Comparative Analysis of the Bilateral Quadrilateral and Constant Strain Triangle Element Meshing Of A Square Plate Using Finite Element Method

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ABSTRACT

In my work, a square plate is analysed by using finite element method. firstly a square plate of simple meshing of four noded isoparamatric element is done in MATLAB and obtain the value of deflection and then start refining the square plate by using mesh automatic refinement and increase the no of nodes and obtain the value of deflection and this fashion is follow for two more time and analyze the amount of convergence in deflection.

Same manner will follow for constant strain triangle element and analyse the convergence in result.

Validate the result by using FEAST Software of same square plate

Keywords :- four noded isoparametric element, constant strain triangle element, FEAST,MATLAB

INTRODUCTION-

Two dimensional elements comprises of three or more nodes in a plane and are connected at common nodes along common edges to make one complete continuous structure. Compatibility between nodal displacements is then enforced during the formulation of the equilibrium equations for two dimensional elements. Two dimensional elements are important for carrying out plane stress analysis which includes plate problems.

This chapter comprises the development of stiffness matrix for 3 node constant strain triangle (CST) and 4 node bilinear quadrilateral element (Q4). Finally, a plane stress problem for cantilever beam is solved using automatic mesh generator developed in MATLAB for node coordinates and nodal connectivity to determine which element shape should be preferred for the faster and accurate analysis with mesh refinement

4.2 Plane Stress Problem

A thin planar body subjected to in plane loading is a two dimensional problem of plane stress. The body is X-Y plane and the thickness which is small and uniform is in Z direction. In the Z direction the body is free to deform and will not have any stresses in that direction.

Plane stress is defined as the state of stress in which normal stress σz and the shear stresses σxz and σyz, perpendicular to x-y plane are zero, (Bhavikatti, 2005)

So that $\sigma_z = \tau_{xz} = \tau_{yz} = 0$ and $\gamma_{xz} = \gamma_{yz} = 0$ 4.1

And the elasticity matrix as

$$D_b = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}$$

4.3. Types of Elements

4.3.1 Constant Strain Triangle (CST)

The CST is simplest element to formulate and has been extensively used for the two dimensional problem. The stiffness matrix of this element is obtained rather directly, without going through the integration process. As the strain within the element is constant, the performance of this element is poor and a huge number of elements are required to obtain reasonably accurate solutions.

We will let u and v are the nodal displacements in the x and y direction, respectively. Here counterclockwise system is considered for labeling of nodes in all formulations.

The nodal displacement matrix is given by

$$\{\delta\} = [u1\ v1\ u2\ v2\ u3\ v3]T \qquad 4.3$$

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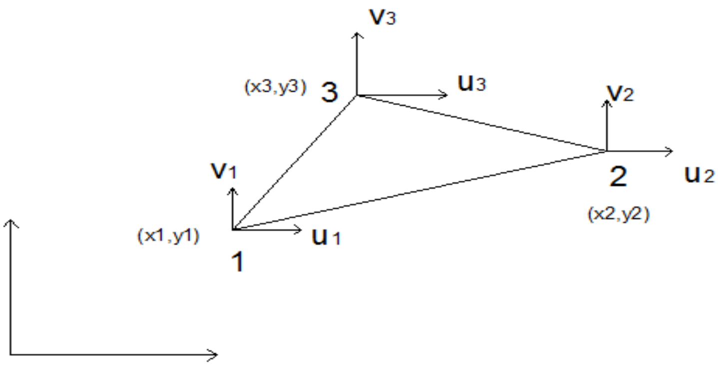

Fig. 4.2 Constant strain triangle

Shape functions for CST element

$$N1 = \xi$$

$$N2 = \eta \qquad (4.4)$$

$$N3 = 1 - \xi - \eta$$

For plane stress condition, Material matrix is given by

$$D_b = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}$$

Strain displacement matrix

$$[B] = \begin{bmatrix} \frac{\partial N1}{\partial x} & 0 & \frac{\partial N2}{\partial x} & 0 & \frac{\partial N3}{\partial x} & 0 \\ 0 & \frac{\partial N1}{\partial y} & 0 & \frac{\partial N2}{\partial y} & 0 & \frac{\partial N3}{\partial y} \\ \frac{\partial N1}{\partial y} & \frac{\partial N1}{\partial x} & \frac{\partial N2}{\partial y} & \frac{\partial N2}{\partial x} & \frac{\partial N3}{\partial y} & \frac{\partial N3}{\partial x} \end{bmatrix}$$

We find that the derivatives of shape functions are constant and hence [B] matrix does not depend on the position within the element. That is why this element is referred as CST element and there is no requirement of using the integration points within the element.

Jacobian matrix

$$[J] = \begin{bmatrix} \frac{\partial N1}{\partial \xi} & \frac{\partial N2}{\partial \xi} & \frac{\partial N3}{\partial \xi} \\ \frac{\partial N1}{\partial \eta} & \frac{\partial N2}{\partial \eta} & \frac{\partial N3}{\partial \eta} \end{bmatrix} \begin{bmatrix} x1 & y1 \\ x2 & y2 \\ x3 & y3 \end{bmatrix}$$

Area of triangle

$$\Delta = \frac{1}{2|J|} \begin{bmatrix} y2 - y3 & y3 - y1 \\ x3 - x2 & x1 - x3 \end{bmatrix}$$

Stiffness matrix of CST element

$$[ke] = [B]^T.[D].[B].t.\Delta$$

4.3.2. Bilinear Quadrilateral Element (Q4)

Let us consider the 4 noded quadrilateral to have overall eight degrees of freedom, u1, v1, ..., u4, and v4 associated with the global x and y directions.

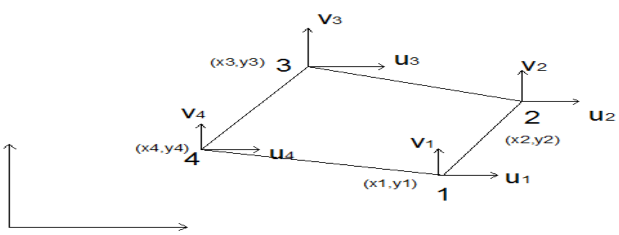

Fig. 4.3 Bilinear quadrilateral element

Shape functions for Q4 element

$$N_1 = \frac{1}{4}(1 - \xi)(1 - \eta)$$

$$N_2 = \frac{1}{4}(1 + \xi)(1 - \eta)$$

$$N_3 = \frac{1}{4}(1 + \xi)(1 + \eta)$$

$$N_4 = \frac{1}{4}(1 - \xi)(1 + \eta)$$

For plane stress, Material matrix is given by

$$D_b = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}$$

Strain displacement matrix

$$B_b^e = \begin{bmatrix} 0 & \frac{-\partial N1}{\partial x} & 0 & 0 & \frac{-\partial N2}{\partial x} & 0 & 0 & \frac{-\partial N3}{\partial x} & 0 & 0 & \frac{-\partial N4}{\partial x} & 0 \\ 0 & 0 & \frac{-\partial N1}{\partial y} & 0 & 0 & \frac{-\partial N2}{\partial y} & 0 & 0 & \frac{-\partial N3}{\partial y} & 0 & 0 & \frac{-\partial N4}{\partial y} \\ 0 & \frac{-\partial N1}{\partial y} & \frac{-\partial N1}{\partial x} & 0 & \frac{-\partial N2}{\partial y} & \frac{-\partial N2}{\partial x} & 0 & \frac{-\partial N3}{\partial y} & \frac{-\partial N3}{\partial x} & 0 & \frac{-\partial N4}{\partial y} & \frac{-\partial N4}{\partial x} \end{bmatrix}$$

Jacobian matrix

$$[J] = \begin{bmatrix} \frac{\partial N1}{\partial \xi} & \frac{\partial N2}{\partial \xi} & \frac{\partial N3}{\partial \xi} & \frac{\partial N4}{\partial \xi} \\ \frac{\partial N1}{\partial \eta} & \frac{\partial N2}{\partial \eta} & \frac{\partial N3}{\partial \eta} & \frac{\partial N4}{\partial \eta} \end{bmatrix}\begin{bmatrix} x1 & y1 \\ x2 & y2 \\ x3 & y3 \\ x4 & y4 \end{bmatrix}$$

4.4 Numerical Integration

Gauss quadrature method is used as numerical integration for calculating stiffness matrix. The Gauss-quadrature integration is summarized below

$$\iint_{-1-1}^{1\,1} f(\xi,\eta)\ \partial\xi\,\partial\eta = \sum_i \sum_j f(\xi_i,\eta_i)\text{wij}$$

Where,

m = number of sampling points

ξi, ηj = location of sampling points

wij = weighting coefficient

It should be remembered that m sampling points a polynomial of degree (2m-1) can be integrated exactly. For example, if we take m=2 then function having cubic polynomial can be integrated exactly. Generally, less number of sampling points are used to reduce the computation effort

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(Ghali & Neville, 2017) . In quadrilateral elements, 2x2 sampling points are used frequently [Neville].

Table 4.1: Gauss sampling points

m	ξi, ηj	wij
1	0	2
2	+0.57735 -0.57735	1 1

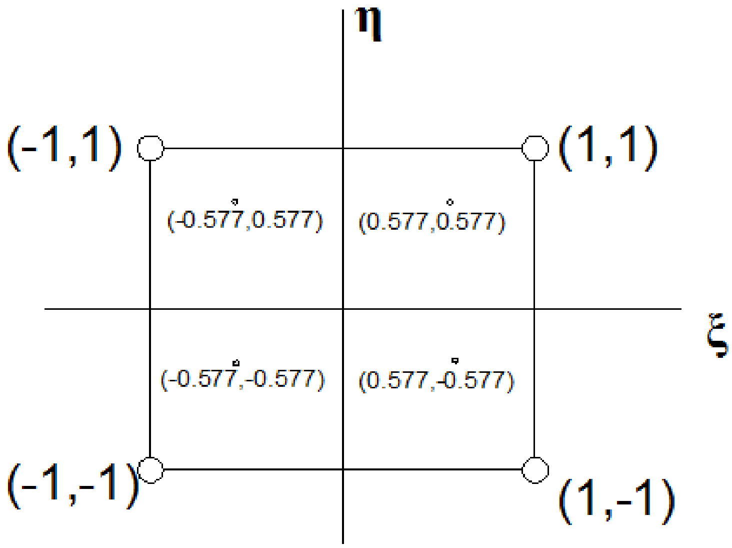


Fig. 4.4 Gaussian points location in element

Stiffness matrix of Q4 element

$$[K]= wi.wj. [B]^T .[D].[B].t. |J|$$

RESULT AND DISCUSSION

A square plate is subjected to a uniformly distributed load And fixed at the two opposite ends. Find the deflection of the plate using 4 four-node isoparametric elements and constant strain triangle element. The size of the plate is 5m. by 5m. and its thickness is 0.1 m. It is made of steel and the applied force is 10000 N at the centre.

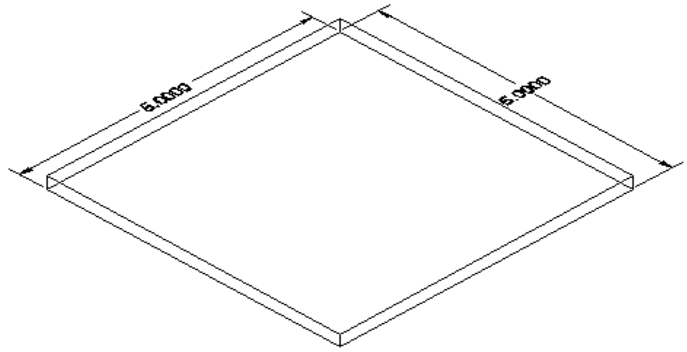
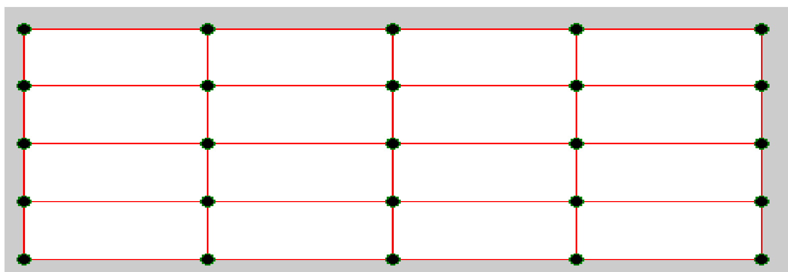
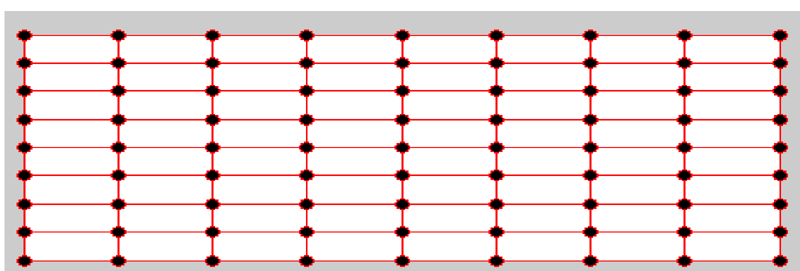


Fig. 4.5 Square plate problem

Bilateral quadrilateral element meshing



(a) meshing of square plate of 4 noded element(1st degree meshing)



(b) meshing of square plate of 4 noded element (2nd degree meshing)

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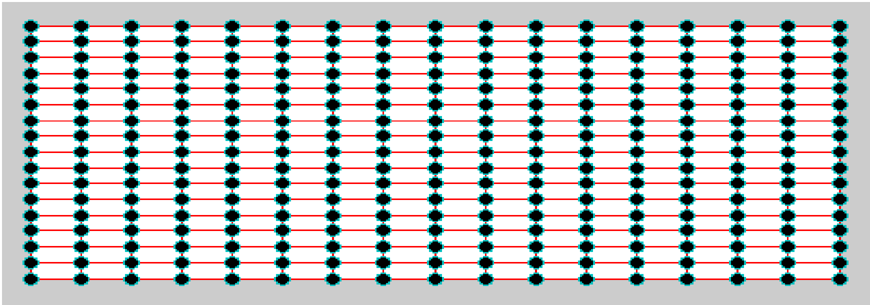


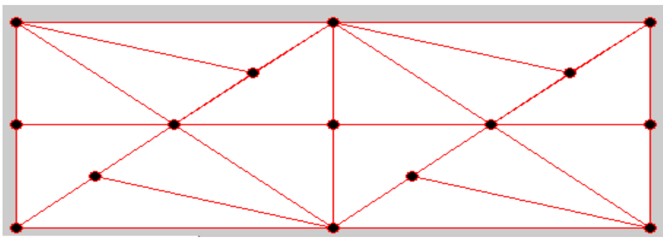
(c) meshing of square plate of 4 noded element (3rd degree meshing)

Fig. 4.11 4noded element mesh for plate (Q4_rows-colums) in MATLAB

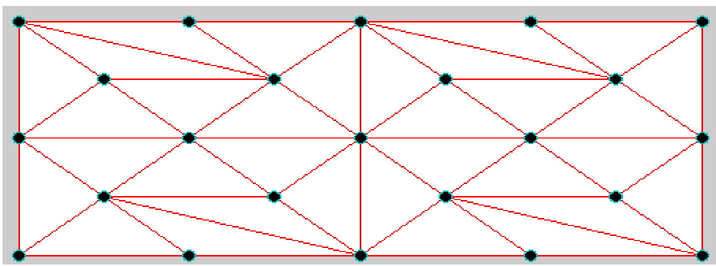
Table 4.2 Deflections results for plate with four noded element in MATLAB

Element	No. of nodes	DOF	Maximum Deflection (m)
1st degree meshing	25	75	6.12998×10^{-5}
2nd degree meshing	81	243	6.6213×10^{-5}
3rd degree meshing	289	867	6.9054×10^{-5}

Constant strain triangle element meshing



(a) meshing of square plate of CST element(1st degree meshing)



(b) meshing of square plate of CST element(2nd degree meshing)

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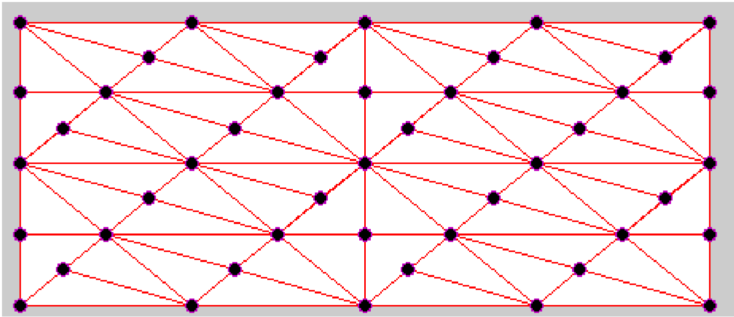
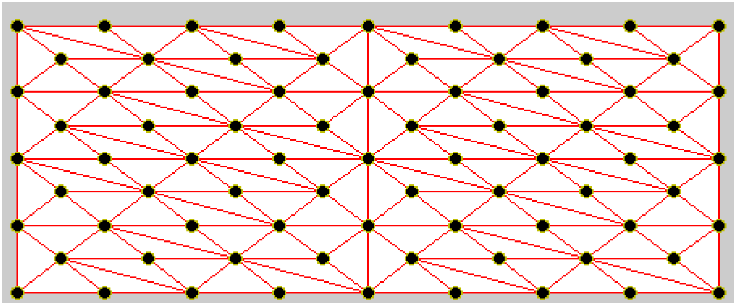


(c) meshing of square plate of CST element(3rd degree meshing)

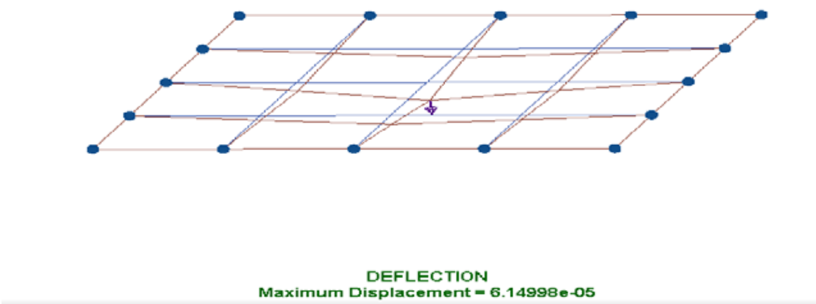


(d) meshing of square plate of CST element(4th degree meshing)

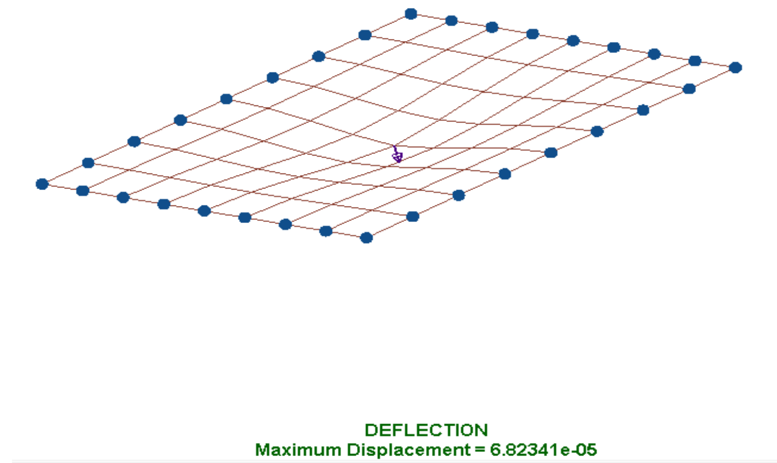
Fig. 4.11 CST element mesh for plate (Q4_rows-colums) in MATLAB

Table 4.5 Deflections results for plate with four noded element in MATLAB

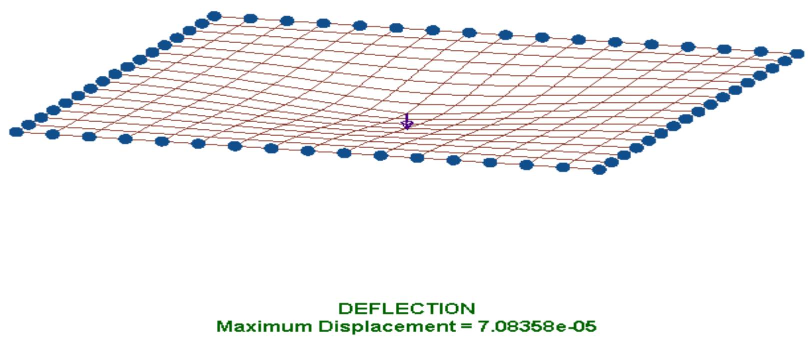
CST Element	No. of nodes	DOF	Maximum Deflection (m)
1st degree meshing	15	45	3.9298×10^{-5}
2nd degree meshing	23	69	4.6213×10^{-5}
3rd degree meshing	45	135	5.9423×10^{-5}
4th degree meshing	77	231	6.8054×10^{-5}



(a) 1st degree mesh in FEAST



(b) 2nd degree meshing in FEAST



(c) 3rd degree meshing in FEAST

Fig. 4.13 4noded element mesh for plate (Q4_rows-colums) in FEAST

Table 4.6Deflections results for plate with four noded element in FEAST

Element	No. of nodes	DOF	Maximum Deflection (m)
1st degree meshing	25	75	6.14998×10^{-5}
2nd degree meshing	81	243	6.82341×10^{-5}
3rd degree meshing	289	867	7.08368×10^{-5}

Conclusion

It is observed from the table 4.4 and table 4.5 that the CST element meshes are converging faster for same degree of refinement containing approx no of nodes as that of 4noded element meshes. But the values have much error in CST element as compared to Q4 element .Whereas, Q4 element mesh is converging slower as well giving negligible error compared to FEAST result. Overall, it can be understood that the 4 node element should be preferred for the analysis as it gives better results in less number of elements compared to 3 noded meshing which will greatly reduce the computational effort.

Whereas 3 noded element is tougher to analyse as compared to 4 noded element. For example liveload computation where the load is moving with respect to time .it is very difficult to extract the element of the moving load for the computation.Now as we know that the 4 node element perform better so the further analysis will be carried out using this element only.