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Stress Field in Functionally Graded Cylindrical Structures Under Thermo-Mechanical Loading

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Abstract

By using composite Cylinders Assemblage (CCA) model for a three-phase system consisting of a metallic and ceramic phase joined together by an FGM phase of arbitrary composition (gradation), a generalized formulation for the micro stress analysis of functionally graded cylindrical structure under axisymmetric mechanical and/or thermal loading is presented.FGM phase is divided into an arbitrary number (n) of concentric Cylinder (phase) between metal and ceramics phases. Each of the n concentric FGM phases has different properties in proportion to the average volume fractions of the constituents in that phase. All the phases are assumed to be transversely isotropic with respect to their stiffness and thermal expansion coefficient. Temperature field can be different in each phase, but here only a uniform field has been considered. The formulation presented is expected to be useful in the analysis of cylindrical FGM structure subjected to thermo-mechanical loading.

Introduction

In engineering applications, individual elements are exposed to local loading conditions, which can vary greatly with location. In cases, we use different materials such as metal and ceramic. The abrupt transition in microstructure and composition between the different materials often result in high residual stress regions and local stress concentrations which can lead to the subsequent nucleation of micro cracks at or near the biomaterial interface. The intensity existence of the stress concentration effects due to large dissimilarity in properties can be substantially reduced if the microstructure is gradually varied from that of the metal to that of ceramic. Relatively new classes of materials known as functionally graded materials (FGMs) have been developed to minimize such property mismatch effects.

The composition and of an FGM varies with location, resulting in spatially dependent properties. The composition profile can take different forms depending upon the required performance. These materials have potential to enjoy a wide range of thermal and structural applications, including thermal gradient structures, wear and corrosion resistant coatings and metal ceramic joining.

Compositions of FGMs are continuous or step-wise. FGM is not the one made simply by affixing different materials of different characteristics. FGM can be made by continuously inter-mixing several materials without generating a boundary. Thus, if was named for this feature of the gradual changes (gradient) of functions. The changes can be made to go in any direction, along thickness or along the plane. Properties may be quite different from one face to the other face of the material. For example, in a metal-ceramic FGM, the metal rich side is typically placed in the regions where mechanical properties, such as toughness, need to be high. In contrast, the ceramic-rich side, which has lower thermal conductivity and can withstand high temperatures, is placed in regions where there are potentially sharp temperature gradients.

The advantages of using FGM as an alternative to two dissimilar materials joined directly together include: smoothing of thermal stress distributions across the layers, minimization or elimination of stress concentrations, increase in biomaterial bonding strength, and improved fracture toughness compared to that of monolithic ceramics, as a result of the plastic deformation of the metallic phase.

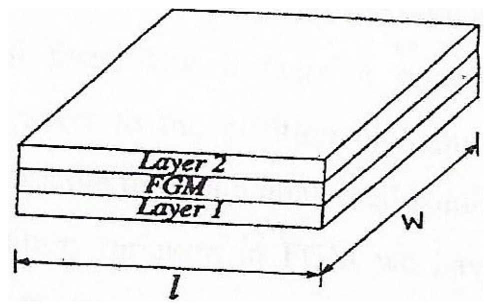


Fig.1.Schematic view of three-layered model

Applications of FGM and the Present Works:

In case of an artificial implant for a bone, its design changes from a dense, stiff external structure (the cortical bone) to a porous internal one (the chancellors bone). An artificial Hip joint is an ideal example of this kind of structure. Early methods of correcting the diseased or fractured hip joints only involved the ace tabular cup or femoral head. One technique of restoring hip joint function is to place a cup over the femoral head while the surface of the acetabulum is also respected to fit the cup. The implant serves as a mold interposing the two surfaces, which eventually adapt according to the function of the joint. The wide variety of implants reflects the limited knowledge of the function of the joints and the ability of the joint to accommodate any insult imposed upon it by the various implants. Most femoral head replacements are performed with the installation of an ace tabular cup. This is the so-called hip joint replacement, which is frequently performed bilaterally. The various types of hip implants can be grouped into ball and socket, retained ball and socket, turning bearing, and floating acetabulum and double cup.

The single most difficult problem of hip joint as well as other joint replacements is fixation of the implants. This is due to the fact that the implant lies on the cancellers bone, which has few trabeculae to support the large load imposed. Also, the stress concentration of the implant at point of sharp contact, such as the cal car region and the end of the femoral stem, makes the already weakened bone more necrotic. So for this purpose we use bone cement as a fixation device for the implants.

Materials with changing composition, microstructure, or porosity across the volume of the material are referred to as the functionally graded material (FGM) .Functionally graded materials (FGMs) are designed with changing properties over the volume of the bulk material, with the aim of performing a set of specified functions . The properties of material in FGMs are not uniform across the entire material, and the properties depend on the spatial position of the material in the bulk Structure of the material. Functionally graded materials are designed with varying properties that include changing chemical properties, changing mechanical, magnetic, thermal, and electrical properties. There are FGMs that are designed as stepwise-graded structures, and some are designed to be continuous-graded structures, depending on the areas of application .There are different types of areas, in which FGMs are now being used that are different from the initial area of application, for which the material was invented .In this chapter, the different types of FGMs and their areas of application are presented. The different types of FGMs include porosity and pore size gradient structured FGMs, chemical gradient-structured FGMs, and micro structural gradient-structured FGMs. These different types of functionally graded materials are presented in the next sections.

Types of Functionally Graded Materials

- Chemical Composition Gradient Functionally Gradient Materials
- Porosity Gradient Functionally Gradient Materials
- Microstructure Gradient Functionally Gradient Materials

Problem Description

A cylindrical structure containing a functionally graded phase sandwiched between a metallic and a ceramic phase has been considered. The composition and microstructure of an FGM varies with location, resulting in spatially dependent properties. The composition profile can take different forms depending upon the required performance. It may be of linear, cubic, parabolic or any arbitrarily varying profile.Analytical solutions are possible only for the linear composition profile or any other profile which can be expressed in functional analytical form. But if there is an abrupt change in the composition with respect to the position or some arbitrary variations exist then it is not possible to obtain analytical solution. So for this kind of arbitrary composition variation in FGM we have developed a new algorithm using a concentric cylindrical assemblage (CCA) model. In this approach we assume the FGM phase as consisting of a number of concentric layers, each one having distinct value of volume fraction. Mechanical properties such as elastic modulus, coefficient of thermal expansion, modulus of rigidity, all depends upon the volume fraction. So each concentric layer of FGM phase has also distinct values of mechanical properties. Thus, the problem is converted into an assemblage of a number of concentric cylinders each with distinct mechanical and thermal property, and bonded perfectly to each other.

Problem Formulation

As mentioned earlier, formulation procedure is given for a Cylindrical FGM structure here a metallic and a ceramic Phase is joined together by an FGM phase of arbitrary profile. It is assumed that FGM phase has arbitrary compositional gradation along the radial direction. Its composition may be linear, cubic, parabolic, and quadratic or whatever the design problem would need. We assume that FGM phase consists of a number of concentric layers, each one having distinct value of volume fraction. Mechanical properties such as elastic modulus E, coefficient of thermal of thermal expansion $\propto$, modulus in FGM. So each concentric layer of FGM phase has also distinct values of mechanical properties. So it is considered that more than one type of concentric phases exists between the central core and the outermost material.

In the CCA model used here, we have taken total P number of concentric circular cylindrical layers. The different phases are: (i) outermost cylinder with the properties of ceramic, (ii) FGM phase having number of concentric layers, (iii) innermost metallic cylinder. These phases are denoted by domain 1, 2, 3… P respectively, and their limiting outermost radii by r_1, $r_{2,}r_3$, …………r_p, respectively.

To make the formulation more general, all these phases, confined between the cylinders, are assumed to be transversely isotropic with regard to their stiffness and thermal expansion coefficient. The (t,θ) plane, which is also the cross-sectional plane of the cylindrical system represents this transverse plane of isotropy.

Basic Assumptions:

Apart from taking all the phases to be transversely isotropic they are taken to be linearly isotropic. It is also assumed that

- No voids exist in any of the phases,
- A perfect bond exists at the common boundary of the phases,

Boundary Conditions:

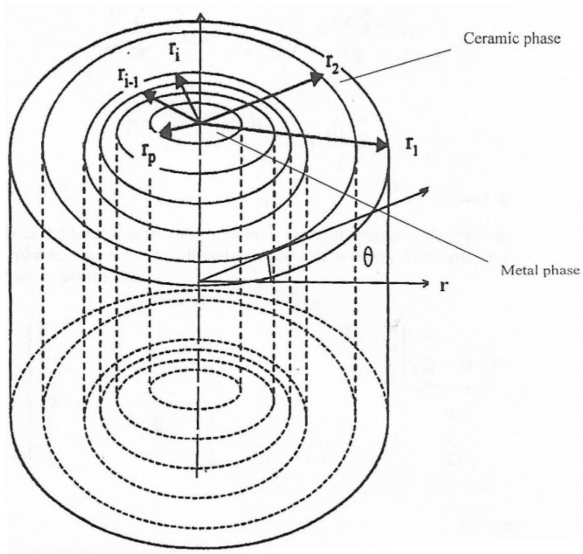
The entire system is subjected to three independent boundary conditions;

- Uniaxial applied stress $\sigma_{oz,}$
- Biaxial applied stress $\sigma_{or,}$and
- Axisymmetric temperature change ΔT

Where r and z are respectively, the radial and axial directions referred to cylindrical coordinates (r, θ,z) shown in Fig.6.

Analytical Derivation:

Referring to Fig.6, for any domain (say nth phase/domain), the stress (σ_{ij}) equilibrium equations may be written as



$$\frac{1}{r}\frac{\partial}{\partial r}(r\sigma_{rr}^{n}) + \frac{1}{r}\frac{\partial \sigma_{r\theta}^{n}}{\partial \theta} + \frac{\partial \sigma_{rz}^{n}}{\partial z} - \frac{\partial \sigma_{\theta\theta}^{n}}{r} = 0$$

$$\frac{1}{r}\frac{\partial}{\partial r}(r^{2}\sigma_{\theta r}^{n}) + \frac{1}{r}\frac{\partial \sigma_{\theta\theta}^{n}}{\partial \theta} + \frac{\partial \sigma_{\theta z}^{n}}{\partial z} = 0$$

$$\frac{1}{r}\frac{\partial}{\partial r}(r\sigma_{zr}^{n}) + \frac{1}{r}\frac{\partial \sigma_{z\theta}^{n}}{\partial \theta} + \frac{\partial \sigma_{zz}^{n}}{\partial z} = 0 \quad \text{………………} (1)$$

Throughout the text, a superscript n or n in the subscript will represent the nth phase. Treating the domain to be transversely isotropic, the stress-strain relations are

$$\begin{bmatrix}\sigma_{rr}^{n}\\ \sigma_{\theta\theta}^{n}\\ \sigma_{zz}^{n}\\ \sigma_{\theta z}^{n}\\ \sigma_{zr}^{n}\\ \sigma_{r\theta}^{n}\end{bmatrix} = \begin{bmatrix} c_{11}^{n} & c_{12}^{n} & c_{13}^{n} & 0 & 0 & 0\\ c_{12}^{n} & c_{11}^{n} & c_{13}^{n} & 0 & 0 & 0\\ c_{13}^{n} & c_{13}^{n} & c_{33}^{n} & 0 & 0 & 0\\ 0 & 0 & 0 & c_{44}^{n} & 0 & 0\\ 0 & 0 & 0 & 0 & c_{44}^{n} & 0\\ 0 & 0 & 0 & 0 & 0 & \frac{c_{11}^{n}-c_{12}^{n}}{2}\end{bmatrix} \begin{bmatrix} e_{rr}^{n} - \alpha_{nT}^{n}T_{n}\\ e_{\theta\theta}^{n} - \alpha_{nT}^{n}T_{n}\\ e_{zz}^{n} - \alpha_{nT}^{n}T_{n}\\ 2e_{\theta z}^{n}\\ 2e_{zr}^{n}\\ 2e_{r\theta}^{n}\end{bmatrix}$$

This can alternatively be written in the form

$$\sigma_{rr}^{n} = C_{11}^{n}e_{rr}^{n} + C_{12}^{n}e_{\theta\theta}^{n} + C_{13}^{n}e_{zz}^{n} - \gamma_{1}^{n}T_{n}$$

$$\sigma_{\theta\theta}^{n} = C_{12}^{n}e_{rr}^{n} + C_{11}^{n}e_{\theta\theta}^{n} + C_{13}^{n}e_{zz}^{n} - \gamma_{1}^{n}T_{n}$$

$$\sigma_{zz}^{n} = C_{13}^{n}e_{rr}^{n} + C_{13}^{n}e_{\theta\theta}^{n} + C_{33}^{n}e_{zz}^{n} - \gamma_{3}^{n}T_{n}$$

$$\sigma_{\theta z}^{n} = 2C_{44}^{n} e_{\theta z}^{n}$$

$$\sigma_{zr}^{n} = 2C_{44}^{n} 2e_{zr}^{n}$$

$$\sigma_{r\theta}^{n} = (C_{11}^{n} - C_{12}^{n}) e_{r\theta}^{n} \qquad \text{……….. (2)}$$

Where

$$\gamma_{1}^{n} = (C_{11}^{n} + C_{12}^{n}) \alpha_{nT}^{n} + C_{13}^{n} \alpha_{nL}^{n}$$

$$\gamma_{3}^{n} = 2C_{13}^{n} \alpha_{nT}^{n} + C_{33}^{n} \alpha_{nL}^{n} \qquad \text{….. (3)}$$

e_{ij} and C_{ij} and are respectively, strain components and the elastic constants. $\propto_{nT}$ and $\propto_{nL}$ are the coefficient of thermal expansion in transverse and longitudinal direction respectively. Due to ax symmetry, the displacement field (u_i) in the n-th domain can be expressed as

$$u_{r}^{n} = u_{n}(r)$$

$$u_{\theta}^{n} = 0$$

$$u_{z}^{n} = w_{n}(z) \qquad \text{……………. (4)}$$

This consequently gives

$$e_{rr}^{n} = \frac{\partial u_{n}}{\partial r}$$

$$e_{\theta\theta}^{n} = \frac{u_{n}}{r}$$

$$e_{zz}^{n} = \frac{\partial w_{n}}{\partial z}, \qquad \text{and}$$

$$e_{zr}^{n} = e_{\theta z}^{n} = e_{r\theta}^{n} \qquad \text{… ……….. (5)}$$

The stress-strain relations expressed in Equations (2) are now written, using equation (5), as

$$\sigma_{zz}^{n} = C_{13}^{n} \frac{\partial u_{n}}{\partial r} + C_{13}^{n} \frac{u_{n}}{r} + C_{33}^{n} \frac{\partial w_{n}}{\partial z} - \gamma_{3}^{n} T_{n}$$

$$\sigma_{rr}^{n} = C_{11}^{n} \frac{\partial u_{n}}{\partial r} + C_{12}^{n} \frac{u_{n}}{r} + C_{13}^{n} \frac{\partial w_{n}}{\partial z} - \gamma_{1}^{n} T_{n}$$

$$\sigma_{\theta\theta}^{n} = C_{12}^{n} \frac{\partial u_{n}}{\partial r} + C_{11}^{n} \frac{u_{n}}{r} + C_{13}^{n} \frac{\partial w_{n}}{\partial z} - \gamma_{1}^{n} T_{n}$$

$$\sigma_{rz}^{n} = 0, \qquad \sigma_{\theta z}^{n} = 0 \quad , \qquad \sigma_{r\theta}^{n} = 0$$

…………….. (6)

From equation (1) and (6), the governing differential equations are then obtained in terms of displacements in the form:

$$\frac{\partial^2 u_n}{\partial r^2} + \frac{1}{r}\frac{\partial u_n}{\partial r} - \frac{u_n}{r^2} = K_n \frac{\partial T_n}{\partial r} \qquad \text{………… (7)}$$

$$\frac{\partial^2 w_n}{\partial z^2} = 0 \qquad \text{… ……. (8)}$$

With

$$K_n = \gamma_1^n / C_{11}^n$$

The boundary conditions and continuity conditions of stresses and displacements at the interfaces of the concentric cylinders (phases) are given as the following

$$\sigma_{rr}^1 = \sigma_{0r} \qquad r = r_1 \qquad \dots\dots(9)$$

$$\sigma_{rr}^2 = \sigma_{rr}^1, u_1 = u_2, w_1 = w_2 \qquad at \qquad r = r_2 \qquad \dots.(10)$$

$$\sigma_{rr}^3 = \sigma_{rr}^2, u_2 = u_3, w_2 = w_3 \qquad at \qquad r = r_3 \qquad \dots.(11)$$

$$\sigma_{rr}^i = \sigma_{rr}^{i+1}, u_i = u_{i+1}, w_i = w_{i+1} \qquad at \quad r = r_{i+1} \quad for \quad i = 1\ to\ (p-1) \qquad \dots.(12)$$

$$\int_0^p \sigma\,{}^p_{zz}\, r\partial r + \int_{r_p}^{r_{p-1}} \sigma_{zz}^{p-1}\, r\partial r + \cdots\dots\dots + \int_{r_i}^{r_{i-1}} \sigma_{zz}^{i-1}\, r\partial r + \cdots\dots. + \int_{r_2}^{r_1} \sigma_{zz}^1\, r\partial r = \int_0^{r_1} \sigma_{0z}\, r\partial r \qquad \dots\dots(13)$$

General solution to equation number (7) and (8) can be given as;

$$u_n(r) = A_n r + \frac{B_n}{r} + rS_n(r) + \frac{1}{r}D_n(r) \qquad \text{………………. (14)}$$

$$w_n(z) = E_n z + F_n \qquad \dots\dots\dots\dots\dots\dots(15)$$

Where A_n, B_n, E_n and F_n are unknown constants to be determined by utilizing the boundary conditions, and

$$f_n(r) = K_n \frac{\partial T_n}{\partial r}$$

$$S_n(r) = \frac{1}{2}\int f_n(r)\partial r,$$

$$D_n(r) = -\frac{1}{2} r^2 f_n(r)\partial r$$

It may be noted that in case of Tn being a constant (i.e., not a function of r) over the domain (phase) n, fn will be zero and consequently, S_n (r) and D_n (r) also become zero.

The constant F_n in equation (15) can be made zero because a constant value of it would mean a rigid body displacement in the z-direction, i.e,

$$F_1 = F_2 = \cdots \dots \dots \dots \dots \dots . = F_P = 0$$

Now by using the boundary conditions, i.e. the condition of continuity in equation number (15), gives

$$E_1 = E_2 = E_3 = \dots\dots = E_P = E$$

Therefore, equation (15) yields

$$w_1(z) = w_2(z) = \dots\dots\dots\dots\dots w_p(z) = E_z \qquad \dots \dots .. (16)$$

Where E is a constant. Now substituting the expression (14) and (15), into equation (1) gives

$$\sigma_{rr}^n = C_{11}^n[A_n - \frac{B_n}{r^2} + S_n(r) - \frac{1}{r^2}D_n(r)] + C_{12}^n[A_n + \frac{B_n}{r^2} + S_n(r)\frac{1}{r^2}D_n(r)]$$

$$+ C_{13}^n E - \gamma_1^n T_n(r)$$

$$\sigma_{\theta\theta}^n = C_{12}^n[A_n - \frac{B_n}{r^2} + S_n(r) - \frac{1}{r^2}D_n(r)] + C_{11}^n[A_n + \frac{B_n}{r^2} + S_n(r)\frac{1}{r^2}D_n(r)]$$

$$+ C_{13}^n E - \gamma_1^n T_n(r)$$

$$\sigma_{zz}^n = 2C_{13}^n[A_n + S_n(r) + C_{33}^n E - \gamma_3^n T_n(r)$$

$$\dots \dots \dots \dots \dots \dots .. (17)$$

Thus, for getting the stresses and displacements in all the P-phases through equations (14),(16) and (17), (2p+1) Constants (p values of each An and Bn, and e) must be determined. Since the radial displacement at the center of the core (i.e., Up (o)) cannot be in finite, Bp must be zero. Therefore only 2p constants are left which are determined with the help of the boundary conditions. Equations (10),(11) and (12) furnish a total of 2(p-1) Conditions. The remaining two Conditions needed for Finding out the values of 2p constants are given be Equations (9) and (13). Thus substitution of the Expression of radial displacement at from Equations (14) into Equations (10),(11)and (12), and stress Equations (17) into Equations (9) and (13) gives a set of 2p simultaneous Equations, which can be put in a Matrix form

$$[a_{ij}][A_1, A_2, A_3, A_4, \dots . A_P, B_1, \dots . B_{p-1}, E] = \begin{pmatrix} b_1 \\ b_2 \\ \cdot \\ \vdots \\ b_{2p} \end{pmatrix}$$

$$\dots\dots\dots\dots\dots . (18)$$

In the matrix equation (18) the first (p-1) equations correspond to continuity of displacement conditions (10)… (12), p^{th} equation is obtained by imposing the condition (9) of the applied radial stress. The next (p-1) equations came through the condition of continuity of the radial stress. The last ($2p^{th}$) equation results from the balancing of the axial forces using equation (13). In fact, by applying the conditions in this order, the elements of the square matrix [a_{ij}] and the vector {b_{ij}}are observed to follow a definite pattern and the matrix expands in a particular manner as the number of phases (p) increases.

Particular Case:

Some of the non-zero coefficients of the matrices [aij] and {bij} given above, get further simplified for the case when we take material as isotropic and a constant temperature over entire structure. So it is obvious that for T_n being constant value (i.e., not a function of radius) for the entire phase n,

$$S_n = D_n = P_n = Q_n = 0$$

This simplifies the stress equation (17) as

$$\sigma_{rr}^n = C_{11}^n(A_n - \frac{B_n}{r^2}) + C_{12}^n(A_n + \frac{B_n}{r^2}) + C_{13}^n E - \gamma_1^n T_n$$

$$\sigma_{\theta\theta}^n = C_{12}^n\left(A_n - \frac{B_n}{r^2}\right) + C_{11}^n\left(A_n + \frac{B_n}{r^2}\right) + C_{13}^n E - \gamma_1^n T_n$$

$$\sigma_{zz}^n = 2C_{13}^n A_n + C_{33}^n E - \gamma_3^n T_n$$

The displacement equations (14) and (15) become

$$U_n(r) = A_n r + \frac{B_n}{r}$$

$$W_n(z) = Ez$$

In the matrix equation (18) the coefficient of (b_{ij}), get simplified as

$$b_1 = 0 \qquad for\ i = 1\ to\ (p-1)$$

$$= \sigma_{or} + \gamma_1^1 \mathrm{T} \qquad \text{for i=p}$$

$$= (\gamma_1^{i-p} - \gamma_1^{i-p+1})T \qquad for\ i = (p+1)to(2p-1)$$

$$= \sigma_{oz} r_1^2 + \sum_{j=1}^{p} \gamma_3^j\, T\left(r_j^2 - r_{j+1}^2\right) \qquad for\ \ i = 2p$$

Here we have assumed that material is isotropic, so

$$C_{11}^n = C_{22}^n = C_{33}^n$$

and

$$C_{12}^n = C_{13}^n$$

$$C_{44}^n = \frac{C_{11}^n - C_{12}^n}{2}$$

$$C_{11}^{n} = \frac{(1 - v_n)E_n}{(1 + v_n)(1 - 2v_n)}$$

$$C_{12}^{n} = \frac{v_n E_n}{(1 + v_n)(1 - 2v_n)}$$

Here the volume fraction of metallic core V_m and ceramic layer V_c can be given as

$$V_m = (\frac{r_p}{r_1})^2$$

$$V_c = 1 - (\frac{r_2}{r_1})^2$$

Where,

r_p = radius of metalic core
r_1 = radius of the ceramic phase
r_2 = Effective radius of the FGM phase

Description of the Cylindrical FGM System

The analytical formulation developed in the previous chapter has been demonstrated to find out the internal micro-stresses developed in the functionally graded cylindrical structure when it is subjected to the mechanical loading, thermal loading or the combination of both kinds of loadings. Results have been shown through figures for different combinations of the loadings.

In our particular case we have taken the cylindrical system of Ni-FGM-$A1_2O_3$ with all the phases assumed to be isotropic, free of damage and having the temperature independent properties given in the Table 1. A constant temperature of 300^oC is considered throughout the region. Here we have assumed a linear profile of the gradation of the properties in the functionally graded phase.

The compositional gradation of FGM is defined by the volume fraction of the ceramic phase. Let the volume fraction of the ceramic material within the FGM vary as a function of the coordinate, r, and be arbitrarily defined by a generic function, V (r) which satisfies the following conditions at the interfaces which the FGM makes with homogeneous phases on its two sides.

$$V(r) = \begin{cases} 0 & at\ r = r_p \\ 1 & at\ r = r_2 \end{cases}$$

The elastic properties of the FGM in concentric cylinder (phases) n is given by its Young's modulus, E_n, and Poisson's ratio, v_n , together with the coefficient of thermal expansion, a_n. These are assumed to vary according to the rule of mixture. Then,

$$E_n(r) = E_2 + (E_1 - E_2)V(r)$$
$$v_n(r) = v_2 + (v_1 - v_2)V(r)$$
$$\alpha_n(r) = \alpha_2 + (\alpha_1 - \alpha_2)V(r)$$

Where,

$$V(r) = (\frac{r-r_p}{r_2-r_p})$$

Table 1. Thermo-Elastic Properties for the Metallic (Ni) And Ceramic ($Al_2 2_3$) Phases:

Property	Metallic Core (Ni)	Ceramic Phase(AL_2O_3)
Elastic modulus (GPa)	214	380
Poisson ratio	0.31	0.25
Coefficient of thermal Expansion, $\alpha(0_{C^{-1}})$	$15.4 * 10^{-6}$	$7.4 * 10^{-6}$
Volume Fraction	0.12	0.38

Here we assume that radius of the metallic core r_p is 4 mm. We have shown the results for bi-axial loading, uniaxial loading and for thermal loading by taking different number of concentric cylinders in FGM phase.

Value of uniaxial applied stress= 3.65 GPa

Value of Biaxial applied stress=2.63 GPa

Results and Discussion

Firstly, some solutions are obtained for the bi-axial loading conditions when properties of both the inner and outer cylinders are taken to be identical so that the system works as a solid cylinder. Values of the radial and circumferential stresses are found to be equal to the external applied load, and also would be uniform throughout the region. We can see that the value of axial stress is zero.

To demonstrate the effect of number of phases 'p' in which FGM is divided, results have been shown for p=12 and p= 100. The FGM phase should be divided into a large number of concentric cylinders to get smooth and converging value of stresses. For the case of bi-axial loading at the outer surface of the assembly, stress variations are shown for the radial, circumferential and the axial component of the stresses. the stresses developed in the axial direction are of significance. Its value in the metallic core takes a constant negative value and also in some of the FGM region it is negative while in the region closer to the ceramic phase it becomes positive. In case of unidirectional axial loading only tensile stresses develop.

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