IJISET - International Journal of Innovative Science, Engineering & Technology, Vol. 6 Issue 11, November 2019
ISSN (Online) 2348 – 7968 | Impact Factor (2019) – 6.248
www.ijiset.com

Applied Stochastic System State Modeling and Forecasting under Uncertainty

Angel Tanev[1]

[1] Probability, Statistics and Operations Research, Sofia University "St. Kliment Ohridski", Sofia, 1000, Bulgaria

Abstract
The stochastic systems state modeling and forecasting is an important subject in the today's designed systems subjected to uncertainty in the field. The system is considered as a composition of a deterministic and a stochastic part. A theoretical background is given with a stochastic differential equation (SDE) which describes a random process of degradation. Such description allows more detailed understanding of how the stochastic system's state variable can be forecasted in the field. A practical application is given on stochastic system's state forecasting.
Keywords: *Stochastic System, State, Stochastic Differential Equation, Forecasting.*

1. Introduction

Suppose a stochastic system under consideration in this study shown below on Fig.1:

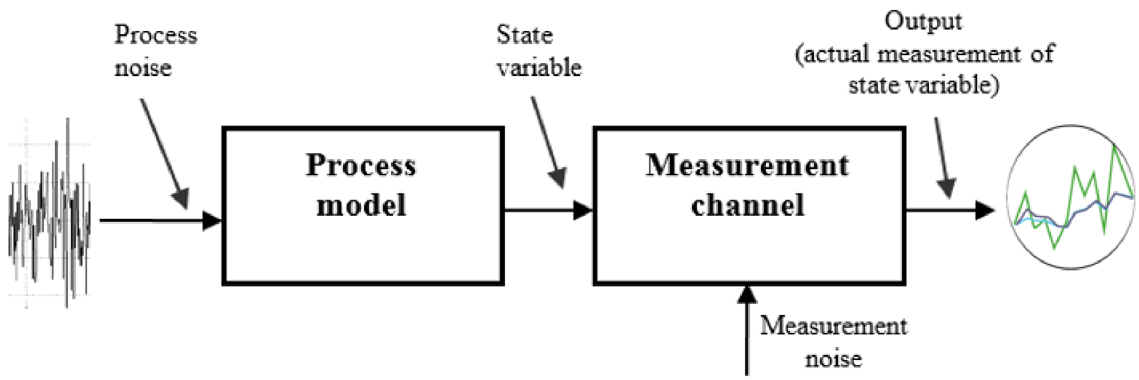

Fig. 1 Stochastic System overview.

Such systems (see Fig.1) can be found in many domains like aviation (e.g. radionavigation and radiolocation systems); air armament control systems; automotive systems, etc. The systems state modeling and forecasting under uncertainty is an important part of studying these systems. The forecasting will allow for these systems to predict the future state and to make some further decisions on predictive maintenance (i.e. to have an idea when given parameter could exceed some predefined threshold like upper drift, faulty state, etc.). Some authors have worked on the theory and researched similar systems- see papers [2,3,5-11].

2. Theoretical Background

The stochastic system (for example, radionavigation system's measurement channel) can be represented with the stochastic differential equation (SDE) given below [4]:

$$dx = \mu dt + \sigma dw \quad (1)$$

$$dw = \varepsilon\sqrt{dt} \quad (2)$$

where:

- dw is a generalized Wiener process and ε is Normal(0,1) distribution.

Some other, more general, version of Eq.(1) and Eq.(2) can be found as the following expression [1]:

$$x(t) = \int_0^t g(\tau)\, d\tau + \int_0^t f(\tau)\, dw(\tau) \quad (3)$$

Consider now the following expression:

$$x_{t+1} = x_t + Normal(\mu, \sigma) \quad (4)$$

Here Eq.(4) suggests that the state variable's value changes in one unit of time by an amount that is normally distributed with mean μ and variance σ^2. The normal distribution is a reasonable assumption for a lot of variables (due to the central limit theorem) that the variable x is a sum of many independent random variables [2]. One can generalize to any time interval T:

$$x_{t+T} = x_t + Normal\left(\mu T, \sigma\sqrt{T}\right) \quad (5)$$

Therefore, the equation Eq.(5) allows to keep using normal distributions and to make a prediction between any time intervals we choose in advance. Since the above equation Eq.(5) considers discrete units of time, one can be written as well as in a continuous time form, where one can consider any small time interval Δt:

$$\Delta x = Normal\left(\mu(\Delta t), \sigma\sqrt{\Delta t}\right) \quad (6)$$

Equation (4) has the important property of being memoryless, i.e. to perform a prediction of the value of x some time T from now, we only need to know the current value of x, not anything about the path it took to get to the present value. We can use equations (1) and (2) to model the system's state:

$$\frac{dS}{S} = \mu dt + \sigma dw \quad (7)$$

Applying some further transformations, the Eq.(7) be rewritten as:

$$dS = \mu S dt + \sigma S dw \quad (8)$$

The parameters μ and σ are the drift parameter and volatility parameter respectively [1]. Then, following the Itô's lemma which suggests that for a function F of a stochastic variable X that follows the Itô's process of the form $dx(t)=a(x,t)dt+b(x,t)dw$, then we have:

$$dF = \left(\frac{\partial F}{\partial t} + a(x,t)\frac{\partial F}{\partial x} + \frac{1}{2}b(x,t)^2\frac{\partial^2 F}{\partial x^2}\right)dt + b(x,t)\frac{\partial F}{\partial x}dw_t \quad (9)$$

Applying some further substitutions, such as $F(S)=log(S)$ and as well as taking into account the Eq.(7) where $x=S$, $a(x,t)=\mu$ and $b(x,t)=\sigma$:

$$d(\ln(S)) = \left(\frac{\partial \ln(S)}{\partial t} + \mu \frac{\partial \ln(S)}{\partial S} + \frac{1}{2}\sigma^2 \frac{\partial^2 \ln(S)}{\partial S^2}\right)dt + \sigma \frac{\partial \ln(S)}{\partial S} dw_t \qquad (10)$$

$$d(\ln(S)) = \left(\mu - \frac{\sigma^2}{2}\right)dt + \sigma dw \qquad (11)$$

Applying an integration on Eq.(11) over time T, one can obtain the value of the state variable $S_{(t+T)}$ when having some initial value of S_t :

$$S_{t+T} = S_t exp\left(Normal\left(\left(\mu - \frac{\sigma^2}{2}\right)T, \sigma\sqrt{T}\right)\right) \qquad (12)$$

The last Eq.(12) is a Geometric Brownian Motion (GBM) model- the "geometric" part is due to the fact that we are effectively multiplying lots of distributions together (they were added in log space). Next, GBM approach is very easy to be reproduced with a spreadsheet- see next section for practical application of the Eq.(1)-(12).

3. Illustrative Example

Consider again the stochastic system shown on Fig.1. In the real practice, for example, we may be interested in studying such systems when a drift of some basic parameter could exceed some predefined threshold values. Suppose our basic parameter of interest is 5V supply voltage degradation process (this block is important part of GPS navigation system). The stochastic process has the following characteristics (estimated from related reliability test): μ=-1e-06; σ=1.6e-02. Running the stochastic process simulation by using the process parameters μ and σ stated above, we obtain the following 50 possible scenarios of Geometric Brownian motion (see Fig.2). The applied spreadsheet stochastic process simulation is done by using the formula: " =Xt+NORMINV((RAND(), μ*T, σ*SQRT(T))".

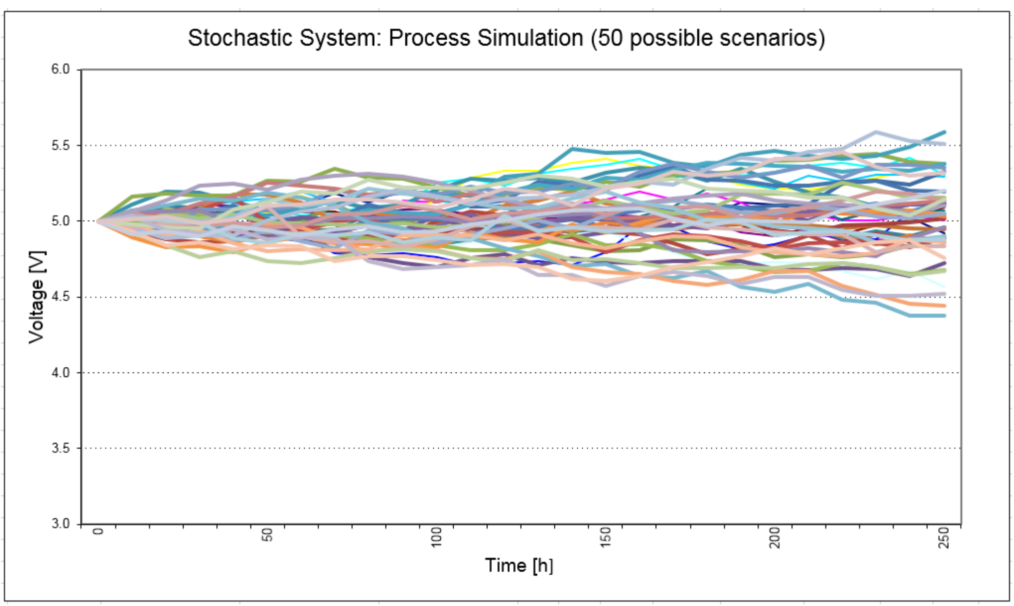

Fig. 2 Stochastic Process Simulation with X_0=5V; μ= -1e-06; σ=1.6e-02

From robustness point of view, the supply voltage critical lower and upper values needed for this system still to operate is 4.5V/5.5 V. Looking at the performed simulation (Fig.2)- the expected moment when the power supply voltage will exceed the upper and lower threshold values is after 225 hours in the field under the worst usage conditions. This study allows a forecasting to be done based on the GBM modeling.

4. Conclusions

The following major outcomes have been summarized based on the performed study:

- The GBM modeling (Eq.(4)-Eq.(5)) allows to do some prediction over time T of the state variable value of x due to its memoryless property.
- Stochastic system state forecasting was performed considering the deterministic part and the uncertainty part of the process.
- The modeling was successfully applied on real stochastic system and the results can be used further on the purpose of making some decisions (maintenance planning, etc.).

Acknowledgments

The author would like to thank to all colleagues for their valuable remarks during that study.

References

[1] Gindev E., Introduction of Reliability Engineering. Sofia: Marin Drinov, 2001.
[2] Petrov N., Operational reliability of technical systems at risk. Publishing house “Uchkov”, Bourgas, Asen Zlatarov University, 2000.
[3] Andonov A., Tanev A., Statistical Hypothesis Testing and Their Application in Risk Technical systems, Academic Journal “Mechanics, Transport, Communications”, issue 3/2008, art. ID: 303, pp. VIII-19-VIII-22, 2008.
[4] Nenkov Y., Stochastic System State Recognition Incorporating Cost of Missed Detection and False Alarm, International Journal of Innovative Science, Engineering and Technology (IJISET), vol. 6, issue 8, August 2019, pp. 12-14.
[5] Wang Y., Liu L., Study on life assessment technology for marine intermediate bearing, Safety and Reliability of Complex Engineered Systems, Proceedings of the 25th European Safety and Reliability conference, ESREL 2015, Zürich, Switzerland, 7–10 Sept 2015, Taylor & Francis Group, pp. 1785-1790, 2015.
[6] Wang, H., The Application of Inverse Gaussian distribution in Reliability. In K. Chen(ed.), Journal of Guizhou University (Natural Science)18:18–29, 2001.
[7] Huang, W., Reliability Analysis Considering Product Performance Degradation. Arizona: The University of Arizona, 2002.
[8] Ahmed T.A., Van Casteren J.A., Precise Estimates for The Solution of Stochastic Functional Differential Equations With Discontinuous Initial Data (Part 1), International Journal of Innovative Science, Engineering and Technology (IJISET), vol. 1, issue 8, October 2014, pp. 179-191.
[9] Petrov N., An approach for determining the periodicity of regulation work of risk technical systems, International Journal of Computing, vol.2, issue 3, pp.74-76, 2003.
[10] Petrov N., Panayotova G., Research for distribution of data from risk technical systems, International Journal of Computing, vol.8, issue 3, pp.32-35, 2009.
[11] Lalov D., Suboptimal filtration in complex processing of information, International Scientific Conference 11-12 Apr 2019, Aviation Faculty, Vasil Levski National Military University, Dolna Mitropolia, pp. 153-158 (in Bulgarian).

Angel Tanev received MSc degree in Automation and System Engineering in 2002 and MSc degree in Operational Research, Probability and Statistics at Faculty of Mathematics and Informatics at Sofia University “St Kliment Ohridski” in 2012. He also obtained a PhD degree in the field of Reliability engineering. His research interests include reliability, operational research, dependability, statistics, risk theory, optimal control.