

Output Feedback Controller Design for Damping Power System Oscillations

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Abstract

This paper presents output feedback controller design for damping power system oscillation. The design method does not need the specification of weighting matrices. The eigenvalues of electromechanical mode would be shifted to a pre-specified vertical strip. For practical implementation, the design method only uses partial output feedback. Effectiveness of this controller is evaluated and example, the interconnected power system, is given to illustrate the advantages and effectiveness of the proposed method.

Keywords: *Dynamic stability, strip eigenvalue assignment, output feedback controller, multimachine system.*

1. Introduction

Maintaining system stability has been one of the major concerns for integrated operation of power systems. Modern power systems have been growing in size and complexity with increasing interconnection between systems. An increase in the damping of the system response is desirable, not only because it reduces the fluctuations in the controlled variables and hence improving the quality of the electric service, but mainly because this damping is translated into an increase in the power transmission stability limits. Higher stability limits bring significant economic savings as the need for the expansion of the transmission lines can be postponed [1-5].

Optimal control theory has been widely used in industrial applications. In recent years, the modal control design has been used in power systems to shift the dominant eigenvalues. Different methods have been proposed to assign eigenvalues by modifying the weighting matrix of the quadratic performance index. Optimal and sub-optimal control strategies on the basis of linear system theory using various system states and measurable output as input to the controller have also been attempted [6].

Although the closed-loop system constructed by using the optimal control theory has some advantages, they are still many problems to solve. One of the most serious is that it

is rather difficult to specify the control performance described in terms of a quadratic performance index. However, the design of the linear quadratic regulator has one major difficulty: how to specify the matrices Q and R , where Q and R are the weighting matrices of the quadratic terms for the state vector and control vector [7-9]. The weighting matrices usually would be decided based on trial and error to give satisfactory performance.

This paper is presented for finding a linear quadratic controller such that the optimal closed-loop system has eigenvalues lying within a vertical strip in the complex s-plane [10]. Aiming at improving system stability the design method does not need the specification of the weighting matrices. In this work, the desired positions of the eigenvalues are achieved without convergence problems. One basic difficulty of the state feedback control is that it is usually impractical since some of system states cannot be measured. An output feedback controller is preferred. The output gains are obtained from a transformation of the state feedback matrix to minimize the time integral of the deviation of the state variable. For demonstrating the effectiveness of damping enhancement, eigenvalue analysis simulation result is used to show that the proposed controller gives significant improvement in the dynamic performance of the interconnected power system.

2. Problem Formulation

2.1. Strip Eigenvalue Assignment

Consider a linear time-invariant controllable system that is described in the state space by [11]:

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (1)$$

$$y(t) = Cx(t) \quad (2)$$

where $x(t)$, $u(t)$, and $y(t)$ are the $n \times 1$ state vector, $m \times 1$ input vector, and $p \times 1$ output vector, respectively. A , B , and C are constant matrices of appropriate dimensions.

In the design of a conventionally optimal control system, the control vector is given by

$$u(t) = -Kx(t) \quad (3)$$

where K is the mxn state feedback control matrix designed to minimize the following quadratic performance index:

$$J = \frac{1}{2} \int_0^{\infty} (x^T Q x + u^T R u) dt \quad (4)$$

In (4) the weighting matrices Q and R are nxn non-negative and mxm positive definite symmetric matrices, respectively. The feedback gain in (3) is $K (=R^{-1}B^T P)$ with P being a symmetric positive definite matrix, which is solution of the following algebraic Riccati equation (ARE),

$$A^T P + P A - P B R^{-1} B^T P + Q = 0_n \quad (5)$$

and the eigenvalues of A-BK, denoted by $\Lambda(A-BK)$, will lie in the open left half plane of the complex s-plane.

In conventionally optimal system analysis, the gain in (3) is designed by roughly selecting weighting matrices according to physical reasoning. Because of complexity, the matrices Q and R are commonly chosen as diagonal matrices. The eigenvalues of the closed-loop system are denoted by $\Lambda(A-BK) = [\lambda_1, \lambda_2, \dots, \lambda_m, \lambda_{m+1}, \dots, \lambda_n]$. In order to improve the system performance, the eigenvalue λ_1 through λ_m will be selected and shifted to a desired region. To achieve result the weighting matrix R in the (5) is set to be an identity matrix for equal weighting of the m control inputs, and the weighting matrix Q and R must be given. But in large power system, it is not easy to determine those weighting matrices. The weighting matrices usually are determined by trial and error to obtain satisfactory performances. To overcome this difficulty, a novel approach for designing the optimal eigenvalues assignment will be proposed in the following discussion. The design method in this paper shifts the closed-loop eigenvalues to a pre-specified vertical strip without the need of weighting matrices.

Let (A, B) be the pair of the open-loop system matrices in (1) and h ≥ 0 represent the prescribed degree of relative stability. Then the closed-loop matrix $A_c = A - B R^{-1} B^T \tilde{P}$ has all its eigenvalues lying on the left side of the -h vertical line as shown in Fig. 1(a), where the matrix \tilde{P} is the solution of the following ARE [12]:

$$(A + hI_n)^T \tilde{P} + \tilde{P}(A + hI_n) - \tilde{P} B R^{-1} B^T \tilde{P} + Q = 0_n \quad (6)$$

Note that in (6) with $Q = 0_n$, the unstable eigenvalues of A + hI_n are shifted to their mirror image positions with respect to the -h vertical line, which are the eigenvalues of the closed-loop system matrix A_c.

Assume that h₁ and h₂ are two positive real values to determine an open vertical strip of [-h₂, -h₁] on the

negative real axis as shown in Fig. 1(b) and given nxn matrix $\tilde{A} = A + h_1 I$. The control law changed to be

$$u(t) = -\rho \tilde{K} x(t) \quad (7)$$

with the feedback gain $\tilde{K} = R^{-1} B^T \tilde{P}$. The matrix \tilde{P} is the solution of the following modified ARE:

$$\tilde{A}^T \tilde{P} + \tilde{P} \tilde{A} - \tilde{P} B R^{-1} B^T \tilde{P} = 0_n \quad (8)$$

The gain ρ is selected by

$$\rho = \frac{1}{2} + \frac{h_2 - h_1}{2 \cdot \text{tr}(\tilde{A}^+)} = \frac{1}{2} + \frac{h_2 - h_1}{\text{tr}(B \tilde{K})} \quad (9)$$

where $\text{tr}(\tilde{A}^+) = \sum_{i=1}^{n^+} \lambda_i^+ = \frac{1}{2} \text{tr}(B \tilde{K})$ and $\lambda_i^+ (i = 1, 2, \dots, n^+)$ are

the eigenvalues of \tilde{A} in the right half-plane of the complex s-plane. The optimal closed-loop system becomes

$$\dot{x}(t) = (A - \rho B \tilde{K}) x(t) \quad (10)$$

Equation (10) consists of a set of eigenvalues which lie inside the vertical strip of the [-h₂, -h₁] as shown in Figure 1(b). In (8) for equal weighting of the m control inputs, we can let R be unity matrix. These solving the ARE (8) does not need a Q matrix, so it is easy to design an optimal controller for power system oscillation damping.

2.2. Output Feedback Controller

In practice it is not easy to observe all the state variables. Therefore, we assume that only

$$y(t) = C x(t) \quad (11)$$

is observable.

The control vector u(t) can be generated in various ways from output signal y(t) [13, 14]. In here we determine it so as to minimize the time integral of the deviation of the state variable from its truly optimal trajectory x(t). This method requires no iterative calculation. The control is,

$$u(t) = -F^o y(t) = -F^o C x(t) \quad (12)$$

Here F^o is to be determined.

Minimizing the state deviation exciting vectors, it can be show that the matrix F^o is obtained as follows:

$$F^o = F^* V C^T (C V C^T)^{-1} \quad (13)$$

where $F^* = \rho \tilde{K}$ from (7) and V is obtained from the matrix equation

$$(A - B F^*) V + V (A - B F^*)^T + I = 0 \quad (14)$$

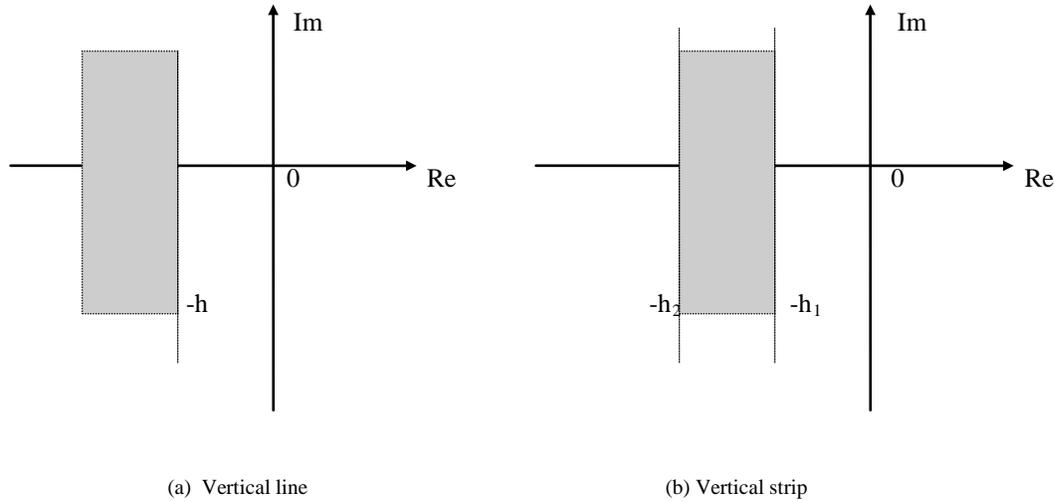


Fig. 1 Complex s-plane

3. Simulation Results

The system shown in Fig. 2, taken from [15], is studied. Consider a linear time-invariant multivariable system is described in the state space form as,

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \quad (15)$$

with $x_i = [\Delta\omega_i, \Delta\delta_i, \Delta e_{qi}, \Delta e_{fdi}]^T$ and $y_i = \Delta\omega_i$ for $i = 1, 2$ where $[A, B, C]$ are constant matrices.

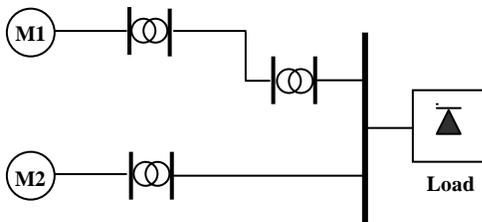


Fig. 2 Multimachine system

For the system as shown in Fig. 2, the system eigenvalues without control are tabulated in the first column of Table 1. It is clear that the electromechanical modes are poorly damped. The minimal damping ratio of electromechanical mode is 0.0092, so that is not good enough. To improve

the system dynamic stability, those modes should be shifted toward certain desirable locations. In the eigenvalue assignment, if we choose $h_1 = 2.5$ and $h_2 = 3.5$, the electromechanical modes with absolute real parts less than $h_1 = 2.5$, will be shifted to the vertical strip of $[-h_2, -h_1] = [-3.5, -2.5]$. The other modes will not be changed.

Two control schemes are compared: (1) optimal control, and (2) proposed method. The minimal damping ratio of that mode is improved to be 0.267 in this proposed method that is inside the acceptable range. It is shown from Table 1, that the relative stability of the proposed method is much better than optimal control [15].

Table 1: System eigenvalues

Open-Loop	Optimal Control	Proposed Method
-0.0904±j9.8430	-0.7325±j9.8778	-2.8378±j10.2313
-0.0006	-1.9077±j1.8517	-2.7331±j2.3279
-0.2443	-25.1741±j67.8188	-22.3665±j66.8444
-25.1741±j67.8187	-25.2385±j30.3070	-22.6888±j28.2189
-25.2329±j30.3073		

Vertical strip in $h_2 = 3.5$; $h_1 = 2.5$ and $\rho = 0.5522$

The transient responses of the angular frequencies to a 5 % change in the mechanical torque of machine 1 are shown in Fig. 3.

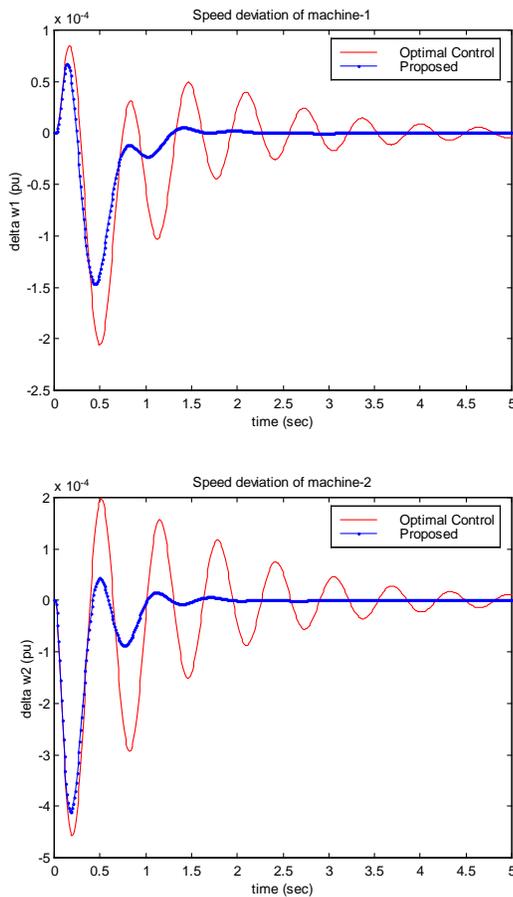


Fig. 3 Transient responses on the angular frequencies to a 5% change in the mechanical torque of machine 1.

4. Conclusions

An output feedback controller is designed to increase the stability of interconnected power system. The electromechanical mode can be shifted to a pre-specified vertical strip without effect the other modes. The design method is very simple and avoids the difficulty of choosing weighting matrices. The results obtained from the simulation show that the proposed controller can effectively damp the power system oscillations under disturbances.

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