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ANY l – STATE SOLUTIONS OF THE SCHRÖDINGER EQUATION INTERACTING WITH CLASS OF YUKAWA - ECKART POTENTIALS

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Abstract

The approximate analytical solutions of the Schrödinger equation is obtain with a class of Yukawa plus Eckart potentials, with the Greene-Aldrich approximation in the centrifugal term. The energy eigenvalues and the corresponding normalized wave function expressed in terms of Jacobi polynomial are calculated using the Nikiforov-Uvarov method. The energy eigenvalues for three special cases of Hellmann, Coulomb and inversely quadratic Yukawa potentials were obtained. The bound state energy eigenvalues expressions and numerical computations agreed with the already existing literature.

Keywords: Schrödinger equation; class of Yukawa potential; Eckart potential; Nikiforov-Uvarov method

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1.0 INTRODUCTION

In the recent time, Physicists have developed much interest in searching for exponential-type potentials. The reason is that most of the exponential-type potentials play an essential role in physics for example, Yukawa potential is used in solid-state, plasma and atomic physics [1]. A few of these potentials have been solved exactly while others can only be solved approximately [2,3] with the use of different approximation schemes [4,5]. Different methods have been employed to obtain the solution of the non-relativistic wave equations with chosen interacting potential models. These include the factorization method [6], functional analysis approach [7], super symmetry quantum mechanics (SUSYQM) [8], asymptotic iteration method (AIM) [9], exact and proper quantization rules [10], Nikiforov-Uvarov method (NU) [11, 12] and others. With the above different methods, many researchers have devoted much interest in solving the radial Schrödinger equation (SE) to obtain bound state solutions for some physical potential models [13-16]. For instance, Edet et al. [17] obtained bound state solutions of the Schrödinger equation for the modified Kratzer potential plus screened Coulomb potential including a centrifugal term. Hamzavi et al. [18], obtained an approximate bound states solution of the Hellmann potential. Edet et al. [19] obtained any l-state solutions of the SE interacting with Hellmann-Kratzer potential model. The class of Yukawa potential is greatly important with applications, cutting across nuclear physics and condensed matter physics [20]. Eckart potential is one of the most important exponential-type potential in atomic physics and chemical physics. Recently, there has been great interest in combining two or more potentials in both the relativistic and non-relativistic regime [12, 17]. The essence of combining two or more physical potential models is to have a wider range of applications. Hence, motivated by the success of the combination of exponential-type potentials, we seek to investigate the bound state solutions of the SE with combined (class of Yukawa and Eckart) potentials of the form [9, 14].

$$V(r)=-\frac{br+cre^{-\delta r}-ae^{-2\delta r}}{r^{2}}-\frac{de^{-\delta r}}{1-e^{-\delta r}}+\frac{pe^{-\delta r}}{\left(1-e^{-\delta r}\right)^{2}} \tag{1}$$

where a, b, c, d, and p are potential strength, δ is the screening parameter. It can be deduced that when $d = p = 0$, Eq.(1) reduces to class of Yukawa potential. Also when $a = d = p = 0$, Eq. (1) reduces to Hellmann potential. When $a = c = d = p = \delta = 0$, Eq.(1) reduces to Coulomb potential. Also when $a = b = c = 0$,Eq. (1) reduces to Eckart potential. It is noted that the exact solution of the SE with the combined potentials in Eq. (1) is not possible due to the presence of the inverse square term or centrifugal term. Therefore, to obtain approximate solutions, we employ a suitable approximation scheme. It is found that such approximation proposed by Greene and Aldrich [21]

$$\frac{1}{r^2} \approx \frac{\delta^2}{\left(1-e^{-\delta r}\right)^2} \tag{2}$$

Is a good approximation to the centrifugal or inverse square term which is valid for δ<<1 for a short range potential. The paper is organized as follows: In section 2, the NU method is reviewed, in section 3 the bound state energy eigenvalues and corresponding normalized wave function are calculated ,in section 4 the results are discussed. In section 5, conclusion is presented.

2.0 REVIEW OF NIKIFOROV-UVAROV (NU) METHOD

The NU method was proposed by Nikiforov and Uvarov [22] to transform Schrödinger-like equations into a second-order differential equation via a coordinate transformation $s = s(r)$, of the form

$$\psi''(s)+\frac{\tilde{\tau}(s)}{\sigma(s)}\psi'(s)+\frac{\tilde{\sigma}(s)}{\sigma^2(s)}\psi(s)=0 \tag{3}$$

where $\tilde{\sigma}(s)$ *and* $\sigma(s)$ are polynomials, at most second degree and $\tilde{\tau}(s)$ is a first-degree polynomial. The exact solution of Eq.(3) can be obtain by using the transformation.

$$\psi(s)=\phi(s)\chi(s) \tag{4}$$

This transformation reduces Eq. (3) into a hypergeometric-type equation of the form

$$\sigma(s)\chi''(s)+\tau(s)\chi'(s)+\lambda\chi(s)=0 \tag{5}$$

The function $\Phi(s)$ can be defined as the logarithm derivative

$$\frac{\phi'(s)}{\phi(s)}=\frac{\pi(s)}{\sigma(s)} \tag{6}$$

With $\pi(s)$ being at most a first-degree polynomial. The second part of $\psi(s)$ being $\chi(s)$ in Eq. (5) is the hypergeometric function with its polynomial solution given by Rodrigues relation as

$$\chi_n(s)=\frac{B_n(s)}{\rho(s)}\frac{d^n}{ds^n}\left[\sigma^n(s)\rho(s)\right] \tag{7}$$

where B_n is the normalization constant and $\rho(s)$ the weight function which satisfies the condition below;

$$\frac{d}{ds}\left(\sigma(s)\rho(s)\right)=\tau(s)\rho(s) \tag{8}$$

where also

$$\tau(s)=\tilde{\tau}(s)+2\pi(s) \tag{9}$$

For bound solutions, it is required that

$$\frac{d\tau(s)}{ds}<0 \tag{10}$$

The eigenfunctions and eigenvalues can be obtained using the definition of the following function $\pi(s)$ and parameter λ, respectively:

$$\pi(s)=\frac{\sigma'(s)-\tilde{\tau}(s)}{2}\pm\sqrt{\left(\frac{\sigma'(s)-\tilde{\tau}(s)}{2}\right)^2-\tilde{\sigma}(s)+k\sigma(s)} \tag{11}$$

and

$$\lambda=k+\pi'(s) \tag{12}$$

The value of k can be obtained by setting the discriminant in the square root in Eq. (11) equal to zero. As such, the new eigenvalues equation can be given as

$$\lambda+n\tau^{'}(s)+\frac{n(n-1)}{2}\sigma^{"}(s)=0,(n=0,1,2,...) \tag{13}$$

3.0 APPROXIMATE SOLUTION OF THE SCHRÖDINGER EQUATION WITH CLASS OF YUKAWA PLUS ECKART POTENTIALS

The SE for two particles interacting via a spherically symmetric potential $V(r)$ in three dimensional space, takes the form

$$\frac{d^2R(r)}{dr^2}+\frac{2\mu}{\hbar^2}\left[E_{nl}-V(r)-\frac{\hbar^2}{2\mu}\frac{l(l+1)}{r^2}\right]R(r)=0 \tag{14}$$

where $l,\ \mu,\ r\ and\ \hbar$ are the angular momentum, the reduced mass for the particle, inter-particle distance and reduced plank constant respectively.

By substituting Eq. (1) and Eq. (2) into Eq. (14) we obtain

$$\frac{d^2R(r)}{dr^2}+\frac{2\mu}{\hbar^2}\begin{bmatrix}E_{nl}+\frac{b\delta}{1-e^{-\delta r}}-\frac{ce^{-\delta r}\delta}{1-e^{-\delta r}}+\frac{ae^{-2\delta r}\delta^2}{\left(1-e^{-\delta r}\right)^2}+\frac{de^{-\delta r}}{1-e^{-\delta r}}\\ -\frac{pe^{-\delta r}}{\left(1-e^{-\delta r}\right)^2}-\frac{\hbar^2}{2\mu}\frac{l(l+1)\delta^2}{\left(1-e^{-\delta r}\right)^2}\end{bmatrix}R(r)=0 \quad (15)$$

By using coordinate transformation

$$s(r)=e^{-\delta r} \quad (16)$$

We obtain the differential equation of the form

$$\frac{d^2R(s)}{ds^2}+\frac{1}{s}\frac{dR(s)}{ds}+\frac{1}{s^2}\begin{bmatrix}\frac{2\mu E}{\delta^2\hbar^2}+\frac{2\mu b}{\delta\hbar^2(1-s)}-\frac{2\mu cs}{\delta\hbar^2\left(1-s\right)}+\frac{2\mu as^2}{\hbar^2\left(1-s\right)^2}\\ +\frac{2\mu ds}{\delta^2\hbar^2\left(1-s\right)}-\frac{2\mu ps}{\delta^2\hbar^2\left(1-s\right)^2}-\frac{l(l+1)}{\left(1-s\right)^2}\end{bmatrix}R(s)=0 \quad (17)$$

Let,

$$\left.-\varepsilon=\frac{2\mu E}{\delta^2\hbar^2},\alpha_1=\frac{2\mu b}{\delta\hbar^2},\ \alpha_2=\frac{2\mu c}{\delta\hbar^2},\ \alpha_3=\frac{2\mu a}{\hbar^2},\ \alpha_4=\frac{2\mu d}{\delta^2\hbar^2},\ \alpha_5=\frac{2\mu p}{\delta^2\hbar^2}\ ,\gamma=l(l+1)\right\} \quad (18)$$

By substituting Eq. (18) into Eq. (17) we obtain the differential equation of the form

$$\frac{d^2R(s)}{ds^2}+\frac{1-s}{s(1-s)}\frac{dR(s)}{ds}+\frac{1}{s^2\left(1-s\right)^2}\begin{bmatrix}-\left(\varepsilon-\alpha_2-\alpha_3-\alpha_4\right)s^2+\\ \left(2\varepsilon-\alpha_1-\alpha_2+\alpha_4-\alpha_5\right)s-\left(\varepsilon-\alpha_1+\gamma\right)\end{bmatrix}R(s)=0 \quad (19)$$

Comparing Eqs. (19) and (3) ,we have the following parameters

$$\left.\tilde{\sigma}(s)=-\left(\varepsilon-\alpha_2-\alpha_3-\alpha_4\right)s^2+\left(2\varepsilon-\alpha_1-\alpha_2+\alpha_4-\alpha_5\right)s-\left(\varepsilon-\alpha_1+\gamma\right),\ \sigma(s)=s\left(1-s\right),\ \tilde{\tau}(s)=1-s\right\} \quad (20)$$

Substituting Eq. (20) into Eq. (11) ,we obtain $\pi(s)$ to be

$$\pi(s)=\frac{-s}{2}\pm\sqrt{\left(A-k\right)s^2+\left(k+B\right)s+C} \quad (21)$$

where

$$\left.A=\frac{1}{4}+\varepsilon-\alpha_2-\alpha_3-\alpha_4,\ B=-\left(2\varepsilon-\alpha_1-\alpha_2+\alpha_4-\alpha_5\right),\ C=\varepsilon-\alpha_1+\gamma\right\} \quad (22)$$

To find the constant k , the discriminant of the expression under the square root of Eq. (21) must be equal to zero. As such we have that

$$k=\alpha_1-\alpha_2+\alpha_4-\alpha_5-2\gamma\pm 2\sqrt{\varepsilon-\alpha_1+\gamma}\sqrt{\gamma-\alpha_3-2\alpha_4+\alpha_5+\frac{1}{4}} \tag{23}$$

Substituting Eq. (23) into Eq. (21) we obtain

$$\pi(s)=-\frac{s}{2}\pm\left(\sqrt{\varepsilon-\alpha_1+\gamma}+\sqrt{\gamma-\alpha_3-2\alpha_4+\alpha_5+\frac{1}{4}}\right)s-\sqrt{\varepsilon-\alpha_1+\gamma} \tag{24}$$

With $\tau(s)$ being obtained as

$$\tau(s)=1-2s-2\sqrt{\varepsilon-\alpha_1+\gamma}s-2\sqrt{\gamma-\alpha_3-2\alpha_4+\alpha_5\frac{1}{4}}s+2\sqrt{\varepsilon-\alpha_1+\gamma} \tag{25}$$

Referring to Eq. (12),we define the constant λ as

$$\begin{aligned}\lambda=&-\frac{1}{2}-\sqrt{\varepsilon-\alpha_1+\gamma}-\sqrt{\gamma-\alpha_3-2\alpha_4+\alpha_5+\frac{1}{4}}-(\alpha_1-\alpha_2+\alpha_4-\alpha_5-2\gamma)\\&-2\sqrt{\varepsilon-\alpha_1+\gamma}\sqrt{\gamma-\alpha_3-2\alpha_4+\alpha_5+\frac{1}{4}}\end{aligned} \tag{26}$$

Taking the derivative of Eq. (25) with respect to s , we have

$$\tau^{'}(s)=-2-2\left(\sqrt{\varepsilon-\alpha_1+\gamma}+\sqrt{\gamma-\alpha_3-2\alpha_4+\alpha_5+\frac{1}{4}}\right) \tag{27}$$

And also taking the derivative of $\sigma(s)$ with respect to s from Eq. (20), we have

$$\sigma^{''}(s)=-2 \tag{28}$$

Substituting Eqs. (25) and (28) into Eq. (13),we obtain

$$\lambda_n=n^2+n+2n\left(\sqrt{\varepsilon-\alpha_1+\gamma}+\sqrt{\gamma-\alpha_3-2\alpha_4+\alpha_5+\frac{1}{4}}\right) \tag{29}$$

Equating Eqs. (26) and (29) and substituting Eq.(18) yields the energy eigenvalues equation of the class of Yukawa plus Eckart potentials in the form

$$E_{nl}=\frac{\delta^2\hbar^2 l(l+1)}{2\mu}-\delta c-\frac{\delta^2\hbar^2}{8\mu}\left[\frac{\left(n+\frac{1}{2}\right)^2+2\left(n+\frac{1}{2}\right)\sqrt{l(l+1)-\frac{2\mu a}{\hbar^2}-\frac{4\mu d}{\delta^2\hbar^2}+\frac{2\mu p}{\delta^2\hbar^2}+\frac{1}{4}}-Q}{n+\frac{1}{2}+\sqrt{l(l+1)-\frac{2\mu a}{\hbar^2}-\frac{4\mu d}{\delta^2\hbar^2}+\frac{2\mu p}{\delta^2\hbar^2}+\frac{1}{4}}}\right]^2 \tag{30}$$

where,

$$Q=\frac{2\mu b}{\delta\hbar^2}+\frac{2\mu c}{\delta\hbar^2}-\frac{2\mu d}{\delta^2\hbar^2}+\frac{2\mu p}{\delta^2\hbar^2}+l(l+1) \tag{31}$$

Special cases

1. When we set $a=d=p=0$ in Eq. (30), we obtain the energy eigenvalues for Hellmann potential

$$E_{nl}=\frac{\delta^2\hbar^2 l(l+1)}{2\mu}-\delta c-\frac{\delta^2\hbar^2}{8\mu}\left[\frac{(n+l+1)^2+\frac{2\mu}{\delta\hbar^2}(c-b)+l(l+1)}{(n+l+1)}\right]^2 \tag{32}$$

Equation (32) is in agreement with Eq. (38) of Ref. [12]

2. When we set $a=c=d=p=\delta=0$ in Eq. (30), we obtain the energy eigenvalues for Coulomb potential

$$E_{nl}=-\frac{\mu b^2}{2\hbar^2(n+l+1)^2} \tag{33}$$

The result of Eq. (33) is consistent with the result obtained in Eq. (36) in Ref. [17]

3. When we set $b=c=d=p=0$ in Eq. (30), we obtain the energy eigenvalues for inversely quadratic Yukawa potential

$$E_{nl}=\frac{\delta^2\hbar^2 l(l+1)}{2\mu}-\frac{\delta^2\hbar^2}{8\mu}\left[\frac{\left(n+\frac{1}{2}\right)^2+2\left(n+\frac{1}{2}\right)\sqrt{l(l+1)-\frac{2\mu a}{\hbar^2}+\frac{1}{4}}+l(l+1)}{n+\frac{1}{2}+\sqrt{l(l+1)-\frac{2\mu a}{\hbar^2}+\frac{1}{4}}}\right]^2 \tag{34}$$

To obtain the corresponding wavefunction, we consider Eq. (6) and upon substituting Eqs. (20) and (24) and integrating, we get

$$\phi(s)=s^{\sqrt{\varepsilon+\gamma}}(1-s)^{\frac{1}{2}+\sqrt{\frac{1}{4}+\gamma}} \tag{35}$$

To get the hypergeometric function considering Eq. (4), we first determine the weight function of Eq. (8) , upon differentiating the left hand side we obtain

$$\frac{\rho'(s)}{\rho}=\frac{\tau(s)-\sigma'(s)}{\sigma(s)} \tag{36}$$

Substituting Eqs. (24) and (20) into Eq. (36) and integrating ,thereafter simplify we obtain

$$\rho(s)=s^{2\sqrt{\varepsilon+\gamma}}(1-s)^{2\sqrt{\frac{1}{4}+\gamma}} \tag{37}$$

By substituting Eqs. (24) and (37) into Eq. (7) we obtain the Rodrigue's equation as

$$\chi_n(s) = B_{nl} s^{-2\sqrt{\varepsilon+\gamma}} (1-s)^{-2\sqrt{\frac{1}{4}+\gamma}} \frac{d^n}{ds^n}\left[s^{n+2\sqrt{\varepsilon+\gamma}} (1-s)^{n+2\sqrt{\frac{1}{4}+\gamma}} \right] \quad (38)$$

where B_{nl} is normalization constant.

Equation (38) is a equivalent to

$$P_n^{\left(2\sqrt{\varepsilon+\gamma},2\sqrt{\frac{1}{4}+\gamma}\right)}(1-2s) \quad (39)$$

where $p_n^{(\alpha,\beta)}$ is Jacobi Polynomial

The wave function is given by

$$\psi_{nl}(s) = B_{nl} s^{\sqrt{\varepsilon+\gamma}} (1-s)^{\frac{1}{2}+\sqrt{\frac{1}{4}+\gamma}} P_n^{\left(2\sqrt{\varepsilon+\gamma},2\sqrt{\frac{1}{4}+\gamma}\right)}(1-2s) \quad (40)$$

Using the normalization condition, we obtain the normalization constant as follows

$$\int_0^\infty |\psi_{nl}(r)|^2 dr = 1 \quad (41)$$

From our coordinate transformation of Eq. (16), we have that

$$-\frac{1}{\alpha s}\int_1^0 |\psi_{nl}(\mathrm{s})|^2 ds = 1 \quad (42)$$

By letting, $y = 1-2s$, we have

$$\frac{B_{nl}^2}{\alpha}\int_{-1}^{1}\left(\frac{1-y}{2}\right)^{2\sqrt{\varepsilon+\gamma}}\left(\frac{1+y}{2}\right)^{1+2\sqrt{\frac{1}{4}+\gamma}}\left[P_n^{\left(2\sqrt{\varepsilon+\gamma},2\sqrt{\frac{1}{4}+\gamma}\right)} y\right]^2 dy = 1 \quad (43)$$

Let

$$\left.\mu = 1+2\sqrt{\frac{1}{4}+\gamma},\ \mu-1 = 2\sqrt{\frac{1}{4}+\gamma},\ u = 2\sqrt{\varepsilon+\gamma}\right\} \quad (44)$$

By substituting Eq. (44) into Eq. (43) we have

$$\frac{B_{nl}^2}{\alpha}\int_{-1}^{1}\left(\frac{1-y}{2}\right)^{u}\left(\frac{1+y}{2}\right)^{\mu}\left[P_n^{(2\mathrm{u},\mu-1)} y\right]^2 dy = 1 \quad (45)$$

According to Onate and Ojonubah [16] , integral of the form in Eq. (45) can be expresses as

$$\int_{-1}^{1}\left(\frac{1-p}{2}\right)^{x}\left(\frac{1+p}{2}\right)^{y}\left[P_n^{(2x,2y-1)}p\right]^2 dp=\frac{2\Gamma(x+n+1)\Gamma(y+n+1)}{n!x\Gamma(x+y+n+1)} \tag{46}$$

Hence, comparing Eq. (45) with the standard integral of Eq. (46), we obtain the normalization constant as

$$B_{nl}=\sqrt{\frac{n!u\alpha\Gamma(\mathrm{u}+\mu+\mathrm{n}+1)}{2\Gamma(u+n+1)\Gamma(\mu+n+1)}} \tag{47}$$

4.0 RESULTS AND DISCUSSION

Table 1

Bound states energy for the class of Yukawa plus Eckart Potential with fitting obtained from Ref. [9,14] $\delta=0.005, d=0.00005, a=0.0001, b=2, c=-1, p=0.0002, \hbar=\mu=1$

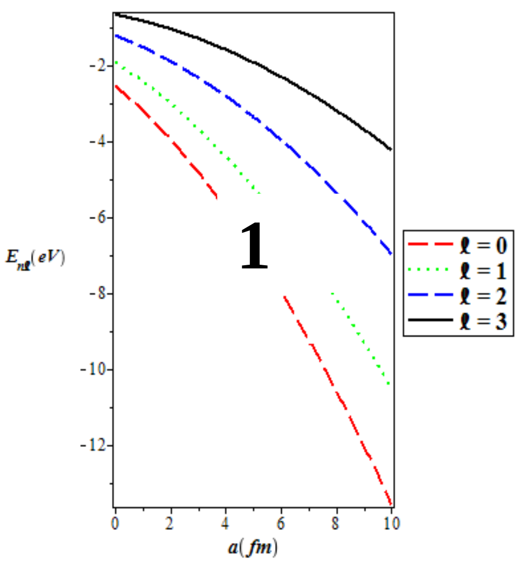
n	l	E_{nl}^{CYEP}	n	l	E_{nl}^{CYEP}	n	l	E_{nl}^{CYEP}
0	0	-0.3806267099	3	0	-0.09773557870	4	0	-0.06993974555
1	0	-0.2214129270		1	-0.08685177275		1	-0.06280847999
	1	-0.1891279009		2	-0.07126864750		2	-0.05232891302
2	0	-0.1424952261		3	-0.05610003578		3	-0.04178710049
	1	-0.1246616810					4	-0.03256295866
	2	-0.1000007070						

Table 2

Comparison of energy eigenvalues (eV) for special case of Hellmann potential as a function of the screening parameter δ with $\hbar=2\mu=1$ for $a=d=p=0,\ b=2,\ c=1$

State	δ	Present method	(SUSY) [13]	(AP) [18]	(PT) [15]
1S	0.001	−0.2515002500	-0.251 500	- 0.250969	-0.250999
	0.005	−0.2575062500	-0.257 506	-0.254 933	-0.254 963
	0.01	−0.2650250000	-0.265 025	-0.259 823	-0.289 852
2S	0.001	−0.06400100000	-0.064 001	-0.063 243	-0.063 494
	0.005	−0.07002500000	-0.070 025	-0.067 106	-0.067 353
	0.01	−0.07760000000	-0.077 600	-0.071 689	-0.071 928
2P	0.001	−0.06375025000	-0.063 750	-0.063 495	-0.063 495
	0.005	−0.06875625000	-0.068 756	-0.067 377	-0.067 377
	0.01	−0.07502500000	-0.075 025	-0.072 020	-0.072 020
3S	0.001	−0.02928002778	-0.029 280	-0.028 283	-0.028 764
	0.005	−0.03533402778	-0.035 334	-0.031 993	-0.032 457
	0.01	−0.04300277778	-0.043 003	-0.036 142	-0.036 557
3P	0.001	−0.02916802778	-0.029 169	-0.028 765	-0.028 765
	0.005	−0.03475625000	-0.034 756	-0.032 480	-0.032 480
	0.01	−0.04180277778	-0.041 803	-0.036 648	-0.036 644

3d	0.001	−0.02894469445	-0.028 945	-0.028 767	-0.250 833
	0.005	−0.03361736111	-0.033 617	-0.032 526	-0.254 151
	0.01	−0.03946944445	-0.039 469	-0.036 813	-0.258 269
4S	0.001	−0.01712900000	-0.017 129	-0.016 130	-0.016 601
	0.005	−0.02322500000	-0.023 225	-0.019 646	-0.020 077
	0.01	−0.03102500000	-0.031 025	-0.023 289	-0.023 551
4p	0.001	−0.01706556250	-0.017 066	-0.016 602	-0.016 602
	0.005	−0.02288906250	-0.022 889	-0.020 100	-0.020 098
	0.01	−0.03030625000	-0.030 306	-0.023 711	-0.023 641
4d	0.001	−0.01693906250	-0.016 939	-0.016 604	-0.016 604
	0.005	−0.02222656250	-0.022 227	-0.020 142	-0.020 142
	0.01	−0.02890625000	-0.028 906	-0.023 857	-0.023 814
4f	0.001	−0.01675025000	-0.016 750	-0.016 607	-0.016 607
	0.005	−0.02125625000	-0.021 257	-0.020 206	-0.020 206
	0.01	−0.02690000000	-0.026 900	-0.024 072	-0.024 056



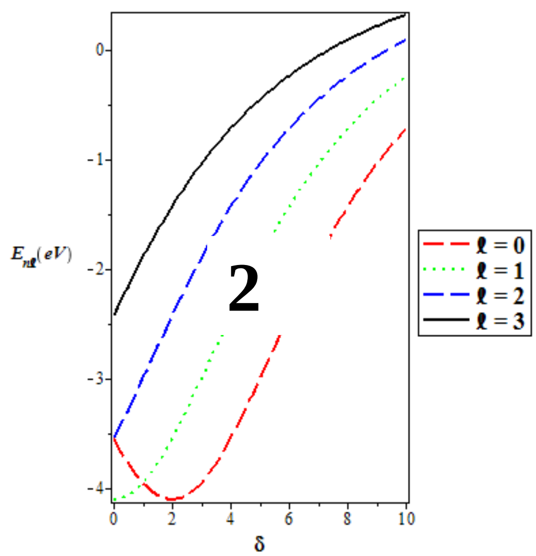
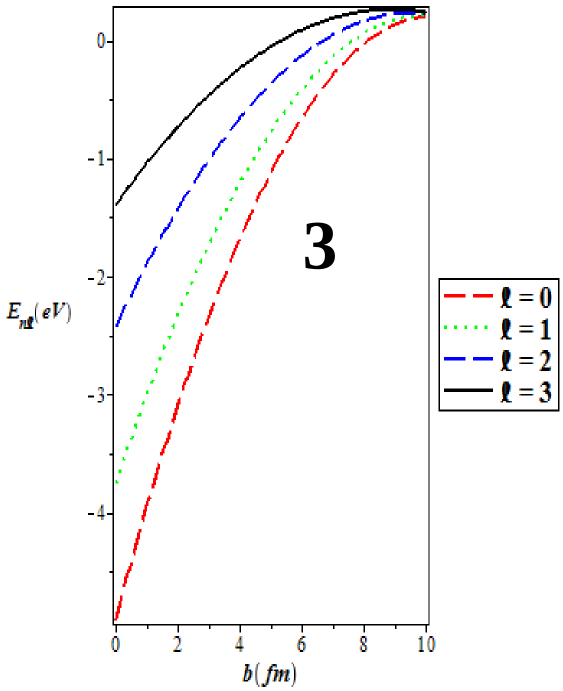


Fig1: Energy eigenvalues variation with parameter (a) for various vibrational quantum numbers
Fig 2: Energy eigenvalues variation with screening parameter (δ) for various vibrational quantum numbers



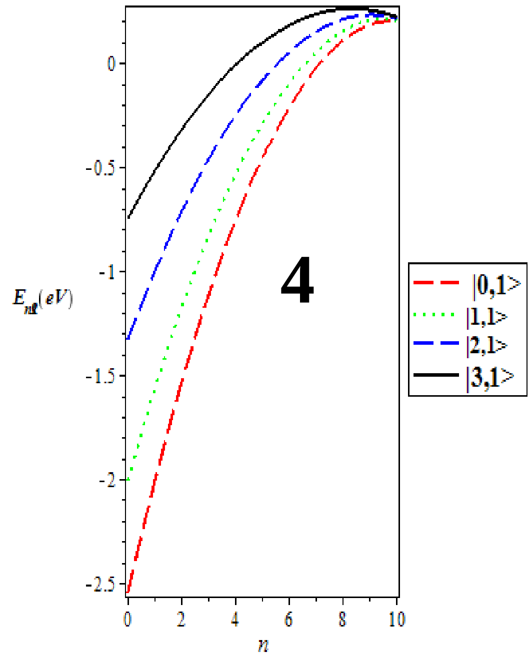


Fig 3: Energy eigenvalues variation with parameter (b) for various vibrational quantum numbers.

Fig. 4 : Energy eigenvalues variation with quantum number for various vibrational quantum number**.**

4.1 DISCUSSION

In Table 1, we numerically reported the energy eigenvalues for class of Yukawa plus Eckart potentials $\left(a = 0.00001, b = 2, c = -1, d = 0.00005, p = 0.0002\right)$. The numerical energy eigenvalues for Hellmann potential is presented in Table 2 for $\left(a = d = p = 0, b = 2, c = 1\right)$. The result showed good agreement with the results of other researchers like; Ref. [18] with AP method, SUSY method of Ref. [13] and PT method of Ref. [15].
The energy increases as n increases.We have plotted the energy eigenvalues with different parameters of combined potential such as a , b and δ as shown in Figs.1-3, for various values of vibrational quantum number. We also plotted energy eigenvalues with quantum number as shown in Fig.4. They are an increase in energy eigenvalues as the quantum number increases.

5.0 CONCLUSIONS

In this study, we obtain the bound state solutions of the Schrödinger equation by combining class of Yukawa and Eckart potentials using the Nikiforov-Uvarov method. After making appropriate approximation to the centrifugal term, we obtain the energy eigenvalues and the corresponding normalized wave function. We obtain three special cases of Hellmann, Coulomb and inversely Yukawa potential. In table 1 and table 2, we have numerically obtained the energy eigenvalues for the combined potential and a special case of Hellmann potential respectively. We found that the result of the Hellmann potential is in excellent agreement with works of other researchers. The variation in the combined energy eigenvalues with some potential parameters was also plotted. It was discovered that the energy eigenvalues increases as the vibrational quantum number increases.

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