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Numerical Solution of Imbibition Phenomenon Using PDQM in Vertical Downward Direction

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Abstract: In this paper, we have discussed counter current imbibition in water-oil flow through homogeneous porous medium in the vertical downward direction. Counter current imbibition occurring in secondary oil recovery method. We obtained numeric solution of the non-linear PDE by Polynomial based differential quadrature method. Numerical solution and Graphs are prepared by using MATLAB code.

Keywords: Imbibition Phenomenon, Vertical downward direction, Polynomial based differential quadrature method.

Introduction: We have discussed counter-current imbibition phenomenon in water-oil flow over homogeneous porous medium in the vertical downward direction. In the process of secondary oil recovery this phenomenon arises. This phenomenon occurs as a result of inequality in saturation capability of the two immiscible fluids flowing in the medium. Due to the nature of flow of fluids, Imbibition phenomenon divided into two parts (1) "Counter current" and (2) "Co-current" imbibition. If both the fluids flowing in opposite direction then this process is known as Counter-current and if both the fluids flow in same direction is known as Co-current imbibition phenomenon.

Many authors and research scholar has been investigated this phenomenon in the different point of view. Many researchers have been obtained numerical solution of this phenomenon in Homogeneous, Heterogeneous and Cracked porous medium. Brownscombe, Dyes, Scheideggar, Mehta and Verma, Graham and Richardson did a remarkable work and played a very important role in this research field.

Bokserman, Zheltov and Kocheskov characterized "physics behind of water-oil flow in cracked and heterogeneous porous medium". Bourbiux and Kalaydjian [3] observed "the Co-current and Counter-current spontaneous imbibition on water-wet core and concluded that the recovery rate be more in Co-current as compared to counter-current flow." Zhou [13] inspected "the effect of wettability on oil recovery and concluded that wettability has a significant impact on the

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imbibition rate." Firoozabadi [12] designed "the Co and Counter-current imbibition in water-wet rocks and concluded that oil recovery from counter-current imbibition is lower than the one from the co-current." Darvishi et al. [16] inspected "the effect of permeability on spontaneous imbibition using carbonate cores." They noted that "water imbibed more easily in cores with higher permeability, even in oil-wet cores at connate water saturation." Mishra [14] examined this phenomenon in "homogeneous porous medium by using Homotopy perturbation transform method and observed that the saturation of water advances with the time and reaches to a constant value after a very long time." Parikh et al [4] explained it "analytically in a vertical downward direction in homogeneous porous medium."

Basic assumption for Mathematical Formulation: When we performing experimental and practical study, field area is too large. Therefore, we selected small part of it. We also assumed that selected part of that porous media is cylindrical.

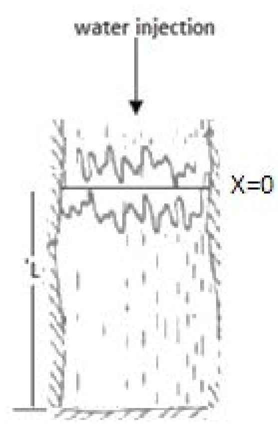

Figure: 1 Imbibition in Vertical downward direction [4]

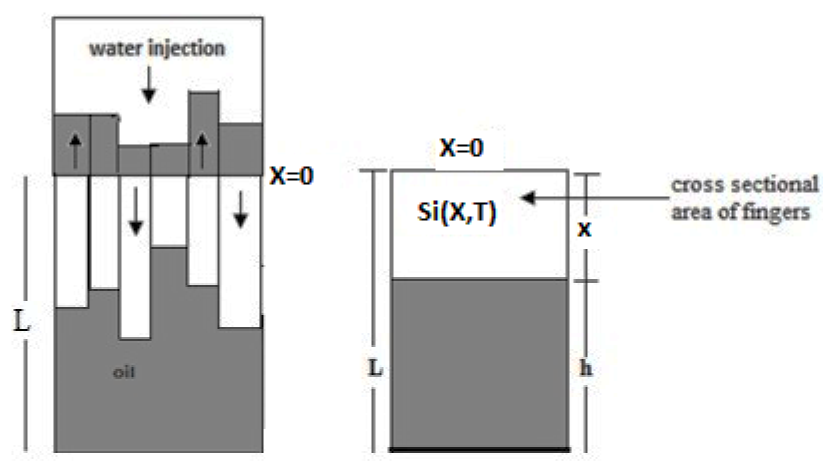

Figure: 2 Schematic diagrams [4]

For mathematical model we have to assume following basic assumptions.

(1) We have selected vertical cross factional range that is in rectangular pattern.
(2) In place of actual fingers arises in uneven pattern, we reflected as proper small rectangular fingers. We considered its average rate for examination of saturation of injected water.
(3) We assume that non-saturating and some saturating fluid completely saturated to the medium.
(4) We consider uniform water injection into an oil saturated vertical downward homogeneous porous media.
Water and oil both are flowing in homogeneous vertical porous media with constant porosity (P) and permeability (K). Reynolds's number law for fluid flow and the Darcy's law will be valid to calculate velocity of water and oil under the effect of gravitational force. The additional gravitational effect will increase velocity of injected water in downward direction hence more oil can be displaced during this process. Spontaneous flow will occurs without any external force.

MATHEMATICAL FORMULATION OF THE PROBLEM

We have to stablish mathematical formulations of the given problems. We need to follow basic governing rules and specific relations. Few of them as mentioned bellows

(1) Accepting limitations of Darcy's law and low Reynolds number, the velocity of injected fluid and native fluid can be written as follows:

$$V_i = -\frac{K_i}{\delta_i} K \left(\frac{\partial P_i}{\partial x} + \rho_i g \right) \tag{1}$$

$$V_n = -\frac{K_n}{\delta_n} K \left(\frac{\partial P_n}{\partial x} - \rho_n g \right) \tag{2}$$

K= permeability of the homogeneous medium ,

K_i = relative permeability's of water and K_n = relative permeability's of oil

$P_i = pressure\ of\ injected\ fluid\ and\ P_n = pressure\ of\ native\ fluid$

$\rho_i = density\ of\ injected\ fluid\ and\ \rho_n = density\ of\ native\ fluid$

δ_i = constant kinematic viscosity of water *and* δ_n = constant kinematic viscosity of oil

g = the acceleration due to gravity

The coordinate x is measured along the vertical axis of cylindrical medium, the origin being at the imbibition surface.

(2) Using continuity equation for given flow problem.

$$P\left(\frac{\partial S_i}{\partial t}\right)+\frac{\partial V_i}{\partial x}=0 \tag{3}$$

$P=P$orosity of the medium

(3) P_c capillary pressure can be defined as

$$P_c=P_n-P_i=f(S_i) \tag{4}$$

(4) We will consider following relationships for "relative permeability-phase saturation and capillary pressure-phase saturation due to Schiedegger and Johnson (1961)".

$$K_i=S_i\,,\ K_n=1-\alpha S_i\ (\alpha=1.11) \tag{5}$$

(5) "As Mehta [16] suggested that capillary pressure is proportional to saturation of injected fluid in opposite direction"

$P_c=-\beta S_i$, "where β is constant of proportionality" (6)

Due to contact of the two phases, there is spontaneous flow in counter-current imbibition phenomenon. Therefore, the sum of the velocities of injected fluid and native fluid is zero.

$$V_i+V_n=0$$

i.e. $\therefore\ V_i=-V_n$, "Scheidegger" [1] (7)

Using equation (1) & (2)

$$\frac{K_i}{\delta_i}K\left(\frac{\partial P_i}{\partial x}+\rho_i g\right)+\frac{K_n}{\delta_n}K\left(\frac{\partial P_n}{\partial x}-\rho_n g\right)=0 \tag{8}$$

Using equation (8) & (4)

$$\left(\frac{K_i}{\delta_i}+\frac{K_n}{\delta_n}\right)\frac{\partial P_i}{\partial x}+\left(\frac{K_n}{\delta_n}\right)\left(\frac{\partial P_c}{\partial x}\right)=-\left(\frac{K_i}{\delta_i}\rho_i-\frac{K_n}{\delta_n}\rho_n\right)g \tag{9}$$

$$\therefore\ \frac{\partial P_i}{\partial x}=-\left[\frac{\left(\frac{K_i}{\delta_i}\rho_i-\frac{K_n}{\delta_n}\rho_n\right)g+\left(\frac{K_n}{\delta_n}\right)\frac{\partial P_c}{\partial x}}{\left(\frac{K_i}{\delta_i}+\frac{K_n}{\delta_n}\right)}\right] \tag{10}$$

Replacing value of $\frac{\partial P_i}{\partial x}$ as of equation (10) into (1), we have

$$V_i = -\left(\frac{K_i}{\delta_i}\right) K \left[\frac{\left(\frac{K_n}{\delta_n}\right)(\rho_i + \rho_n) g - \left(\frac{K_n}{\delta_n}\right)\frac{\partial P_c}{\partial x}}{\left(\frac{K_i}{\delta_i} + \frac{K_n}{\delta_n}\right)} \right] \tag{11}$$

Replacing value of V_i as of equation (11) into (3)

$$P\left(\frac{\partial S_i}{\partial t}\right) + \frac{\partial}{\partial x}\left[K \left(\frac{\left(\frac{K_i}{\delta_i}\right)\left(\frac{K_n}{\delta_n}\right)}{\left(\frac{K_i}{\delta_i} + \frac{K_n}{\delta_n}\right)} \frac{dP_c}{dS_i}\frac{\partial S_i}{\partial x} \right)\right] - \frac{\partial}{\partial x}\left[K(\rho_i + \rho_n) g \frac{\left(\frac{K_i}{\delta_i}\right)\left(\frac{K_n}{\delta_n}\right)}{\left(\frac{K_i}{\delta_i} + \frac{K_n}{\delta_n}\right)} \right] = 0 \tag{12}$$

Due to involvement of water and oil, we assumed (“Schiedegger” [24])

$$\left[\frac{\left(\frac{K_i}{\delta_i}\right)\left(\frac{K_n}{\delta_n}\right)}{\left(\frac{K_i}{\delta_i} + \frac{K_n}{\delta_n}\right)} \right] = \left(\frac{K_n}{\delta_n}\right) = \frac{1-\alpha S_i}{\delta_n} \tag{13}$$

Replacing values as of (13) and (6) in equation (12), we have

$$P\left(\frac{\partial S_i}{\partial t}\right) = \frac{K\beta}{\delta_n}\frac{\partial}{\partial x}\left[\left((1-\alpha S_i)\frac{\partial S_i}{\partial x}\right)\right] + \frac{Kg(\rho_i + \rho_n)}{\delta_n}\frac{\partial}{\partial x}(1-\alpha S_i) \tag{14}$$

Equation (14) representing governing equation of “Counter current imbibition”.

We substituting $1-\alpha S_i = S$ in equation (14), we have

$$P\left(\frac{\partial S}{\partial t}\right) = \frac{K\beta}{\delta_n}\frac{\partial}{\partial x}\left[\left(S\frac{\partial S}{\partial x}\right)\right] - \frac{Kg\alpha(\rho_i + \rho_n)}{\delta_n}\frac{\partial S}{\partial x} \tag{15}$$

We selecting new dimension less variable to change equation (15) into non dimensional form as

$$T = \frac{K\beta}{L^2 P\delta_n} t, \; X = \frac{x}{L}$$

$$\left(\frac{\partial S}{\partial T}\right) = S\frac{\partial^2 S}{\partial X^2} + \left(\frac{\partial S}{\partial X}\right) - A\left(\frac{\partial S}{\partial X}\right) \tag{16}$$

Where $A = \frac{Lg\alpha(\rho_i + \rho_n)}{\beta}$, $S(X,T) = 1-\alpha S_i(X,T)$

SOLUTION BY POLYNOMIAL BASED DIFFERENTIAL QUADRATURE METHOD

In the numerical experiments, we have used uniform grid points to obtain the numerical solution of the Governing equation.

$$x_i = x_1 + ih,\ i = 1, 2, 3, ..., N \quad where\ \ x_1 = a\ \ and\ \ h = \frac{b-a}{N} \tag{17}$$

Discretization of Differential quadrature method:

Discretize equation (16) using the expression (18) to (20)

$$\frac{\partial S}{\partial T} = \frac{dS_l}{dT},\ l = 1, 2, 3, ..., N, \quad \text{Where } S_l = S(x_l) \tag{18}$$

$$\frac{\partial^2 S}{\partial X^2} = \sum_{m=1}^{N} d_{lm} S_m,\ \ l = 1, 2, 3, ..., N \tag{19}$$

$$\frac{\partial S}{\partial X} = \sum_{m=1}^{N} c_{lm} S_m,\ \ l = 1, 2, 3, ..., N, \text{ where } S_m = S(x_m), \tag{20}$$

Using 1st and 2nd order partial derivative in equation (16) by (18) to (20), we have systems of 1st order ODE.

$$\frac{dS_i}{dT} = \left(\sum_{m=1}^{N} c_{lm} S_m\right)^2 + S_l\left(\sum_{m=1}^{N} d_{lm} S_m\right) - A\left(\sum_{m=1}^{N} d_{lm} S_m\right) \tag{21}$$

Let we assume initial condition as

$$S_0(X) = e^{-X},\ \ for\ any\ X > 0 \tag{22}$$

The boundary conditions

$$S(0,T) = 0,\ T > 0 \qquad S(1,\ T) = 1,\ T > 0 \tag{23}$$

The value of some constant is taken from well-known article as follows:

$$L = 1, g = 9.8, \rho_n = 0.3, \rho_i = 0.1, \beta = 0.1, \alpha = 1.11$$
$$\Rightarrow A \approx 0.43$$

Weighting Coefficient can be obtained by using following formula

$$c_{lm} = \frac{M^{(1)}(x_l)}{(x_l - x_m)M^{(1)}(x_m)},\ l \neq m,\ l,\ m = 1,2,3...,N$$

And $c_{ll} = -\sum_{m=1}^{N} c_{lm}\ ,\ l = 1,2,3....,N$

Similarly

$$d_{lm} = 2c_{lm}\left[a_{ll} - \frac{1}{x_l - x_m}\right],\ for\ l \neq m.$$

And

$$d_{ll} = -\sum_{m=1,\, m\neq l}^{N} d_{lm}$$

Numerical analysis of water-oil flow through Homogeneous Porous medium in vertical downward direction by PDQM

TABLE: 1 SATURATION VS DISTANCE

Time T	0.5	0.6	0.7	0.8	0.9	1
Depth X						
0	0.9562	0.8824	0.8011	0.7009	0.6007	0.5003
0.1	0.85877	0.79094	0.70308	0.61525	0.52737	0.43942
0.2	0.76595	0.68933	0.61277	0.53617	0.45953	0.38295
0.3	0.66067	0.59465	0.52853	0.46257	0.39645	0.33034
0.4	0.56261	0.50635	0.45017	0.39388	0.33768	0.28137
0.5	0.47128	0.42415	0.37708	0.32995	0.28286	0.23564
0.6	0.38613	0.34757	0.30895	0.27037	0.23178	0.19314
0.7	0.30699	0.27628	0.24556	0.21484	0.18418	0.15345
0.8	0.23306	0.20978	0.18647	0.16319	0.13986	0.11656
0.9	0.16418	0.1478	0.13139	0.11498	0.09856	0.08218
1	0.10012	0.09014	0.08009	0.07007	0.06005	0.05002

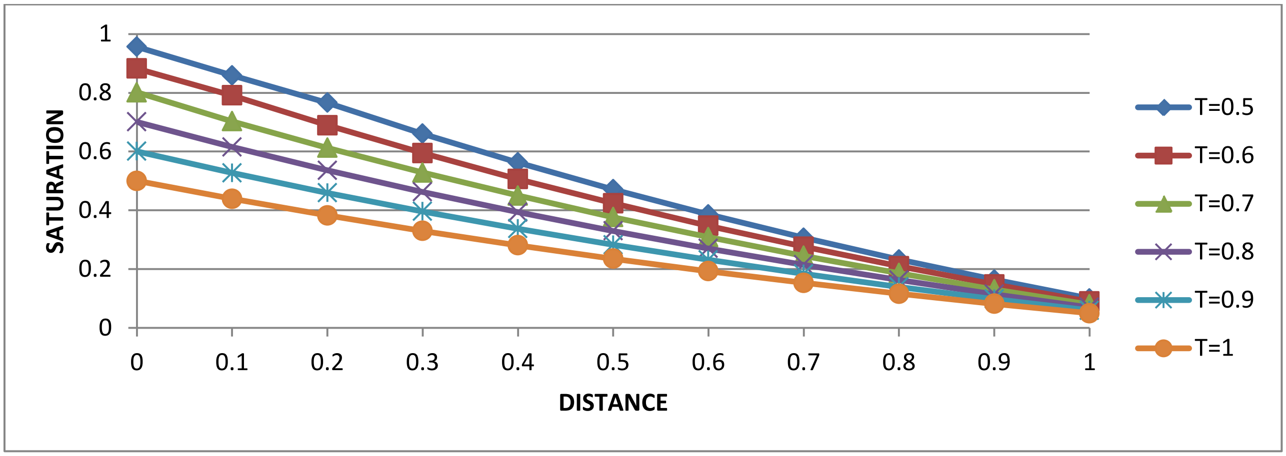

FIGURE: 3 SATURATION VS DISTANCE

TABLE: 2 SATURATION VS TIME

Depth X	0.5	0.6	0.7	0.8	0.9	1
Time T						
0.5	0.47128	0.38613	0.30699	0.23306	0.16418	0.10012
0.6	0.42415	0.34757	0.27628	0.20978	0.1478	0.09014
0.7	0.37708	0.30895	0.24556	0.18647	0.13139	0.08009
0.8	0.32995	0.27037	0.21484	0.16319	0.11498	0.07007
0.9	0.28286	0.23178	0.18418	0.13986	0.09856	0.06005
1	0.23564	0.19314	0.15345	0.11656	0.08218	0.05002

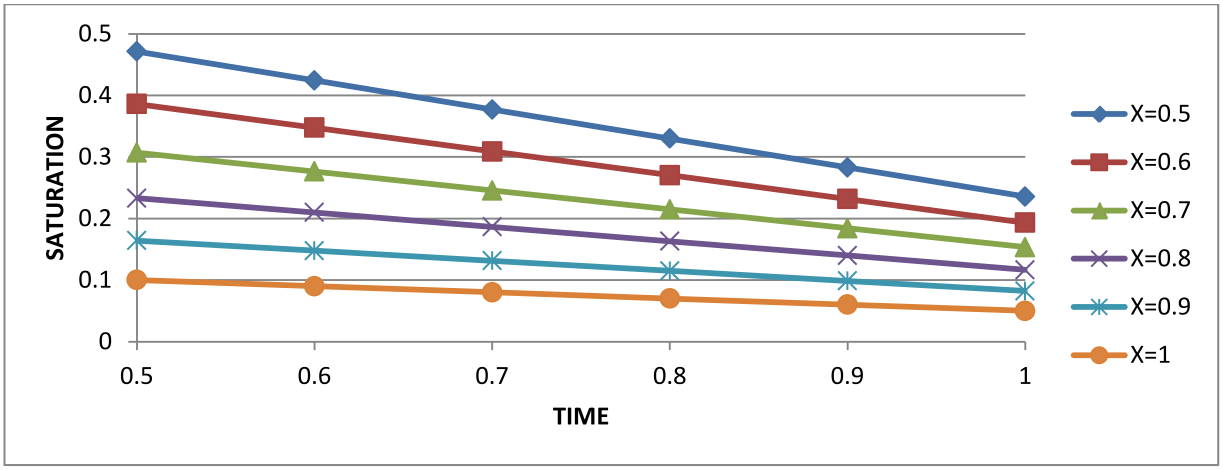

FIGURE: 4 SATURATION VS TIME

Conclusion: In this paper, we studied an analytical method based on Polynomial differential quadrature for solving the second order one dimensional non-linear partial differential equation arising in secondary oil recovery method in the vertical direction. This method gives numerical solutions closed to the exact solutions. The graphical representation and numerical values are given by using MATLAB coding.

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