Influence of Multilayers On The Stress Intensity Factors Of A Buried Elliptical Crack.

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Abstract

The study of cracking mechanisms in multilayer structures has caught our attention in this work. In order to do this, we first went through a numerical validation of the method used, which is the boundary elements.

The method used allowed us to improve the computation time and reduce the numerical study of a dimension, it should be noted that this work also allowed to show the effectiveness of the method to treat the problems of partially open cracks. The results presented on the study of the multilayers were obtained following the study of a four-point bending on the one hand and a localized load on the upper face of the domain on the other hand applied on the multilayer in which one took care of buried an elliptical crack. The analysis of the results was done through the stress intensity factors. These simulations allowed us to highlight the influence of the position of the crack in the field of study

Keywords: *Boundary elements method, sub domains, crack front.*

1. Introduction

The study of multilayers today is an important thing with regard to the increasingly important use of composite materials in mechanical engineering (Franck FC, Lawn RB, Mouginot R, Maugis D [1], [2], [3]). To do so, we are interested in numerical methods that make it possible to analyze the behavior of these multilayers through simulations.

This has already been the subject of several researches. We can mention among others the methods of the finite elements and finite elements extended the methods without mesh etc. in this paper it will be necessary to predict the behavior of stress intensity factors along the crack front of a pre-existing crack trapped in a multilayer.

The method developed to achieve this uses the Boundary Element Method (BEM) boundary element method which has already been the object of a study on the behavior of a three-dimensional crack (Bush MB [4], Komvopoulos K [5], Choi HJ [6], eg Cruse [7], Mi et al [8], Domiguez et al [9], Young [10], Bonnet [11], Aliabadi [12]. The principle of unilateral contact is adopted. At first we will present the main lines of the method of resolution. Based on this, we will then define the field of study and the problem to be solved with the BEM, a method whose effectiveness has been shown in several books. (E.g. Man et al [14], Dandekar and Conant [15], Takahashi and Brebbia [16], Olukoko and Becker [17], Karami [18]). We consider a multilayer consisting of three sub-layers to which we will subject stresses; while taking care to consider that there is a pre-existing elliptical crack in the studied massif. Then the analysis of the results will be done through the stress intensity factors along the crack front. It should be noted that the objective is to understand the evolution of an elliptical crack in a multilayer and to highlight the impact of the nature of the stress and the position of the defect in the multilayer.

2. Problem definition

2.1. Statement of the problem

Consider a solid occupying a domain Ω of the frontier spaceΓ_n. We use the Cartesian coordinate system Ox_i with i = 1, 2, 3. We note Γc the surface of the crack. Volume forces are neglected, and Einstein's index notation and summation convention are adopted. The balance of forces and moments in the absence of volume forces is written in the given coordinate system:

$$\sigma_{ij,j} = 0 \quad dans \;\; \Omega \backslash \Gamma_n \cup \Gamma_c \tag{1}$$

With σ the Cauchy stress tensor, which in the case of linear elasticity in small deformations is related to the linearization tensor linearized by Hooke's law:

$$\sigma_{ij} = C_{ijkl}\, \varepsilon_{kl} \tag{2}$$

In relation (2), C_{ijkl} is the fourth order isotropic tensor of the elastic constants of the material. It is expressed as a function of the shear modulus (G) and the Poisson's ratio (ν) by:

$$C_{ijkl} = G\left[\frac{2\nu}{1-2\nu}\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}\right] \tag{3}$$

δ_{ij} is the Kronecker symbol such that: $\delta_{ij} = \begin{cases} 1 & si \;\; i = j \\ 0 & \text{otherwise} \end{cases}$

The tensor of the small deformations $\varepsilon_{kl}$is defined starting from the vector displacement u by:

$$\varepsilon_{kl} = \frac{1}{2}\left(u_{k,l} + u_{l,k}\right) \tag{4}$$

Let's assemble equations (2) and (4) and equation (1) can be rewritten as:

$$C_{ijkl}u_{k,jl} = 0$$

The flow-type boundary conditions associated with this equation are of the form:
The flow-type boundary conditions associated with this equation are of the form:

$$t_i = \sigma_{ij}n_j \tag{5}$$

With $\vec{n}$ the normal vector outside the considered boundary.

1.1 2.2 Method of solution

We use the so-called dual border element method [18]. The boundary of the domain is composed of the outer faces (Γ_n) and the crack Γ_c which is itself made up of two faces $\Gamma_c^+$and Γ_c^- so that $\Gamma_c = \Gamma_c^+ \cup \Gamma_c^-$. We introduce the displacement jump $\Delta u_j(x) = \Delta u_j(x^+) - \Delta u_j(x^-)$ and the constrained vector jump $\Delta t_j(x) = \Delta t_j(x^+) + \Delta t_j(x^-)$; x^+ (or. x^-) being a point, of the face Γ_c^+ (or.Γ_c^-), of geometric coordinate x. We limit ourselves to cases of symmetrically charged cracks. In this case, the discontinuity relation of the constrained vector disappears from the equations.
For any point belonging to the outer boundary of the environment, we adopt the classical integral formulation in displacement, namely:

$$\int_{\Gamma_n} T_{ki}(x,y)[u_i(y) - u_i(x)]dS(y) + \int_{\Gamma_c^+} T_{ki}(x,y)\Delta u_i(y)dS(y) = \int_{\Gamma_n} U_{ki}(x,y)t_i(y)dS(y) \tag{6}$$

$$t_k(x) = \int_{\Gamma_n} \overline{T}_{ki}(x,y)u_i(y)dS(y) + \int_{\Gamma_c^+} \overline{T}_{ki}(x,y)\Delta u_i(y)dS(y) - \int_{\Gamma_n} \overline{U}_{ki}(x,y)t_i(y)dS(y) \tag{7}$$

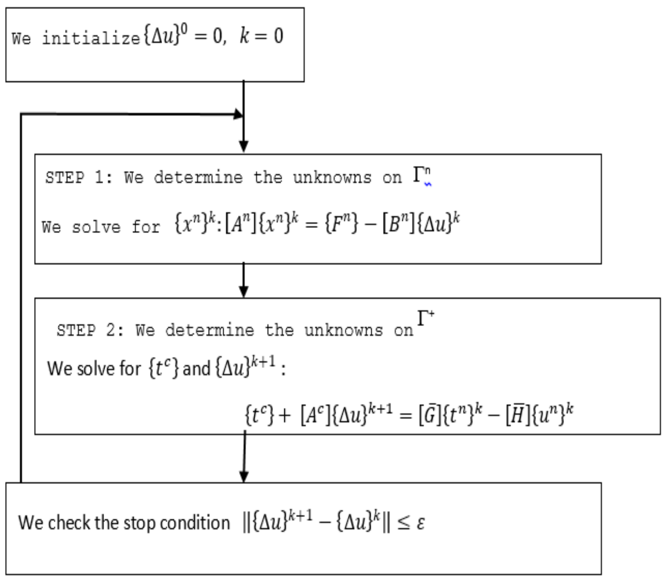
For the points located on the upper face of the crack, we adopt the constraint vector formulation:
In this formulation, the unknowns of the problem are the displacement and constraint vectors on the outer boundary, and displacement jump and stress on the faces of the crack.
The expressions of the influence functions that appear in equations (7) and (8) can be found in many works (e.g. [19, 20]). As already mentioned, the unknowns on the crack surfaces are the displacement jump vector ($\Delta\vec{u}$) and the stress vector $\vec{t}(x^+)$.
The outer boundary and the face of the upper crack are subdivided into a finite number of elements. Nine compliant and / or semi-discontinuous elements are used for the elements of Γn. The elements of the surface of the crack are discontinuous. For the elements of the crack front, the interpolation introduced in Kouitat et al. [21] was adopted.
The discretized form of the boundary equations (eq. (7) and (8)) then leads to systems of equations of the following form:

$$[A^n]\{x^n\} = \{F^n\} - [B^n]\{\Delta u\}$$
$$\{T^c\} + [A^c]\{\Delta u\} = [\bar{G}]\{t^n\} - [\bar{H}]\{u^n\} \quad (8)$$

Where $\{x^n\}$ is the vector of unknown nodal quantities on the outer boundary (displacements and constrained vectors); $\{\Delta u\}$ is the vector of jumps of nodal displacements; {u ^ n} is the vector of nodal displacements on the outer boundary; $\{t^n\}$ is the vector of nodal stress vectors on the outer boundary; $\{T^c\}$ is the vector of the nodal stress vectors on the surface of the crack.
For the envisaged loading, the considered crack can be subjected to a mode of complex solicitation. On the face of the crack, the jumps of displacements and the constrained vectors are not known in advance. The crack may be completely closed, partially closed or completely open. Consequently, it is necessary to impose a unilateral contact condition on the faces of the crack. At a point x of the surface of the crack, we introduce the local coordinate system$(\vec{n}(x), \vec{\tau}(x), \vec{\rho}(x))$. The vector $\vec{n}(x)$ designates the normal external to the surface of the crack at the point x such that $\vec{\tau}(x)$ and $\vec{\rho}(x)$ are two mutually orthogonal vectors, and tangent to the plane of the crack. Normal displacement discontinuity and normal stress must satisfy:

$$\Delta u_j n_j = -\Delta U_n \leq 0,\ t_j n_j = -t_n \leq 0 \quad et \quad t_n \Delta u_n = 0 \quad (9)$$

If the crack opens, then $\Delta U_n \neq 0$ and$t_n = 0$; otherwise $\Delta U_n = 0$ and$t_n \neq 0$.
We assume that the contact on the faces of the crack is frictionless. This indicates that the tangential components $(t_\tau\ et\ t_\rho)$ of the constrained vector are identically zero.
We have chosen to solve equations (8) and (9), combined with the one-sided contact condition, by an iterative scheme whose algorithm is summarized below:



Where ε is the desired precision.

This algorithm can be seen as a different presentation of the superposition method to solve the crack problems presented by Ameen and Raghuprasad [22] and applied to 2D compression fracture problems by Elvin and Leung [23] and by De Bremaecker and al. [24]. In a previous work, compression cracking is formulated as a complementarity problem and solved by the "PATH solver" algorithm. The algorithm has been used successfully by Christensen et al in [25].

The matrix equation system solved in step 1 remains unchanged during the iterative process. The matrix of the system is then factorized once and for all and stored, which allows a computation time saving. In step 2, the system of equations must be completed by the contact equations. For each node of the crack boundary, we write:

$$\begin{cases} min(\Delta u_n, t_n) = 0 \\ t_\tau = 0 \\ t_\rho = 0 \end{cases} \qquad (10)$$

The system obtained is therefore nonlinear and not differentiable in the classical sense. We applied Newton's method using Bouligand's differentiability (B-differentiability) presented in Christensen et al. [22] and Martin Schwartz [26]. We emphasize that the need for a directional derivative is due to the min function in equation (10).

The results presented in the following were obtained using this algorithm. In all the cases tested, the average number of iterations to converge with ε = 10-12 is about 15.

3. Results

We specify that all the results presented in this part were obtained in a numerical experiment where we consider the isotropic domain.

3.1 Validation of the method

To validate our method of calculation we have taken over the digital experience which deals with the problem solved by Sneddon. For the particular case of a circular crack fleeing into an infinite mass subjected to uniform traction on the two lower and upper faces has been treated by Sneddon and the expression of the analytical solution of such a problem is given as follows(Ian Naismith Sneddon 1946; Ian N. Sneddon 1969; Sneddon I.N., s. d.) :

$$K_I = 2\sigma\sqrt{\frac{r}{\pi}} \qquad (11)$$

$$\begin{cases} stress: \sigma \\ radius\ of\ the\ crack:\ r \end{cases}$$

We consider the field of study represented in the following figure which is a parallelepiped. The material parameters used are a Poisson's ratio of ν = 0.3 and a Young's modulus E = 69Gpa. The domain is discretized by elements with nine nodes and consists of 96 elements distributed in 486 border nodes. The crack is circular and horizontal placed in the center in the area (see figure) it is discretized by 16 elements distributed in 288 nodes. The size of the crack and the domain depends on the parameters r and h specified below. The constant and uniform charge density applied is σ = 1.d-2Gpa. The side faces of the domain are free of stress the center of the domain is constrained to axial displacement the upper and lower faces are subjected to uniform traction.

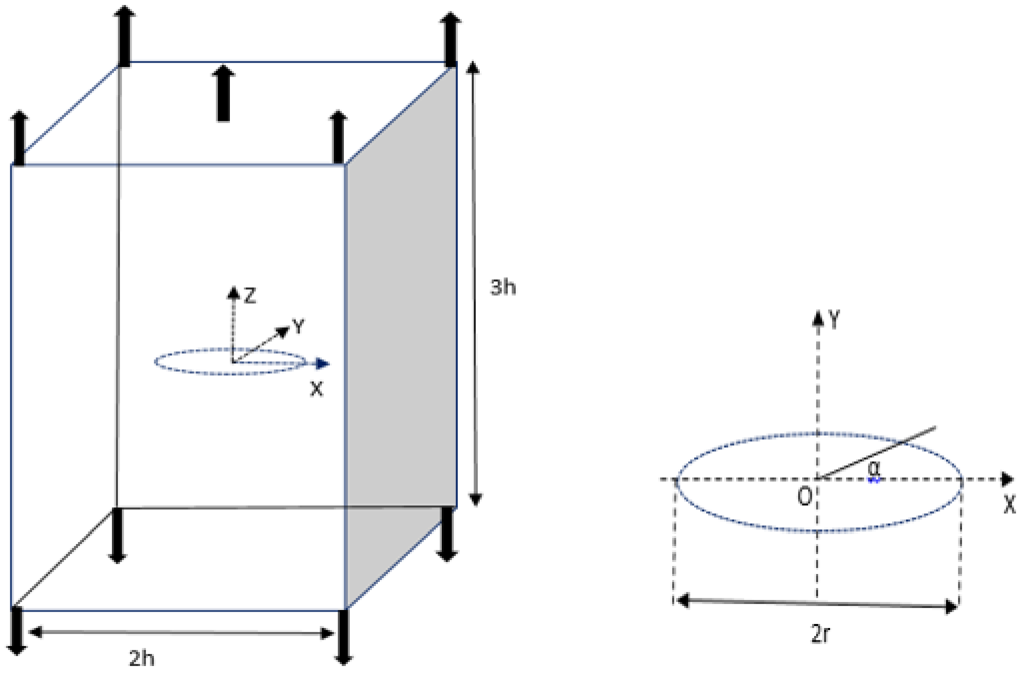


Fig. 1: Geometric representation of the domain and the crack

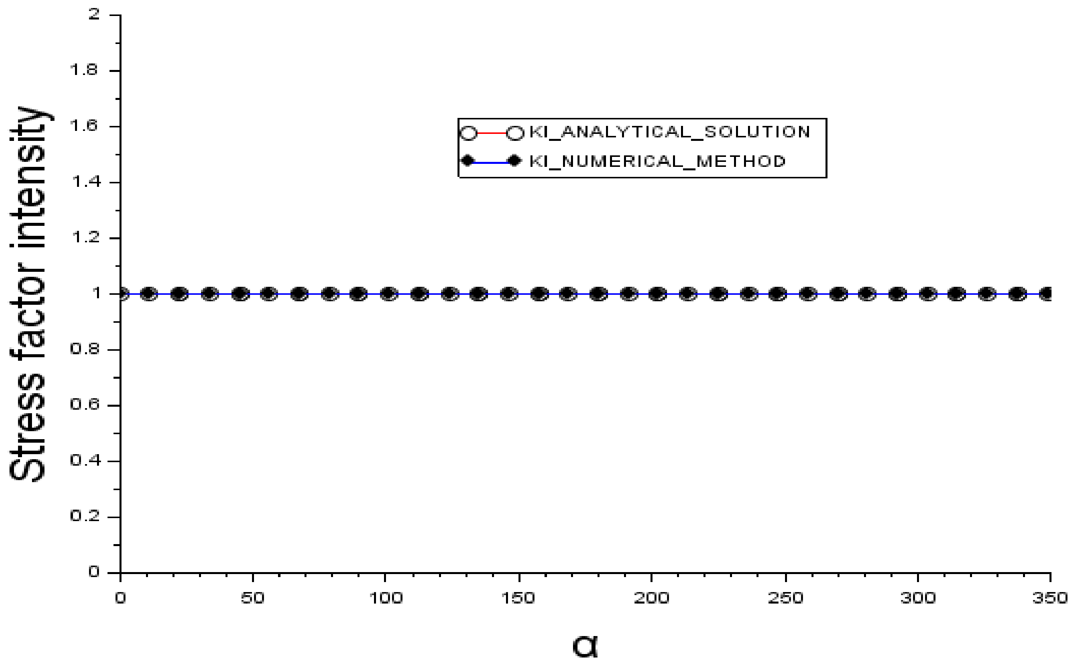


Fig.2: Evolution of KI for the Sneddon problem

Figure 2 shows that there is a good agreement between the numerical computation and the analytical solution of Sneddon. The relative error between the maximum numerical value of the KI and the Sneddon solution is minus 2% (Ian Naismith Sneddon 1946, [28]).

3.2 Case of flexion

The material parameters are a Young's modulus of
E = 69Gpa and a Poisson's ratio of ν = 0.3. All the results presented were obtained in isotropy. For reasons of symmetry we will represent a portion of the crack front in the results of our simulations. The examples presented below are obtained with a uniform charge density applied as a bias. The analysis of the results is with the stress intensity factors (KI, KII and KIII) dimensioned by:

$$K_0 = \sigma\sqrt{2\pi r}$$

The three factors called stress intensity factors determine the displacement of the crack lips by making a combination of three main modes denoted I, II and III.

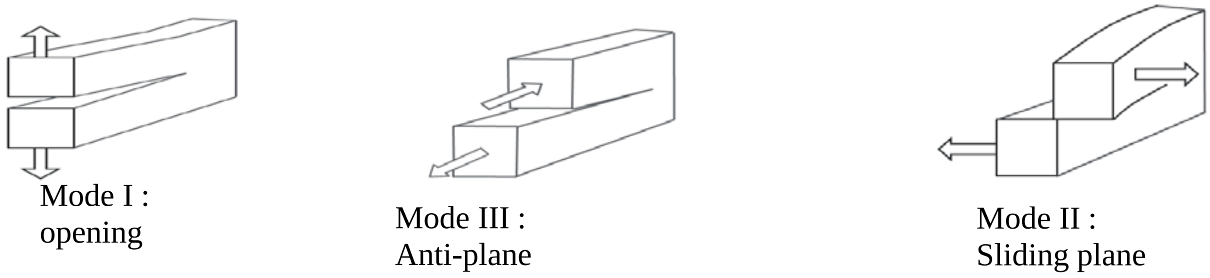


Fig.3: Failure modes of a crack

The solar modules when they are fixed on a support during its operation undergo compression at its ends. In this part, the idea is to reproduce this mechanical action and to analyze its impact on our field of study.
The multilayer consists of three layers, each layer is discretized with 216 elements divided into 1014 border nodes completed by 847 internal nodes. The elliptic crack is placed in the plane (XOZ) centered in the domain following the studied layer and is discretized with 32 elements distributed in 288 nodes. The study marker is a Cartesian coordinate system centered in the field. The lateral faces are free of movement the lower face is in plane support as shown in the following figure. The upper is loaded with a uniform charge density σ at both ends along the axis (OY).

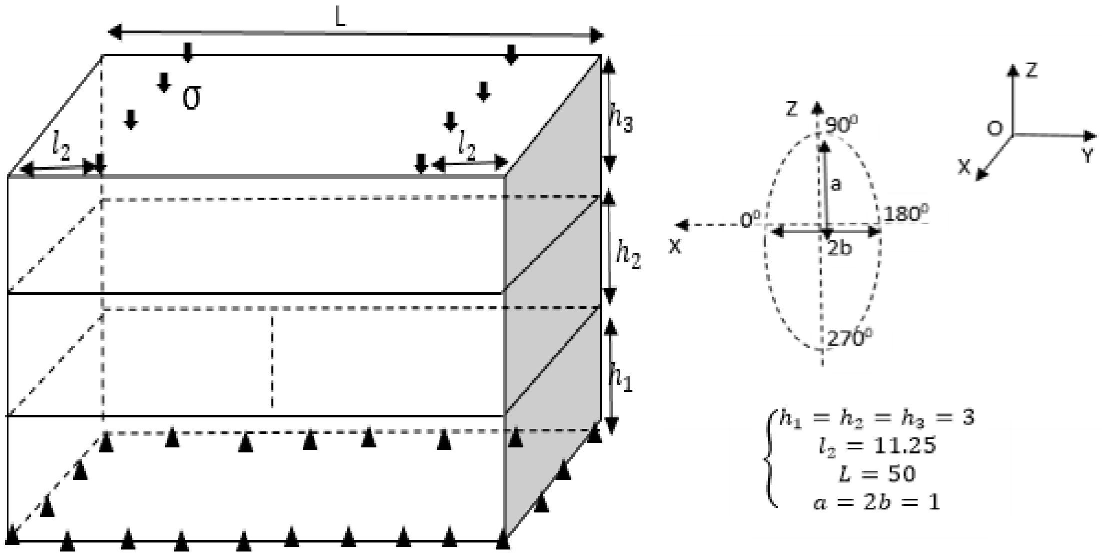


Fig.4:Geometry of domains: elliptic crack, multilayer

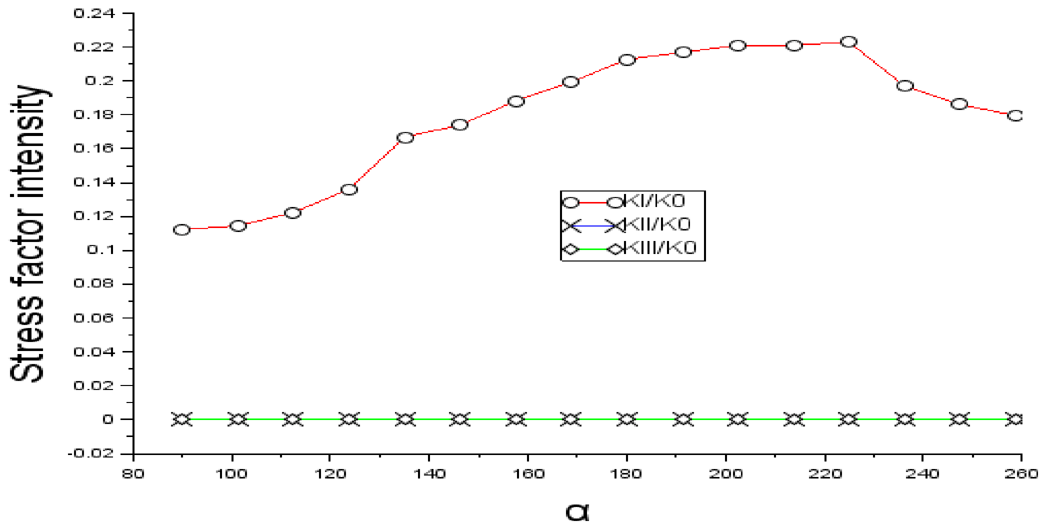


Fig. 5: *Layer1 / evolution of stress intensity factors*

The previous figure above shows the behavior of the stress intensity factors when the crack is in the bottom layer, the result allows to say that this position of the crack is the most dangerous configuration because the crack opens totally because the KI factor is not zero. The other two modes are null (KII and KIII)

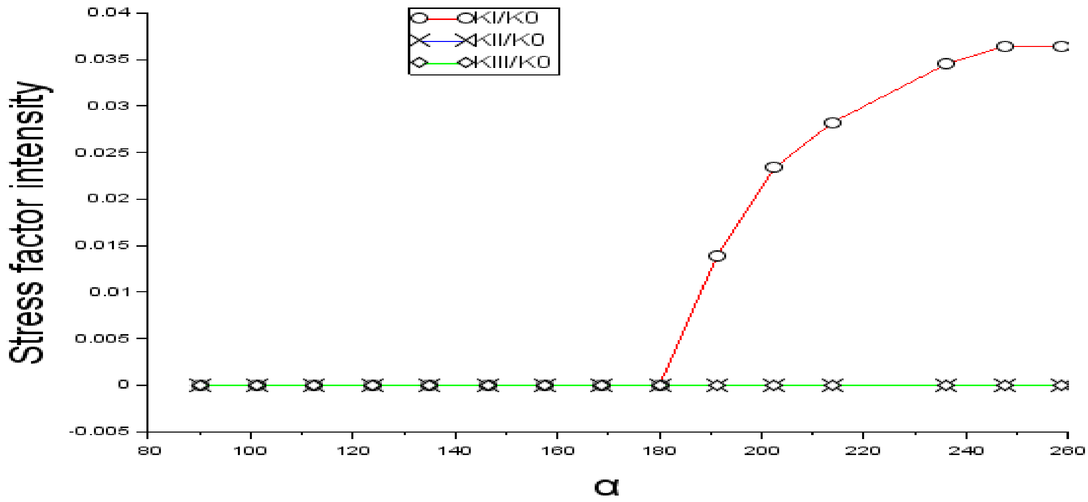


Fig. 6: *Layer2 / evolution of stress intensity factors*

The figure above represents the case where the crack is in the middle layer. Modes II and III are inactivated, only mode I is active, it should also be noted that the opening of the crack is partial. This result makes it possible to show the effectiveness or even the efficiency of the code (BEM) to treat the cases of partially open crack. This is often complex with existing calculation codes.

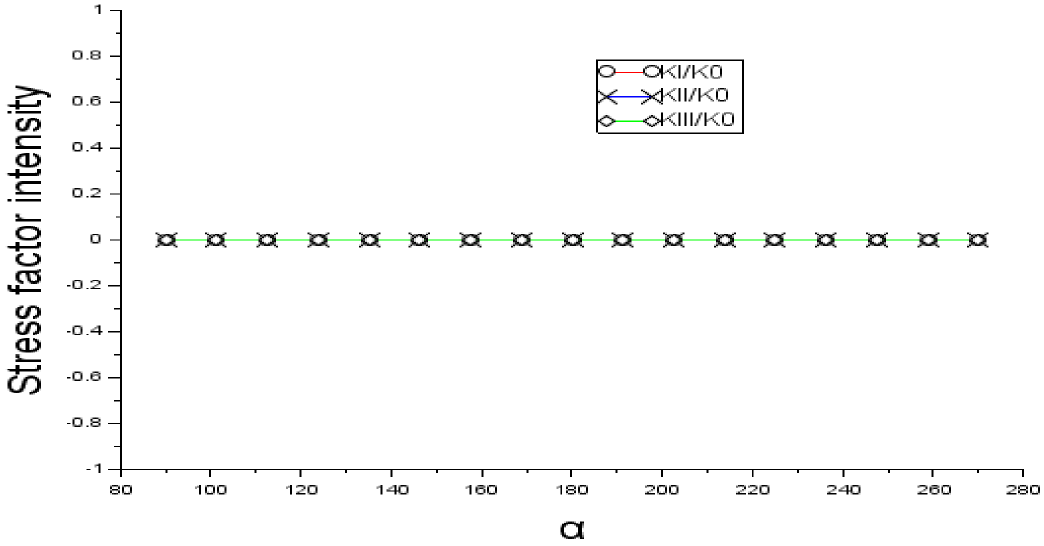
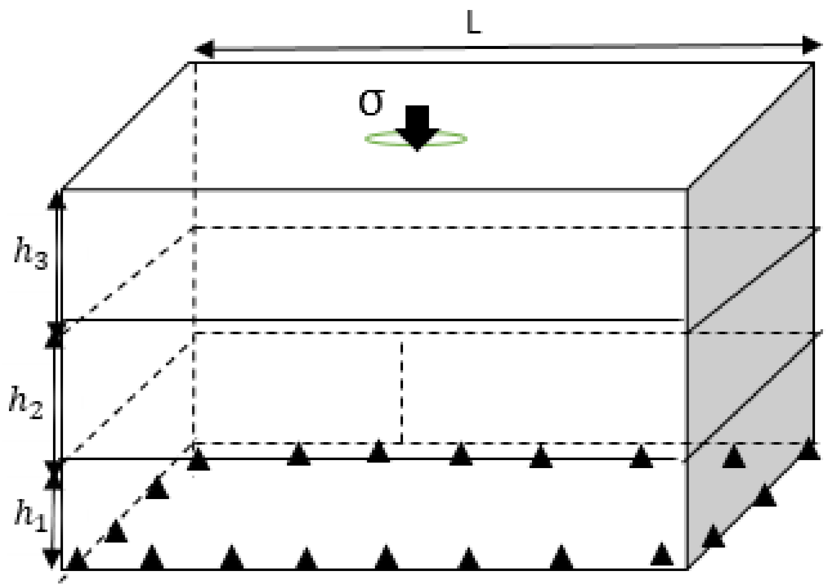


Fig. 7: Layer3 / evolution of stress intensity factors

For this type of loading the crack is in a dangerous configuration when it moves away from the loading face. The results show that the method makes it possible to treat the cases of partially open crack and to study the multilayer structures.

1.1 3.3 Load at a localized point

In this part, we study a particular case of mechanical action that a multilayer system exposed to the actions of nature may encounter. Indeed a projectile can target our system stored in nature. The multilayer consists of three layers, each layer is discretized with 216 elements divided into 1014 border nodes completed by 847 internal nodes. The elliptic crack is placed in the plane (XOZ) centered in the domain following the studied layer and is discretized with 32 elements distributed in 288 nodes. The study marker is a Cartesian coordinate system centered in the field. The lateral faces are free of movement the lower face is in plane support as shown in the following figure. The upper is loaded at the center point of coordinates (0 0 Z).



$$\begin{cases} h_1 = h_2 = h_3 = 3 \\ L = 50 \\ a = 2b = 1 \end{cases}$$

Fig. 8: Geometry of the field of study

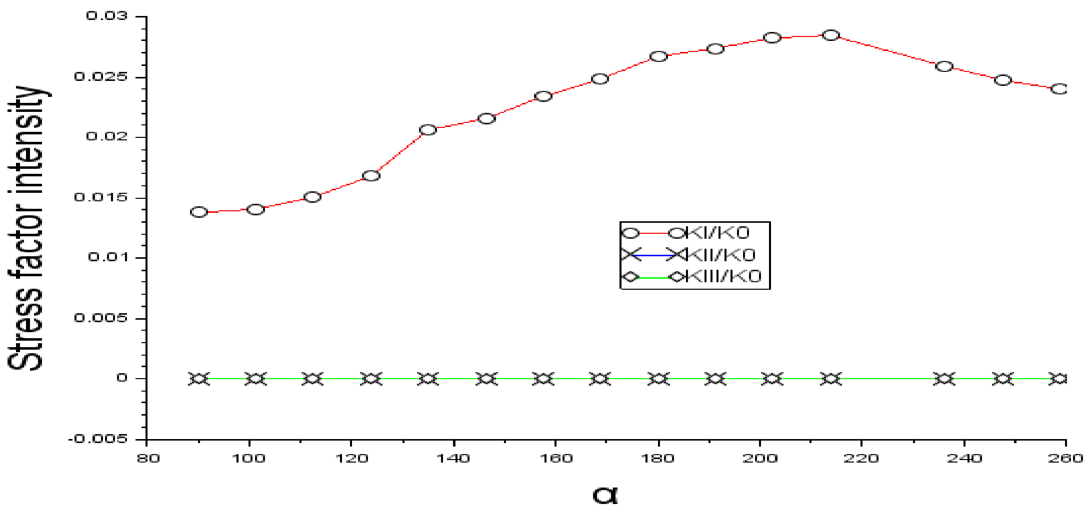


Fig. *9: Layer1 / evolution of stress intensity factors*

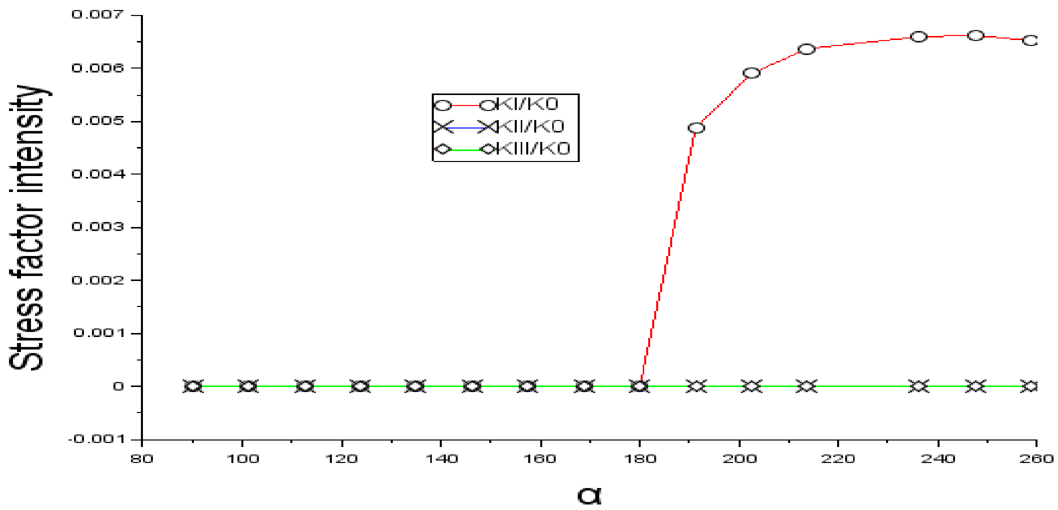


Fig.10: Layer3 / evolution of stress intensity factors

The dangerous configuration always remains the furthest position from the loading face. The shape of the curves is identical to the previous case, except for the changing amplitudes. Indeed the impact of a projectile on a building such as solar modules would be less dangerous than the pressure exerted on its surface during installation or maintenance.

4. Conclusion

In this paper we studied two types of solicitation; and depending on the nature of the solicitation we could see the impact of the multilayer on a pre-existing crack imprisoned in the field of study. We also highlighted once again that the defect position influenced the behavior of stress intensity factors. These results are intended to predict the hazards of a mechanical action on a building (multilayer or composite materials). We made an extension to the solar modules and through our results we showed the effectiveness of the code dealt with the problems of cracking in the multilayer and on the other hand showed that our work could be applied to a particular case of multilayer which are the solar modules. Nevertheless it would be necessary for a very precise work to respect the number of layers constituting a module in order to be able to make the work even more interesting that would be the object of our future investigations.

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