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Method of Cryptography by Applying Laplace Transform To Tangent Trigonometric Function

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Abstract: In this paper we introduced method of cryptography i.e. encryption and decryption by applying Laplace transform and their inverse using tangent trigonometric function by considering series of the form $\mathrm{Gy}^{\mathrm{m}}\tan\mathrm{ny}$.

Key-words: Laplace transform, Inverse Laplace transform, Expansion of tangent trigonometric function, Cryptography, Encryption, Decryption.

Introduction: In mathematics an integral transform plays an important role in the conversion of a function from one function into another function. In this paper we apply Laplace transform to trigonometric tangent function for the method of Cryptography. The Cryptography is the process of converting ordinary plain text into unintelligible text and vice-versa. In present age the network and electronic communication is very important. The use of Cryptography facilities the provision of cash withdrawal from banking, from ATM, pay TV, online purchasing, banking transactions cards, computer passwords, e-commerce transactions, e-Governing, SMS service, e-mails etc. In human life the security of financial information is an essential part. The purpose of using this method is for more security in communication as compared to other methods because cipher text obtained by this method could not be cracked by other persons easily. In the first part we apply Laplace transform to tangent trigonometric function $y\tan y$ then we apply $y^2\tan 2y$ for the same purpose. Finally we conclude by comparing these two functions.

Preliminaries:

Definition: Laplace Transform: The Laplace transform of a function f(y) defined for all real numbers $y \geq 0$, is the function F(s), which is a unilateral transform defined by

$L[f(y)] = F(s) = \int_0^\infty f(y)e^{-sy}dy$ where s is a complex number frequency parameter

Definition: Inverse Laplace Transform: If F(s) is the Laplace transform of f(y) then the inverse Laplace transform of F(s) is given by f(y) and we write $L^{-1}[F(s)] = f(y)$.

Formula:

1) If $f(y) = y^n$ then $L[\,y\,] = \frac{n!}{s^{n+1}}$ and $L^{-1}[\,\frac{1}{s^n}\,] = \frac{t^{n-1}}{(n-1)\,!}$

Main Results:

Encryption: Suppose we want to send the massage "KADA"

In this method we can convert the given plain text in to such a hidden text which could not Possible to crack without key by operating Laplace transforms. Suppose that we are given A

B C D E F G H………….Z. as a plain text. In the first step we have to give the following allotment to letters in the given plain text.

A 0	B 1	C 2	D 3	E 4	F 5	G 6	H 7	I 8	J 9	K 10	L 11	M 12
N 13	O 14	P 15	Q 16	R 17	S 18	T 19	U 20	V 21	W 22	X 23	Y 24	Z 25

Since the plane text is "KADA" it is equivalent to 10,0,3,0.

Let us assume that

$G_0 = 10$, $G_1 = 0$, $G_2 = 3$, $G_3 = 0$ and $G_k = 0$ for $k \geq 4$

We consider the Maclaurin's expansion of $\tan x$ given by

$$\tan x = x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + \frac{17}{315}x^7 + \cdots$$

Using this expansion we write,

$$\tan ny = ny + \frac{1}{3}(ny)^3 + \frac{2}{15}(ny)^5 + \frac{17}{315}(ny)^7 + \cdots \quad \text{------ (1)}$$

Using equation (1) we write,

$$y^m \tan ny = ny^{m+1} + \frac{1}{3}(ny)^{m+3} + \frac{2}{15}(ny)^{m+5} + \frac{17}{315}(ny)^{m+7} + \cdots \quad \text{------ (2)}$$

Consider $f(y) = G\, y^m \tan ny$

$$= G_0\, ny^{m+1} + G_1 \frac{1}{3}(ny)^{m+3} + G_2 \frac{2}{15}(ny)^{m+5} + G_3 \frac{17}{315}(ny)^{m+7} + \cdots$$

$$= 10 \times\ ny^{m+1} + 0 \times \frac{1}{3}(ny)^{m+3} + 3 \times \frac{2}{15}(ny)^{m+5} + 0 \times \frac{17}{315}(ny)^{m+7} + \cdots$$

$$= 10 \times\ ny^{m+1} + 0 \times (ny)^{m+3} + \frac{2}{5} \times (ny)^{m+5} + 0 \times (ny)^{m+7} + \cdots \quad \text{------ (3)}$$

Case I: Put m = 1, n = 1 in equation (3) then $f(y) = G\, y \tan y$

$$\therefore G\, y \tan y = 10 \times\ y^2 + 0 \times\ y^4 + \frac{2}{5} \times\ y^6 + 0 \times y^8 + \cdots \quad \text{-------- (4)}$$

Taking Laplace transform on both sides we get,

$$L[G\, y \tan y] = L\left[10 \times\ y^2 + 0 \times\ y^4 + \frac{2}{5} \times\ y^6 + 0 \times y^8 + \cdots\right]$$

$$= 10 \times L(y^2) + 0 \times L(y^4) + \frac{2}{5} \times L(y^6) + 0 \times L(y^8) + \cdots$$

$$= 10 \times \frac{2!}{s^3} + 0 \times \frac{4!}{s^5} + \frac{2}{5} \times \frac{6!}{s^7} + 0 \times \frac{8!}{s^9} + \cdots$$

$$L[G\, y \tan y] = 20 \times \frac{1}{s^3} + 0 \times \frac{1}{s^5} + 288 \times \frac{1}{s^7} + 0 \times \frac{1}{s^9} + \cdots \quad \text{------------ (5)}$$

Let $r_0 = 20$, $r_1 = 0$, $r_2 = 288$, $r_3 = 0$

Let us calculate the remainders G_i' such that $r_i \equiv G_i'$ mod 26

i.e. $r_0 \equiv G_0'$ mod 26 $\Longrightarrow$ 20 ≡ - 6 mod 26 $\Longrightarrow G_0' = -6$

$r_1 \equiv G_1'$ mod 26 $\Longrightarrow$ 0 ≡ 0 mod 26 $\Longrightarrow G_1' = 0$

$r_2 \equiv G_0'$ mod 26 $\Longrightarrow$ 288 ≡ 2 mod 26 $\Longrightarrow G_2' = 2$

$r_3 \equiv G_3'$ mod 26 $\Longrightarrow$ 0 ≡ 0 mod 26 $\Longrightarrow G_3' = 0$

Hence values 20, 0, 288, 0 are congruent respectively to the remainders - 6, 0, 2, 0 and the quotients 1, 0, 11, 0 form the key.

Using the assignment given in equation (4) the cipher text for given plain text “KADA” will be - 6, 0, 2, 0 i.e. “GACA”. The sender publicity sends the message “GACA” and privately sends the key and the Laplace transform of G y tan y .

Decryption: The receiver receives the message “GACA”.

The equivalent values are

G, A, C, A i.e. 6, 0, 2, 0

and the private key values are 1,0,11,0.

Since r_i = 26 × key + remainder, i.e.

20 = 26 × 1- 6, 0 = 26 × 0 + 0, 288 = 26 × 11 + 2, 0 = 26 × 0 + 0

Thus we get -6, 0, 2, 0 which implies

$$L[G\,y\tan y] = 20\times\frac{1}{s^3} + 0\times\frac{1}{s^5} + 288\times\frac{1}{s^7} + 0\times\frac{1}{s^9} + \cdots \quad \text{------------ } (6)$$

Taking inverse Laplace transform on both sides we get

$$L^{-1}[G\,y\tan y] = L^{-1}\left[20\times\frac{1}{s^3} + 0\times\frac{1}{s^5} + 288\times\frac{1}{s^7} + 0\times\frac{1}{s^9} + \cdots\right]$$

$$\Longrightarrow G\,y\tan y = 20\times L^{-1}\left(\frac{1}{s^3}\right) + 0\times L^{-1}\left(\frac{1}{s^5}\right) + 288\times L^{-1}\left(\frac{1}{s^7}\right) + 0\times L^{-1}\left(\frac{1}{s^9}\right) + \cdots$$

$$= 20\times\frac{y^2}{2\,!} + 0\times\frac{y^4}{4\,!} + 288\times\frac{y^6}{6\,!} + 0\times\frac{y^8}{8\,!} + \cdots$$

$$= 10\times y^2 + 0\times y^4 + \frac{2}{5}\times y^6 + 0\times y^8 + \cdots$$

$$\therefore G\,y\tan y = 10\times y^2 + 0\times y^4 + 3\times\frac{2}{15}y^6 + 0\times y^8 + \cdots$$

In the above expansion the coefficients G_k are 10, 0, 3, 0 which gives the original plain text message as ,

10, 0, 3, 0.

K, A, D, A

Case II : Put m = 2, n = 2 we get, $f(y) = G\, y^2 \tan 2y$

$G\, y^2 \tan 2y = 10 \times y^3 + 0 \times y^5 + \frac{2}{5} \times y^7 + 0 \times y^9 + \cdots$ -------- (7)

Taking Laplace transform on both sides we get,

$$L[G\, y^2 \tan 2y] = L\left[10 \times y^3 + 0 \times y^5 + \frac{2}{5} \times y^7 + 0 \times y^9 + \cdots\right]$$

$$= 10 \times L(y^3) + 0 \times L(y^5) + \frac{2}{5} \times L(y^7) + 0 \times L(y^9) + \cdots$$

$$= 10 \times \frac{3!}{s^4} + 0 \times \frac{5!}{s^6} + \frac{2}{5} \times \frac{7!}{s^8} + 0 \times \frac{9!}{s^{10}} + \cdots$$

$\therefore L[G\, y^2 \tan 2y] = 60 \times \frac{1}{s^4} + 0 \times \frac{1}{s^6} + 2016 \times \frac{1}{s^8} + 0 \times \frac{1}{s^{10}} + \cdots$ ------------ (8)

Let $r_0 = 60$, $r_1 = 0$, $r_2 = 2016$, $r_3 = 0$

Let us calculate G_i' such that $r_i \equiv G_i' \bmod 26$

i.e. $r_0 \equiv G_0' \bmod 26 \Longrightarrow 60 \equiv 8 \bmod 26 \quad \Longrightarrow G_0' = 8$

$r_1 \equiv G_1' \bmod 26 \Longrightarrow 0 \equiv 0 \bmod 26 \quad \Longrightarrow G_1' = 0$

$r_2 \equiv G_0' \bmod 26 \Longrightarrow 2016 \equiv 14 \bmod 26 \quad \Longrightarrow G_2' = 14$

$r_3 \equiv G_3' \bmod 26 \Longrightarrow 0 \equiv 0 \bmod 26 \quad \Longrightarrow G_3' = 0$

Let $G_0' = 8$, $G_1' = 0$, $G_2' = 14$, $G_3' = 0$ and the quotients 2, 0, 77, 0 form the key.

Using the assignment given in equation (7) the cipher text for given plain text “KADA” will be 8, 0, 14, 0 i.e. “IAOA”. The sender publicity sends the message “IAOA” and privately sends the key and the Laplace transform of $G\, y^2 \tan 2y$.

Decryption: the receiver receives the message “IAOA”.

The equivalent values are

I, A, O, A i.e. 8, 0, 14, 0

and the private key values are 2,0,77,0.

Since $r_i = 26 \times \text{key} + \text{remainder}$, i.e.

60 = 26 × 2 + 8,   0 = 26 × 0 + 0,   2016 = 26 × 77 + 14,   0 = 26 × 0 + 0

Thus we get 8, 0, 14, 0 which implies

$L[G\, y^2 \tan 2y] = 60 \times \frac{1}{s^4} + 0 \times \frac{1}{s^6} + 2016 \times \frac{1}{s^8} + 0 \times \frac{1}{s^{10}} + \cdots$ ------------ (9)

Taking inverse Laplace transform on both sides we get

$$L^{-1}[G\, y^2 \tan 2y] = L^{-1}\left[60 \times \frac{1}{s^4} + 0 \times \frac{1}{s^6} + 2016 \times \frac{1}{s^8} + 0 \times \frac{1}{s^{10}} + \cdots\right]$$

$$\Longrightarrow G\, y^2 \tan 2y = 60 \times \frac{y^3}{3!} + 0 \times \frac{y^5}{5!} + 2016 \times \frac{y^7}{7!} + 0 \times \frac{y^9}{9!} + \cdots$$

$$= 60 \times \frac{y^3}{3\,!} + 0 \times \frac{y^5}{5\,!} + 2016 \times \frac{y^7}{7\,!} + 0 \times \frac{y^9}{9\,!} + \cdots$$

$$= 10 \times y^3 + 0 \times y^5 + \frac{2}{15} \times y^7 + 0 \times y^9 + \cdots$$

$$\therefore G\,y^2 \tan 2y = 10 \times y^2 + 0 \times y^4 + 3 \times \frac{2}{15} y^6 + 0 \times y^8 + \cdots$$

In the above expansion the coefficients G_k are 10, 0, 3, 0 which gives the original plain text message as,

10, 0, 3, 0.

K, A, D, A

Summary:

Representation of Encryption process using Laplace transform:

i) Of the function G y tan y

i	G_i	$r_i = (2i)^2(2i-1)^2 G_i$	G_i'	$K_i = \frac{r_i - G_i'}{26}$
0	20	20	-6	1
1	0	0	0	0
2	2	288	2	11
3	0	0	0	0

ii) Of the function $G\,y^2 \tan 2y$

i	G_i	$r_i = (2i)^2(2i-1)^2 G_i$	G_i'	$K_i = \frac{r_i - G_i'}{26}$
0	60	60	8	2
1	0	0	0	0
2	14	2016	14	77
3	0	0	0	0

In this table the values G_i be the coefficients of the function are congruent to the remainders G_i' and the quotients K_i are the private keys to open the message of two different functions.

Conclusion: In this work we introduced a new application of Cryptography by applying Laplace transform to the different function we will obtain the different cipher text for the given plain text. The private key is the quotient of the G_n when divided by 26, the number of multiples of mod 26. For the different functions there are different private keys. It is very difficult to find the private key by any other attack. After producing key we use this key for encryption and decryption that algorithm based on Laplace transformation and modular arithmetic. The same result can be found by considering Laplace transform of trigonometric sine and cosine, hyperbolic sine and cosine functions, exponential function as well as in polynomial function.

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