = A_3 \frac{dF(z)}{dz}\frac{dh}{dy} \qquad 13$$

D. CONSTITUTIVE RELATIONS

In the constitutive equations, there two inplane and three transverse stressess:

$$\sigma_x = \frac{Ez}{1-\mu^2}\left(\left(\frac{-d^2w}{dx^2} + \frac{Fd\theta_{Sx}}{dx}\right) + \mu\left(\frac{d^2w}{dy^2} + \frac{Fd\theta_{Sx}}{dy}\right)\right) \qquad 14$$

That is:

$$\sigma_x = \frac{E}{1-\mu^2}\left[[-A_1z + A_2F(z)]\frac{d^2h}{dx^2} + \mu[-A_1z + A_3F(z)]\frac{d^2h}{dy^2}\right] \qquad 15$$

$$\sigma_y = \frac{Ez}{1-\mu^2}\left(\left(-\frac{d^2w}{dy^2} + \frac{Fd\theta_{Sx}}{dx}\right) + \mu\left(\frac{d^2w}{dx^2} + \frac{Fd\theta_{Sx}}{dy}\right)\right) \qquad 16$$

That is:

$$\sigma_y = \frac{E}{1-\mu^2}\left[\mu[-A_1z + A_2F(z)]\frac{d^2h}{dx^2} + [-A_1z + A_3F(z)]\frac{d^2h}{dy^2}\right] \qquad 17$$

Also, from known state Equation,

$$\tau_{xy} = \frac{E(1-\mu)}{(1-\mu^2)}\left(-\frac{z\partial^2w}{\partial x\partial y} + F\left(\frac{d\theta_{Sx}}{dy} + \frac{d\theta_{Sy}}{dx}\right)\right) \qquad 18$$

That is:

$$\tau_{xy} = \frac{Ez}{2(1-\mu^2)}[-2A_1z + A_2F(z) + A_3F(z)]\frac{d^2h}{\partial x\partial y} \qquad 19$$

$$\tau_{xz} = \frac{E}{2(1+\mu)}.\gamma_{xz} \qquad 20$$

$$\tau_{xzm} = \frac{Ez(1-\mu)}{2(1-\mu^2)}A_2\frac{dF(z)}{dz}\frac{dh}{dx} \qquad 21$$

Also;

$$\tau_{yz} = \frac{E}{2(1+\mu)}.\gamma_{yz} \qquad 22$$

That is:

$$\tau_{yzm} = \frac{Ez(1-\mu)}{2(1-\mu^2)}A_3\frac{dF(z)}{dz}\frac{dh}{dy} \qquad 23$$

Where τ_{xzm} and τ_{yzm} = Maximum vertical shear stresses.

E. TOTAL POTENTIAL ENERGY

Total potential energy is the summation of strain energy, U and external work, V. that's

$$\Pi = U + V \qquad 21$$

Let's define external work as:

$$V = -q\, wyxdxdy \qquad 22$$

Therefore, the non-dimensional form of total potential energy equation for a thick plate of traditional third order shear deformation theory of R and Q coordinates at the span-span aspect ratio, $\propto$ is given as:

Where the span-depth aspect ratio as:

$$\rho = \frac{a}{t}, R = \frac{x}{a}, y = bQ$$

$$\Pi = \frac{D}{2}\int_0^a\int_0^b \left[\left|g_1{A_1}^2\left(\frac{\partial^2 h}{\partial x^2}\right)^2 - 2g_2A_1A_2\left(\frac{\partial^2 h}{\partial x^2}.\frac{\partial^2 h}{\partial x^2}\right) + g_3{A_2}^2\left(\frac{\partial^2 h}{\partial x^2}\right)^2\right| + \left|2g_1\,{A_1}^2\left(\frac{\partial^2 h}{\partial x\partial y}\right)^2 - 2g_2A_1A_2\left(\frac{\partial^2 h}{\partial x\partial y}.\frac{\partial^2 h}{\partial x\partial y}\right) - 2g_2A_1A_3\left(\frac{\partial^2 h}{\partial x\partial y}.\frac{\partial^2 h}{\partial x\partial y}\right)\right| + \left|(1+\mu)g_3A_2A_3\left(\frac{\partial^2 h}{\partial x\partial y}\right)\left(\frac{\partial^2 h}{\partial x\partial y}\right)\right| + \frac{(1-\mu)}{2}\left|g_3{A_2}^2\left(\frac{\partial^2 h}{\partial x\partial y}\right)^2 + g_3{A_3}^2\left(\frac{\partial^2 h}{\partial x\partial y}\right)^2\right| + \left|g_1{A_1}^2\left(\frac{\partial^2 h}{\partial y^2}\right)^2 - 2g_2A_1A_3\left(\frac{\partial^2 h}{\partial y^2}.\frac{\partial^2 h}{\partial y^2}\right) + g_3{A_3}^2\left(\frac{\partial^2 h}{\partial y^2}\right)^2\right| + \left|\frac{(1-\mu)}{2}g_4{A_2}^2\left(\frac{\partial h}{\partial x}\right)^2 + \frac{(1-\mu)}{2}g_4{A_3}^2\left(\frac{\partial h}{\partial y}\right)^2\right|\right]\partial x\partial y - \int_0^a\int_0^b qA_1h\,\partial x\partial y \quad 23$$

F. GENERAL GOVERNING EQUATIONS

The general polynomial deflection function deflection equation of a rectangular plate is defined as (see deflection equation used in Ibearugbulem et al, 2016):

$$w = (a_0 + a_1R + a_2R^2 + a_3R^3 + a_4R^4) \times (b_0 + b_1Q + b_2Q^2 + b_3Q^3 + b_4Q^4) \quad 24$$

G. CRITICAL LOAD BEFORE DEFLECTION REACHES ALLOWABLE LIMIT

The maximum critical thickness on the plate before its deflection reaches allowable value will be determined. This is to ensure that deflection does not exceed specified maximum limit.
To ensure that the critical thickness of the plate is determined before its deflection reaches allowable value;

$$w = A_1\mathrm{h} < \mathrm{w_a} \quad 25$$

That is:

$$w = \frac{qa^4}{D}.k.\mathrm{h} < \mathrm{w_a} \quad 26$$

But;

$$D = \frac{Et^3}{12(1-\mu^2)}$$

Therefore:

$$w = \frac{qa^4}{\frac{Et^3}{12(1-\mu^2)}}.k.\mathrm{h} < \mathrm{w_a} \equiv \frac{12(1-\mu^2)qa^4}{Et^3}.k.\mathrm{h} < \mathrm{w_a} \quad 27$$

Also,

$$q = q_d + q_i \quad 28$$

Putting equation 28 into 27, we have:

$$\frac{12(1-\mu^2)(q_d + q_i)a^4}{Et^3}.k.\mathrm{h} < \mathrm{w_a} \quad 29$$

That is:

$$(q_d + q_i) < \frac{\mathrm{w_a}\,Et^3}{12(1-\mu^2)a^4.k.\mathrm{h}} \quad 30$$

That is:

$$q_i < \left[\frac{\mathrm{w_a}\,Et^3}{12(1-\mu^2)a^4.k.\mathrm{h}}\right] - q_d \quad 31$$

But;

$$q_d = \gamma t \quad 32$$

Where;

$q_d = Self\ weight$ of the plate

$q_i =$ Imposed load of the plate

$\gamma =$ Weight of the plate
$t =$ Thickness of the plate
$\mathrm{w_a} =$ Allowable deflection $= 15$ (BS 8110)
Thus:

$$q_{\,i} < \left[\frac{\mathrm{w_a}\, Et^3}{12(1-\mu^2)a^4.k.\mathrm{h}}\right] - \gamma t \qquad 33$$

Where;

$$\beta = \left[\frac{\mathrm{w_a}\, E}{12(1-\mu^2)a^4.k.\mathrm{h}}\right] \qquad 34$$

Therefore:

$$q_{\,i} < \beta t^3 - \gamma t \qquad 35$$

Rearranging equation 35, we have:

$$\beta t^3 - \gamma t - q_{\,i} > 0 \qquad 36$$

H. DIRECT VARIATION OF TOTAL POTENTIAL ENERGY

This total potential energy contains three unknown coefficients (A_1, A_2 and A_3) for deflection, rotation in x axis and rotation in y axis. In minimization, when the differentiation is done with respect to the coefficient of the displacement, the result is called the direct governing equation. Here, the total potential energy shall be minimized with respect to the coefficient of the deflection, shear deformation along x axis and shear deformation along y axis; A_1, A_2 and A_3. Minimizing or differentiating total potential energy equation with respect to A_1, A_2 and A_3 is said to be the direct variation.
Therefore, by differentiating equation 23 with respect to A_1, A_2 and A_3 is said to be the direct variation we have;

$$\frac{\partial\Pi}{\partial A_1} = \frac{\partial\Pi}{\partial A_2} = \frac{\partial\Pi}{\partial A_3} = 0 \qquad 37$$

$$\begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}\begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix} = \frac{qa^4}{D}\begin{bmatrix} k_q \\ 0 \\ 0 \end{bmatrix} \qquad 38$$

Where;

$$r_{11} = g_1\left(k_1 + \frac{2}{\propto^2}k_2 + \frac{1}{\propto^4}k_3\right) \qquad 39$$

$$r_{12} = -g_2\left(k_1 + \frac{1}{\propto^2}k_2\right) \qquad 40$$

$$r_{13} = -g_2\left(\frac{1}{\propto^2}k_2 + \frac{1}{\propto^4}k_3\right) \qquad 41$$

$$r_{21} = -g_2\left(k_1 + \frac{1}{\propto^2}k_2\right) \qquad 42$$

$$r_{22} = \left(g_3k_1 + \frac{(1-\mu)}{2\propto^2}g_3k_2 + \frac{(1-\mu)}{2}\rho^2 g_4k_4\right) \qquad 43$$

$$r_{23} = g_3\frac{(1+\mu)}{2\propto^2}k_2 \qquad 44$$

$$r_{31} = -g_2\left(\frac{1}{\propto^2}k_2 + \frac{1}{\propto^4}k_3\right) \qquad 45$$

$$r_{32} = g_3\frac{(1+\mu)}{2\propto^2}k_2 \qquad 46$$

$$r_{33} = \left(g_3\frac{(1-\mu)}{2}\left(\frac{1}{\propto^2}k_2 + \frac{1}{\propto^4}k_3\right) + g_4\frac{(1-\mu)}{2\propto^2}\rho^2k_5\right) \qquad 47$$

And;

$$k_1 = \int_0^1\int_0^1\left(\frac{d^2h}{dR^2}\right)^2 dRdQ \qquad 48$$

$$k_2 = \int_0^1 \int_0^1 \left(\frac{d^2h}{dRdQ}\right)^2 dRdQ \quad 49$$

$$k_3 = \int_0^1 \int_0^1 \left(\frac{d^2h}{dQ^2}\right)^2 dRdQ \quad 50$$

$$k_4 = \int_0^1 \int_0^1 \left(\frac{dh}{dR}\right)^2 dRdQ \quad 51$$

$$k_5 = \int_0^1 \int_0^1 \left(\frac{dh}{dQ}\right)^2 dRdQ \quad 52$$

$$k_6 = \int_0^1 \int_0^1 h.dRdQ \quad 53$$

By solving equation 38, we have;

Shear deformation profile of the thick rectangular section of plate used in this study is

$$F(z) = \frac{3}{2}\left(z - 4\frac{z^3}{t^3}\right) \text{ (Okafor et al, 2018)}$$

$g_1 = 1$; $g_2 = 1.2$; $g_3 = 1.33$; $g_4 = 14.4$

By solving equation 33, we got:

$$k_T = k_1 + \frac{2}{\propto^2}k_2 + \frac{1}{\propto^4}k_3 \quad 54$$

And;

$$k_6 = k_{rq} + k_{rN} + k_{rl} \quad 55$$

Therefore:

$$k_{rq} = \frac{q}{A_1}\int_0^1 \int_0^1 h.dRdQ \equiv \frac{q}{A_1}k_q \quad 56$$

$$k_{rq} = \frac{qa^4}{D}k_6 \quad 57$$

But from the matrix above, we got:

$$A_1 = \frac{qa^4}{D}\left(\frac{k_q}{r_{11}U_1 - r_{12}U_2 - r_{13}U_3}\right) \quad 58$$

Let:

$$\overline{A_1} = \left(\frac{k_6}{r_{11}U_1 - r_{12}U_2 - r_{13}U_3}\right) \quad 59$$

That is:

$$A_1 = \overline{A_1}\left(\frac{qa^4}{D}\right) \quad 60$$

Similarly;

$$A_2 = \overline{A_1}\left(\frac{qa^4}{D}\right) \quad 61$$

Similarly;

$$A_3 = \overline{A_3}\left(\frac{qa^4}{D}\right) \quad 62$$

I. DEFINITION OF SOME QUANTITIES

Expressing equation 3 to 20 in the form of non-dimensional form, therefore the displacements of plate under pure bending then be defined as:

$$w = \overline{A_1}h\left(\frac{qa^4}{D}\right) \quad 63$$

From equation 33,

$$q_i < \left[\frac{Et^3}{12(1-\mu^2)a^4}\left(\frac{\mathrm{w_a}}{k.\mathrm{h}}\right)\right] - \gamma t \quad 64$$

That is:

$$q_{\,i} < \left[\frac{D}{a^4}\left(\frac{\mathrm{w_a}}{k.\,\mathrm{h}}\right)\right] - \gamma t \qquad 65$$

Where;

$$q_{\,d} = \gamma t \equiv t = \frac{q_{\,d}}{\gamma} \qquad 66$$

Therefore:

$$\frac{q_{\,i}}{\gamma} < \left[\frac{D}{\gamma a^4}\left(\frac{\mathrm{w_a}}{k.\,\mathrm{h}}\right)\right] - t \qquad 67$$

That is:

$$t_{cri} < \left[\frac{D}{\gamma a^4}\left(\frac{\mathrm{w_a}}{k.\,\mathrm{h}}\right)\right] - \frac{q_{\,i}}{\gamma} \qquad 68$$

That is:

$$t_{cri} < \left[\frac{D}{\gamma a^4}\left(\frac{\mathrm{w_a}}{\overline{A_1}.\,\mathrm{h}}\right)\right] - \frac{q_{\,i}}{\gamma} \qquad 69$$

That is:

$$t_{cri} < \left[\bar{t}\,\frac{D}{\gamma a^4}\right] - \frac{q_{\,i}}{\gamma} \qquad 70$$

Where;

$$\bar{t} = \frac{\mathrm{w_a}}{\overline{A_1}.\,\mathrm{h}} \qquad 71$$

J. NUMERICAL PROBLEM

Determine the deflection at the center of CCSS thick plate. Determine also the critical thickness at the center of the plate. Polynomial displacement function shall be used. The polynomial displacement function, h is given as: $h = (R - 2R^3 + R^4) \times (Q - 2Q^3 + Q^4)$ and $h = (1.5R^2 - 2.5R^3 + R^4) \times (1.5Q^2 - 2.5Q^3 + Q^4)$. The k values herein are given as: $k_1 = 0.2361904761, k_2 0.2359183673, k_3 = 0.2361904761, k_4 = 0.0239002267$, $k_5 = 0.0239002267, k_q = 0.04$ *and* $k_1 = 0.013571428, k_2 = 0.0073469387$, $k_3 = 0.013571428, k_4 = 0.0006462585, k_5 = 0.0006462585, k_q = 0.005625$ respectively

K. RESULTS AND DISCUSSIONS

For thick plates, Kircchopps assumptions of CPT will not be reliable. To improve on CPT, refined plate theories (RPT) evolved. Refined theory that was used here is third order shear deformation theory which was built upon classical plate theory, in the thick-ness coordinate are examined and presented parabolic shear deformation theory with nonlinear variation of inplane displacement and quadratic variation of transverse displacement in thickness coordinate.

A close look at tables 1 to 2 reveals that the values of deflection and length-width ratio increases as the load and span-depth ratio decrease respectively/verse versa. The design factors $\overline{\mathrm{w}}$ for deflection and shear deformation rotation along x coordinate and y coordinate respectively of rectangular plates at varying aspect ratio, ∝ and ρ, are obtained using equations obtained from this study (Tables 1 and 2 as given in the previous sections at the center of the plate). All the work mentioned above and other literature actually aimed at obtaining the displacement (using the assumed displacement functions); maximum deflections, internal forces, moments and shear deformation of the rectangular thick plate, but no work have been done on the limit state analysis of thick rectangular plate to get the actual maximum critical lateral load of the plate before deflection reaches the specified maximum limit Also, much attention has not been given to the study of plate analysis using exact solution approach to determine the actual displacement functions (from the first principle, without assuming displacement functions). This work is very relevant and can be used as a guide in engineering design and construction to avoid structural failure or consequently collapse by ensuring that deflection did not exceed allowable value. It can also be used as a check to achieve most possible economical design in a structure.

Table 1: Critical load before deflection reaches allowable value of SSSS plate for a/t = 4.0

$\propto = \frac{b}{a}$	$A_1 = \overline{A_1}\left(\frac{qb^4}{D}\right)$	$A_2 = \overline{A_2}\left(\frac{qb^4}{D}\right)$	$A_3 = \overline{A_3}\left(\frac{qb^4}{D}\right)$	$w = \overline{w}\left(\frac{qb^4}{D}\right)$	$t_{cri} < \left[\bar{t}\frac{D}{\gamma a^4}\right] - \frac{q_i}{\gamma}$
	$\overline{A_1}$	$\overline{A_2}$	$\overline{A_3}$	$\overline{w}$	$\overline{q_i}$
1	0.21575487	-0.06775304	-0.06775304	0.00337117	4449.494039
1.1	0.2472073	-0.07341237	-0.07023386	0.003862614	3883.380482
1.2	0.28106543	-0.08223772	-0.07542865	0.004391647	3415.574774
1.3	0.30992342	-0.08826042	-0.07790391	0.004842553	3097.539384
1.4	0.33595072	-0.09350837	-0.07968938	0.00524923	2857.561929
1.5	0.35919843	-0.09805226	-0.08093508	0.005612475	2672.617502
1.6	0.37983701	-0.10197395	-0.08176808	0.005934953	2527.399863
1.7	0.39809686	-0.10535571	-0.08229118	0.006220263	2411.473416
1.8	0.41422902	-0.10827427	-0.08258511	0.006472328	2317.558539
1.9	0.42848141	-0.11079813	-0.08271188	0.006695022	2240.470577
2	0.44108594	-0.11298683	-0.0827185	0.006891968	2176.446591

Table 2: Critical load before deflection reaches allowable value of CCSS plate for a/t = 5.0

$\propto = \frac{b}{a}$	$A_1 = \overline{A_1}\left(\frac{qb^4}{D}\right)$	$A_2 = \overline{A_2}\left(\frac{qb^4}{D}\right)$	$A_3 = \overline{A_3}\left(\frac{qb^4}{D}\right)$	$w = \overline{w}\left(\frac{qb^4}{D}\right)$	$t_{cri} < \left[\bar{t}\frac{D}{\gamma a^4}\right] - \frac{q_i}{\gamma}$
	$\overline{A_1}$	$\overline{A_2}$	$\overline{A_3}$	$\overline{w}$	$\overline{q_i}$
1	0.185634408	0.042652653	0.042652653	0.002900538	5171.455072
1.1	0.215802486	0.046810322	0.04461552	0.003371914	4448.512231
1.2	0.245824856	0.0520293	0.047410327	0.003841013	3905.219413
1.3	0.272457992	0.055907114	0.048912189	0.004257156	3523.478946
1.4	0.296476069	0.059274369	0.04997763	0.004632439	3238.035382
1.5	0.317919366	0.062180422	0.050705683	0.00496749	3019.633598
1.6	0.336944449	0.064681168	0.051178844	0.005264757	2849.134339
1.7	0.353765805	0.066831963	0.051462918	0.005527591	2713.659673
1.8	0.36861772	0.068683884	0.051608897	0.005759652	2604.324068
1.9	0.381731489	0.070282142	0.051655516	0.005964555	2514.85672
2	0.393323141	0.071665749	0.051631793	0.006145674	2440.741211

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