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Stochastic System State Recognition Incorporating Cost of Missed Detection and False Alarm

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Abstract

Today the complex systems working modes are subjected to uncertainty. The systems state recognition might become very challenging in case when signal-to-noise ratio is very low. In many domains, such as aviation, automotive, where we have risk systems, it is very important to recognize as precisely as possible the systems state. This paper is proposing one statistical approach for state recognition which is taking into account also the ratio between the cost of missed detection and cost of false alarm.

Keywords: *Stochastic system, Uncertainty, Optimal Boundary, Probability of false alarm, Probability of missed detection.*

1. Introduction

The impact of uncertainty in the today's complex stochastic systems requires a sophisticated analysis. In the recent years, many researchers are investigating the impact of the uncertainty over different type of systems- see, for example, [1,2,3,4,5,6,8]. On one hand, it is important to find the optimal boundary value for state recognition (normal vs faulty), and, on other hand, to minimize the probability of missed detection and false alarm. In risk technical systems, the probability of missed detection may have a significant impact- for instance, in aviation domain this risk may lead to miss a defect detection which could result in significant losses.

2. Theoretical Background

Suppose a stochastic system under uncertainty (see Fig.1).

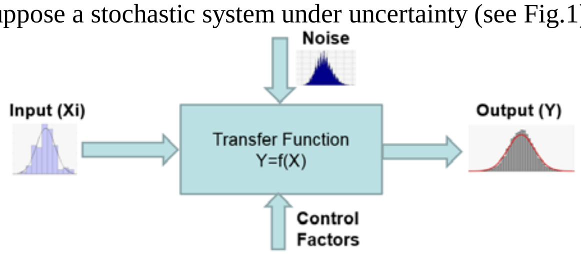

Fig. 1 System diagram overview

The ideal situation is when the output is represented exactly by the transfer function. However, in real life, the system's output (Y) is impacted by some noise variables [7]. Now let's consider the output behavior- the response (Y). Suppose we are observing two different probability density functions (pdfs) on a system's parameter that showing two states: D1- system's normal functioning state and D2- system's faulty state (Fig.2):

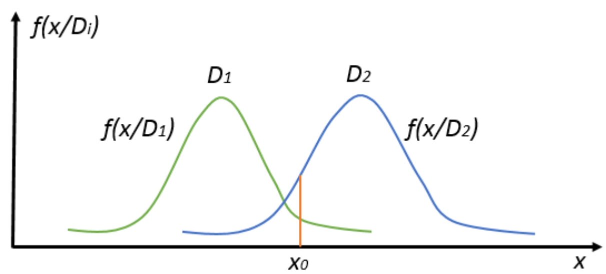

Fig. 2 System under uncertainty with 2 states: D1 and D2.

The probability density functions for each state can be described by the following expressions [2]:

$$f\left(x_0 / D_1\right)=\frac{1}{\sigma_{D_1}\sqrt{2\pi}}e^{\frac{-\left(x_0-\mu(D_1)\right)^2}{2\sigma_{D_1}^2}} \quad (1)$$

$$f\left(x_0 / D_2\right)=\frac{1}{\sigma_{D_2}\sqrt{2\pi}}e^{\frac{-\left(x_0-\mu(D_2)\right)^2}{2\sigma_{D_2}^2}} \quad (2)$$

where: $\mu(D_1)$ - mean value of the system's parameter in normal state; $\mu(D_2)$ - mean value of the system's parameter in faulty state; σ_{D_1} - standard deviation of the system's parameter in normal state; σ_{D_2} - standard deviation of the system's parameter in faulty state.

In the general case, the average risk (the expected loss) is expressed by two components (R_1 and R_2) [9]:

$$R_1 = C_{11}P_1 \int_{-\infty}^{x_0} f(x/D_1)dx + C_{21}P_1 \int_{x_0}^{\infty} f(x/D_1)dx \tag{3}$$

$$R_2 = C_{12}P_2 \int_{-\infty}^{x_0} f(x/D_2)dx + C_{22}P_2 \int_{x_0}^{\infty} f(x/D_2)dx \tag{4}$$

The total average risk is a sum of the two components given above: $R = R_1 + R_2$.

Let's now proceed with a derivative operation with respect to x_0 and then put this derivative equal to 0:

$$\frac{dR}{dx_0} = C_{11}P_1 f(x_0/D_1) - C_{21}P_1 f(x_0/D_1) + \\ + C_{12}P_2 f(x_0/D_2) - C_{22}P_2 f(x_0/D_2) = 0 \tag{5}$$

Processing of Eq.(5) leads to the following expression:

$$\frac{f(x_0/D_1)}{f(x_0/D_2)} = \frac{(C_{12} - C_{22})P_2}{(C_{21} - C_{11})P_1} \tag{6}$$

One should note that in many practical tasks, one can accept the following: $C_{11}=C_{22}=0$ (i.e. assume conservative approach that the profit of correct decisions is 0) [9].

Next step in the analysis is related to minimum risk method [9], i.e. to find an optimal boundary value (x_0) which is minimizing the risk (both missed detection and false alarm). After some transformations (e.g. *ln* transformation on both sides of the Eq.(6)), we obtain the following expression:

$$\frac{-(x_0 - \mu(D_1))^2}{2\sigma_{D_1}^2} + \frac{(x_0 - \mu(D_2))^2}{2\sigma_{D_2}^2} = \ln\left(\frac{C_{12}P_2\sigma_{D_1}}{C_{21}P_1\sigma_{D2}}\right) \tag{7}$$

where: C_{12}- missed detection cost; C_{21}- false alarm cost; P_2- faulty state probability; P_1- normal state probability; x_0- boundary (optimal) value of the parameter for state recognition (normal vs faulty state).

The equation Eq.(7) can be solved for by applying some well-known numerical methods [5].

3. Applications

The measurements (N=34) done on our system under study are showing two separate histograms (i.e. two states, see Fig.3). It is known, by field experience on this system, that beyond 2.8% drift, the system is found in faulty state. However, this boundary is not an optimal value which could result in increasing the risk of missed detection, so there is a need to find an optimal boundary value at which we have a minimum total risk (see Eq.(5) and Eq.(6)).

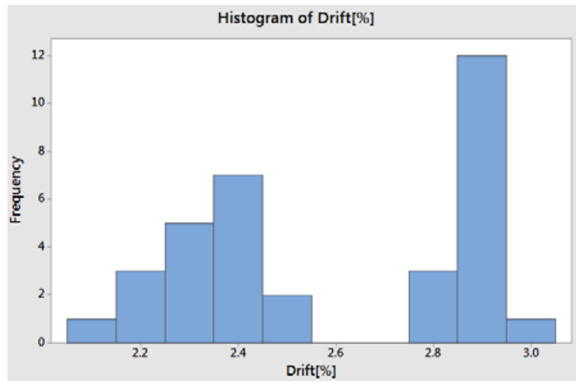

Fig. 3 System states under study.

Then, by assigning Normal probability distributions (see Eq.(1) and Eq.(2)) to the two considered states D1 and D2, the empirical data on Fig.3 can be represented by the next figure below (see Fig. 4):

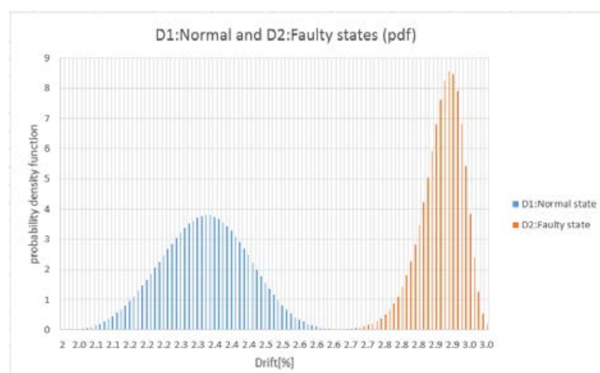

Fig. 4 D1 and D2 data fitting to normal distribution.

The following inputs to our further analysis are: C_{12}- missed detection cost; C_{21}- false alarm cost (assumption: $C_{12}/C_{21}=20/1$); $P_1=0.53$; $P_2=0.47$; (where: $P_1+P_2=1$); D_1(mean value): $\mu(D_1)$ =2.336% with standard deviation σ_{D_1} =0.11%; D_2(mean value): $\mu(D_2)$ =2.891% with standard deviation σ_{D_2} =0.06%. Substituting the above values in the Eq.(7), we obtain the following:

$$\frac{-(x_0-2.336)^2}{2*0.11^2}+\frac{(x_0-2.891)^2}{2*0.06^2}=\ln\left(\frac{20*0.47*0.11}{1*0.53*0.06}\right) \qquad (8)$$

The next step is to find the root of the equation Eq.(8), i.e. to estimate the boundary (optimal) value x_0 where the total risk is at minimum. Applying some numerical methods implemented by Solver function, it allows to find the optimal boundary value which for this case study is x_0=2.66%. In other words, a system drift below 2.66% is considered as normal functioning system's state while system drift beyond 2.66% is associated with faulty state.

Now suppose that the cost ratio is 10 times higher than the 1st case, i.e. C_{12}/C_{21}=200/1, meaning that the cost of missed detection is 200 times higher than the cost of false alarm (especially, in risk systems). Then Eq.(7) becomes the following:

$$\frac{-(x_0-2.336)^2}{2*0.11^2}+\frac{(x_0-2.891)^2}{2*0.06^2}=\ln\left(\frac{200*0.47*0.11}{1*0.53*0.06}\right) \qquad (9)$$

In this case, again, applying some numerical methods implemented by Solver function, it allows to find the optimal boundary value which for this case study is x_0=2.63%. In other words, a system drift below 2.63% is considered as normal functioning system's state while system drift beyond 2.63% is associated with faulty state. Therefore, with more severe cost ratio C_{12}/C_{21}>>>1, the optimal boundary value x_0 becomes more conservative.

4. Conclusions

The following main outcomes from the performed analysis can be summarized in this section:

- It is highly recommended to take into account the cost of missed detection and cost of false alarm in finding the optimal boundary value x_0 related to stochastic system state recognition.
- For more severe cost ratio (C_{12}/C_{21}>>>1), the optimal boundary x_0 becomes more conservative.
- The proposed statistical approach can be applied in risk technical system (e.g. aviation, automotive domain) where the cost of missed detection is considered significantly higher than the cost of false alarm (C_{12}>>C_{21}).

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Yordan Nenkov has obtained PhD degree in 2012 in the field of Communications and Computer sciences. His main research interests are stochastic systems, worst case circuit analysis, mathematical modeling. Currently, he is working in the Research & Development of complex electronic devices for automotive industry.