

# Degree-Edge Based Topological Indices Of Chain Graphs

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## Abstract

Zagreb indices are degree-based indices. If degree and edge of a molecular graph is combined it defines the degree-edge based topological index. The first K-Banhatti index is defined as:

$B_1(G) = \sum_{ue} (d_u + d_e)$ , where  $ue$  means that the vertex  $u$  and edge  $e$  are incident in  $G$ .

In this paper K-Banhatti indices of Penta chain and graphene chain with  $t = 1$  row and  $S$ -benzene rings in a row are investigated.

**Keywords:** Degree, edge, F-index, graphene, K-Banhatti indices, molecular graph, penta chain, Zagreb indices.

## 1. Introduction

A molecular graph is a simple graph such that its vertices correspond to the atoms and the edges to the bonds. Let  $G = (V, E)$  be a simple molecular graph without directed and multiple edges and without loops, the vertex and edge sets of it are represented by  $V = V(G)$  and  $E = E(G)$ , respectively [1-5]. Topological indices describe the structure of molecules numerically and are used in the development of qualitative structure activity relationships (QSARs) [6]. In the QSPR/QSAR study, physicochemical properties and topological indices such as Szeged index, Wiener index, Randic index, Zagreb index, ABC index are used to predict bioactivity of the chemical compounds [7]. In analyzing the structure dependency of  $\Pi$ -electron energy the first and second Zagreb indices are defined as:  $M_1(G) = \sum_{uv \in E(G)} (d_u + d_v)$  and  $M_2(G) = \sum_{uv \in E(G)} (d_u * d_v)$  which are degree based topological indices. The number of edges incident on vertex  $v$  is called degree  $d_v$  of a vertex [8]. According to the above Zagreb indices, the first and second Zagreb polynomials have been defined as:  $M_1(G, x) = \sum_{uv \in E(G)} x^{(d_u + d_v)}$  and

$M_2(G,x) = \sum_{uv \in E(G)} x^{(d_u d_v)}$  [9-11]. Degree based topological indices of Nicotine are studied in [12]. Topological indices of graphene are studied in [13]. The multiplicative Zagreb indices of nanostructures and chains are studied by W.Gao [14]. Redefined Zagreb, Randic, Harmonic and GA indices of graphene are studied by R.Pradeep Kumar [15]. K-Banhatti and hyper K-Banhatti indices of circulant graphs are studied by A.Asghar [16]. K-Banhatti indices of Hexagonal, Honeycomb and derived networks are studied in [17]. ABC K-Banhatti and Augmented K-Banhatti indices of Chemical networks are studied in [18]. K-Banhatti indices of molecular graphs are studied in [19-20].

The first K-Banhatti index  $B_1(G)$  and the second K-Banhatti  $B_2(G)$  of a graph  $G$  are defined as:

$B_1(G) = \sum_{ue} (d_u + d_e)$  and  $B_2(G) = \sum_{ue} (d_u d_e)$ , where  $ue$  means that the vertex  $u$  and edge  $e$  are incident in  $G$  [21].

The first and second multiplicative K-Banhatti indices are defined as:

$PB_1(G) = \prod_{ue} (d_u + d_e)$  and  $PB_2(G) = \prod_{ue} (d_u d_e)$ .

Albertson index  $A(G)$  to determine the irregularity of a graph [22] is defined as:

$A(G) = \sum_{uv} |d_u - d_v|$  and by analogy with this topological index, Albertson K-Banhatti index can be introduced as:  $AB(G) = \sum_{ue} |d_u - d_e|$ .

And reformulated K-Banhatti index is defined as:  $RB(G) = \sum_{uv} [d_u + d_e - 2]^2$ .

The first and second hyper K-Banhatti indices are

$HB_1(G) = \sum_{uv} [d_u + d_e]^2$  and  $HB_2(G) = \sum_{uv} [d_u d_e]^2$ .

We introduce for Randic index, reciprocal Randic index, the K-Banhatti Randic index and K-Banhatti reciprocal Randic index as [8],

K-Banhatti Randic index,

$BR(G) = \sum_{ue} \frac{1}{\sqrt{d_u d_e}}$  and K-Banhatti reciprocal Randic index =  $BRR(G) = \sum_{uv} \sqrt{d_u d_e}$ .

Where  $ue$  means that the vertex  $u$  and edge  $e$  are incident in  $G$  and  $d_u$  is degree of vertex  $u$  in  $G$ .

The Forgotten index is defined as [23-25]:

$F = F(G) = \sum_{uv \in E(G)} d_u^3 = \sum_{uv \in E(G)} [d_u^2 + d_v^2]$ .

We introduce by combining degree and edge of  $G$  the Forgotten K-Banhatti index as:

$FB(G) = \sum_{ue} (d_u^2 + d_e^2)$ .

The notations used in this paper are standard and mainly taken from books [26-30]. In this paper K-Banhatti indices of Penta chain and graphene chain with  $t = 1$  row, S-benzene rings in a row is investigated.

## 2. Results and discussion

The molecular graphs considered in this paper are finite, connected, loop less and without multiple edges. Let  $G = (V, E)$  be a graph with  $n$  vertices and  $m$  edges. The first K-Banhatti index is defined as:  $B_1(G) = \sum_{uv \in E} (d_u + d_v)$ , where  $uv$  denotes the edge of the graph  $G$  connecting the vertices  $u$  and  $v$  and  $d_u$  denotes the degree of the vertex  $u$  and  $d_e = d_u + d_v - 2$ . By combining degree and edge the K-Banhatti indices can be defined for molecular graphs. For formulas of degree-edge based K-Banhatti indices of molecular graph refer to [16-21]. The first and multiple K-Banhatti index, Albertson K-Banhatti index, reformulated K-Banhatti index, first and second hyper K-Banhatti indices, K-Banhatti Randic index, K-Banhatti reciprocal Randic index and Forgotten K-Banhatti index are computed for penta chain and graphene chain. The K-Banhatti indices for penta chain and graphene chain are computed as:

### 2.1 K-Banhatti indices for penta chain

The 2-dimensional graph for double row penta chain  $G(n, S_2)$  is shown in figure (1). It is observed from figure (1) there are four edges as:  $E_{22}$ ,  $E_{23}$ ,  $E_{34}$  and  $E_{44}$ . The number of edges with frequency are represented in table 1.

The first hyper K-Banhatti index is computed as:

$$\begin{aligned} HB_1(G) &= \sum_{uv \in E} (d_u + d_v)^2 \\ &= 4[(2+2)+(2+2)]^2 + (4n)[(2+3)+(3+3)]^2 + (2n)[(3+5)+(4+5)]^2 + (n-2)[(4+6)+(4+6)]^2 \\ &= 1062n - 144 \end{aligned}$$

The second hyper K-Banhatti index is computed as:

$$\begin{aligned} HB_2(G) &= \sum_{uv \in E} (d_u d_v)^2 \\ &= 6[(2*2)*(2*2)]^2 + (4n)[(2*3)*(3*3)]^2 + (2n)[(3*5)*(4*5)]^2 + (n-2)[(4*6)*(4*6)]^2 \\ &= 523440n - 662528. \end{aligned}$$

### 2.2 K-Banhatti indices for graphene chain

Graphene is a two-dimensional layer of pure carbon. It is one atom-thick and single tightly packed layer of carbon atoms that are bonded together in a repeating pattern of hexagon, with each carbon atom covalently bonded to three other carbon atoms. In this section we compute K-Banhatti indices for graphene chain with  $t = 1$  and  $S$  benzene rings in this row. The 2-dimensional graph of graphene with  $t = 1$  row and  $S$  benzene rings is shown in figure (2). The first and second hyper K-Banhatti indices for graphene are computed as:

**First and second hyper K-Banhatti indices for graphene**

It is observed from figure (2), the graphene chain with  $t=1$  and  $S$  benzene rings in this row has three edges:  $E_{22}, E_{23}$  and  $E_{33}$  with  $|d_{2,2}| = 6, |d_{2,3}| = (4S-4)$  and  $|d_{3,3}| = (S-1)$  (table 2).

The first hyper K-Banhatti index is computed as:

$$\begin{aligned} HB_1(G) &= \sum_{u \in E} (d_u + d_v)^2 \\ &= 6[(2+2)+(2+2)]^2 + (4S-4)[(2+3)+(3+3)]^2 + (S-1)[(3+4)+(3+4)]^2 \\ &= 680S-296. \end{aligned}$$

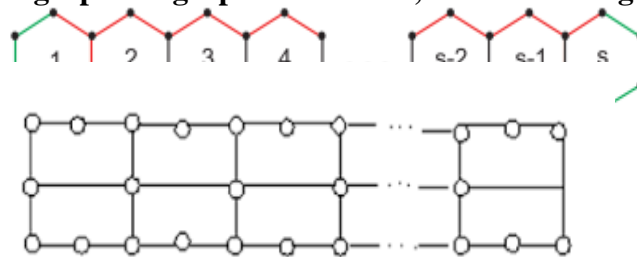
The second hyper K-Banhatti index is computed as:

$$\begin{aligned} HB_2(G) &= \sum_{u \in E} (d_u \cdot d_v)^2 \\ &= 6[(2*2)*(2*2)]^2 + (4S-4)[(2*3)*(3*3)]^2 + (S-1)[(3*4)*(3*4)]^2 = 644972544(4S-4)(S-1). \end{aligned}$$

First and second multiple K-Banhatti indices, AK-Banhatti index, Reformulated K-Banhatti index, First and second hyper K-Banhatti indices, K-Banhatti Randic index, K-Banhatti reciprocal Randic index and Forgotten K-Banhatti index are computed for penta chain and graphene chain and are represented in table number (3).

**Figure no.1: 2-dimensional graph for double row penta chain  $G(n, S_2)$ .**

**Figure no. 2: dimensional graph for graphene  $t=1$ row,  $S$ -benzene rings.**



**Figure: 1**

**Table no. (1): The degree-edge partition for double row penta chain  $G(n, S_2)$ .**

$d_G(u), d_G(v) \setminus e = u v \in E(G)$	(2,2)	(2,3)	(3,4)	(4,4)
$d_G(e)$	2	3	5	6
<b>Number of edges</b>	4	4n	2n	n-2

**Table no. (2): The degree-edge partition for graphene t =1 row, S-benzene rings.**

$d_G(u), d_G(v) \setminus e = u v \in E(G)$	(2,2)	(2,3)	(3,3)
$d_G(e)$	2	3	4
<b>Number of edges</b>	6	(4S-4)	(S-1)

**Table no. (3): K-Banhatti indices for Penta chain and Graphene chain.**

Serial Number	Topological indices	Penta chain	Graphene chain
1.	First multiple K-Banhatti index	$975440(n-2)n^2$	$7392(4S-4)(S-1)$
2.	Second multiple K-Banhatti index	$4777574400(n-2)n^2$	$746496(4S-4)(S-1)$
3.	Albertson K-Banhatti index	$14n-8$	$2S-2$
4.	Reformulated K-Banhatti index	$1098n-504$	$468S-252$
5.	First hyper K-Banhatti index	$1062n-144$	$680S-296$
6.	Second hyper K-Banhatti index	$523440n-662528$	$644972544(4S-4)(S-1)$
7.	K-Banhatti Randic index	$0.982n+0.9002$	$0.6281S+1.4538$
8.	K-Banhatti reciprocal Randic index	$61.589n-32$	$41.394S-17.394$
9.	Forgotten K-Banhatti index	$918n-272$	$522S-330$

### 3. Conclusion

The degree-edge based K-Banhatti indices are studied for penta chain and graphene chain. To compute degree-based topological indices for molecular graph degree of each vertex and edge are important. The first and second multiple K-Banhatti indices, AK-Banhatti index, reformulated K-Banhatti index, first and second hyper K-Banhatti indices, K-Banhatti Randic index, K-Banhatti, reciprocal Randic index and Forgotten K-Banhatti index are computed for penta chain and graphene chain. The degree-edge based K-Banhatti of molecular graphs may be used in QSPR and QSAR studies.

### References

- [1] J. Asadpour, R. Mojard, and L. Safikhani, Computing some topological indices of nanostructure, Digest Journal of Nanomaterials and Biostructures, Vol.6, No.3, July-September 2011, 937-941.
- [2] Y. Gao, M. R. Farahani, M. S. Sarder, and S. Zafar, On the Sanskruti index of circumcoronene series of Benzenoids, Applied Mathematics, 2017, 520-524, ISSN 2152 7393.
- [3] Z. Husain, M. Munir, S. Rafique and S. M. Kang, Topological characterizations and index-analysis of New degree-based descriptors of Honeycomb networks, Symmetry 2018, 10, 478, M.D.P.I.
- [4] A. Usha, P. S. Ranjini, K. M. Devendraiah and V. Loksha, Topological indices of Complete graph with a single rooted vertex, International Journal of Scientific and Engineering Research, Volume 7, Issue 5, may-2016, 22-25.
- [5] H. M. U. Rehman, R. Sardar and A. Raza, Computing topological indices of Hex board and its line graph, Open Journal Mathematical Science, Vol.1 (2017), No.1, 62-71.
- [6] Y. C. Kwun, A. Ali, W. Nazeer, M. A. Chaudhary, and S. M. Kang, M-polynomials and degree based topological indices of triangular, hourglass, and Jagged-Rectangle Benzenoid systems, Hindawi, Journal of Chemistry, Vol.2018, 1-8.
- [7] B. Rajan, A. William, C. Grigoriens and S. Stephen, On certain topological indices of silicate, honeycomb, and hexagonal networks, J. Comp. And Math. Sci. Vol.3(5), 530-535(2012).
- [8] S. S. Shirkol, S. M. Hosamani and S. V. Patil, Degree based topological indices of penta chains, IOSR Journal of Mathematics, Vol.14, Issue 4, Ver.IV, July- August 2018, 26-37.
- [9] M. R. Farahani, Some connectivity indices and Zagreb index of Polyhex nanotubes, Acta Chim. Slov. 2012, 59, 779-783.

- [10]N.K.Raut, Zagreb group indices and polynomials, International Journal of Modern Engineering Research, Vol.6, Issue 6, 2016, 84-87.
- [11]N.K.Raut, F-polynomial and fourth Zagreb polynomial of a molecular graph, International Journal of Science and Research, Vol.7, April-2018, 615-616.
- [12]H.L.Parashivmurthy, M.R.R.Kanna and R.Jagadeesh, Topological indices of Nicotine, IOSR Journal of Engineering, Vol.9, Issue 1, (January 2019) ||V(II)||, 20-28.
- [13]G.Sridhara, M.R.R.Kanna and R.S.Indumathi, Computation of topological indices of graphene, Hindawi Publishing Corporation, Journal of Nanomaterials, Volume 2015, Paper ID:969348, 1-8.
- [14]W.Gao, M.R.Farahani, and M.R.R.Kanna, The multiple Zagreb indices of nanostructures and chains, Open Journal of Discrete Mathematics, 2016, 82-88.
- [15]R.P.Kumar, Soner Nandappa D., and M.R.R.Kanna, Redefined Zagreb, Randic, Harmonic and GA indices of graphene, International Journal of Mathematical analysis, Vol.11, 2017, No.10, 493-502.
- [16]A.Asghar, M.Rafahat, W.Nazeer and W.Gao, K Banhatti and K hyper Banhatti indices of circulant graphs, International journal of advanced and applied sciences, 5(5) 2018, 107-109.
- [17]F.Dayan, M.Javaid, M.Zulqarnain, M.T.Ali, and B.Ahmad, Computing Banhatti indices of Hexagonal, Honeycomb and desired networks, American Journal of Mathematical and Computer modeling, 2018, 3(2) 38-45.
- [18]V.R.Kulli, ABC Banhatti and Augmented Banhatti indices of chemical networks, Journal of Chemistry and Chemical Sciences, Vol.8(8), 1018-1025.
- [19]V.R.Kulli, On K-hyper Banhatti indices and coindices of graphs, International Research Journal of pure Algebra, 6(5), 2016, 300-304.
- [20]V.R.Kulli, On K Banhatti indices of graphs, Journal of Computer and Mathematical Sciences, 7(4), 2016, 213-218. le Benzenoid systems, Hindawi, Journal of Chemistry, Volume 2018, 1-8, <https://doi.org/10.1155/2018/8215950>.
- [21]V.R.Kulli, Computing Banhatti indices of networks, International Journal of Advances in Mathematics, Volume 2018, Number 1, 31-40.
- [22]I.Gutman, Topological indices and irregularity measures, Bulletin of the International Mathematical Institute, Vol.8 (2018), 469-475.

- [23] A. Khaksari, M. Ghorbani, On the forgotten topological index, Iranian J. Math. Chem. 8(3) 327-338, September (2017).
- [24] W. Gao, M. K. Siddiqui, M. Imran, M. K. Jamil, and M. R. Farahani, Forgotten topological index of chemical structure in drugs, Saudi Pharmaceutical Journal, 2016, 258-264.
- [25] G. E. Mehak, A. A. Bhatti, Forgotten topological index of line graph of some chemical structure in drugs, ACTA CHEMICA 1 AS1.26\_2.181-206, (2018).
- [26] R. Todeschini and V. Consonni, Handbook of Molecular Descriptors, Wiley Weinheim, 2000.
- [27] N. Deo, Graph Theory, Prentice-Hall of India, Private Ltd. New Delhi, 2007.
- [28] D. B. West, An Introduction to Graph theory, Prentice-Hall, 1996.
- [29] N. Trinajdtic, Chemical graph theory, CRC press, Boca Raton, FL, 1992.
- [30] J. A. Bondy and U. S. R. Murty, Graph Theory With Applications, Macmillan London and Elsevier, New York, 1976.