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Controllability for Thermal Conductivity Equation with Non-Classical Boundary Conditions

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The article discusses the process described by the equation of thermal conductivity with a non-classical boundary condition.

Controllability for the equation of thermal conductivity with non-classical boundary conditions has been studied [1]. The theories of controllability and observability for ordinary dynamic systems have been sufficiently studied. The necessary and sufficient controllability conditions for the linear system are expressed through the fundamental matrix of the conjugate system. The concept and definition of controllability of a system described by equations with partial derivatives requires expansion [2]. This extension depends on the problem.

Let the control object in the area $D_T = \{(x,t): 0 < x < 1, 0 < t < T\}$ described by the equation

$$\frac{\partial y}{\partial t} = \frac{\partial^2 y}{\partial x^2} + u(x,t) \; in \; D_T \tag{1}$$

with boundary conditions

$$y(0,t) = 0, \; y_x(0,t) = y_x(1,t), \; t \in [0,T] \tag{2}$$

and initial condition

$$y(x,0) = y^0(x), \; x \in [0,1] \tag{3}$$

where $u(x,t)$ - density of heat sources at the point x at time t. There are no management restrictions.

Unlike the finite-dimensional case, in the infinite-dimensional case, controllability is defined as follows. A system whose state is defined as a solution to a problem (1)-(3), called manageable if monitoring $Cy = y_u(x,t)$ notices subspace $\widetilde{L}(D_T)$, dense in space $W_2^{1.0}(D_T)$, when control $u(x,t)$ runs the entire space $L_2(D_T)$. It has been proven that a system whose state is defined as a solution (1)-(3).

Then, the controllability of the system, which is defined as the solution of the equation, is studied.

$$\frac{\partial y}{\partial t} = \frac{\partial}{\partial x}\left(a(x)\frac{\partial y}{\partial x}\right) + b(x,t)y \; in \; D_T \tag{4}$$

with initial condition

$$y(x,0) = u(x) \tag{5}$$

and with boundary conditions

$$y(0,t) = 0,\ y_x(0,t) = y_x(1,t) \tag{6}$$

Here as valid control $u(x)$ will consider functions from space $L_2(0,1)$.

It is proved that if there is an inverse uniqueness of the task (4)-(6), that system whose state is defined as solution of problem (4) - (6) is controlled. In addition, the controlability of the problem

is proved

$$\frac{\partial y}{\partial t} = \frac{\partial}{\partial x}\left(a(x)\frac{\partial y}{\partial x}\right) - b(x)y \ in\ D_T, \tag{7}$$

$$y(x,0) = 0,\ 0 \le x \le 1, \tag{8}$$

$$y(0,t) = 0,\ y_x(1,t) - y_x(0,t) = u(t),\ 0 \le t \le T \tag{9}$$

where $u(t) \in L_2(0,T)$ - control.

The following describes the optimal control in terms of speed, for the equation of thermal conductivity with a non-classical boundary condition. The problem of the existence of optimal controls occupies a fundamental position in the theory of optimal processes and plays an important role in solving practical problems.

Let the control process be described by $y(x,t)$, which is inside the area $D_T = \{(x,t): 0 \le x \le 1,\ 0 \le t \le T\}$ satisfies the equation

$$\frac{\partial y}{\partial t} = \frac{\partial^2 y}{\partial x^2} + u(x,t), \tag{10}$$

with initial condition

$$y(x,0) = y^0(x), \tag{11}$$

and with boundary conditions

$$y(0,t) = 0,\ y_x(0,t) = y_x(1,t) \tag{12}$$

where $y^0(x)$ - given function, $u(x,t)$ - control.

Let the class of valid controls U_∂ convex, closed and bounded subsets of space $L_2(D_T)$.

The theorem is further proved in this section. The problem (10)-(12) unambiguously resolvable $W_{2,0}^{2,1}(D)$ at $\varphi \in \overset{0}{W}{}_2^1(0,1)$, i.e. drains the solution almost everywhere D.

To specify a function $\varphi(x) \in L_2(0,1)$ control must be specified in the selected valid control class $u^*(x,t) \in U_\partial$ such that the appropriate decision $y^*(x,t)$ problem (11)-(12) met a condition

$$y^*(x,\tau_0) = \varphi(x) \tag{13}$$

at the same time $\tau_0 \in (0,T)$ took the lowest possible value.

In this case, the following theorem is proved below. If control exists $u(x,t)\in U_\partial$ such that the appropriate decision $y(x,t)$ the problem satisfies the condition

$$y(x,\tau)=\varphi(x)$$

for some value $\tau\in(0,T)$, then there is control $u^*(x,t)$, optimal in terms of speed, i.e. appropriate solution $y^*(x,t)$ meets a condition

$$y^*(x,\tau_0)=\varphi(x),\ \tau_0=\inf\tau$$

In addition, this article discusses border control.

Let the process be described by the equation

$$\frac{\partial y}{\partial t}=\frac{\partial^2 y}{\partial x^2}+f(x,t), \tag{14}$$

with conditions

$$y(x,0)=y_0(x), \tag{15}$$

$$y(0,t)=0,\ y_x(1,t)-y_x(0,t)=u(t) \tag{16}$$

Allowed control class $u(t)\in U_\partial$ convex, bounded, closed subset space $L_2(0,T)$. Suppose that K - weakly, a closed subset of $L_2(0,1)$.

Let there be control $u(t)\in U_\partial$ such that the appropriate decision $y(x,t)$ problem (14)-(16) meets a condition

$$y(x,\tau)\in K\ \ for\ any\ \tau\in(0,T).$$

Then there is control $u^*(t)\in U_\partial$, optimal in terms of speed

$$y^*(x,\tau_0)\in K,\ \tau_0=\inf\tau\ .$$

Next, the relay of optimal speed control for the controlled process described by the non-classical boundary condition thermal conductivity equation is studied.

Let the state of the system be defined as the solution to the problem

$$\left.\begin{array}{l}\dfrac{\partial y}{\partial t}=\dfrac{\partial^2 y}{\partial x^2}+p(x)u(t)\ in\ D_T\\ y(x,0)=y_0(x),\ 0\le x\le 1,\\ y(0,t)=0,\ y_x(0,t)=y_x(1,t),\ 0\le t\le T\end{array}\right\} \tag{17}$$

where $p(x),\ y_0(x)$ – specified functions from $L_2(0,1)$, and u - control parameter.

Functions are taken for many permissible controls

$$u(t)\in U_\partial=\{u,u(t)\in L_2(0,T),\ |u(t)|\le 1\ almost\ in\ everywhere\}\ .$$

For each allowable control $u(t)$ there is a single generalized solution $y(x,t)$ problem (17), which using the function $G(x,s,t)$ can be represented as $u^*(t)\in U_\partial$,

$$y(x,t)=\int_0^1 G(x,s,t)y_0(s)ds+\int_0^t\int_0^1 G(x,s,t-\sigma)p(s)u(\sigma)dsd\sigma \tag{18}$$

For a given function $\varphi(x)$ from $L_2(0,1)$ need to find a control such that the appropriate solution $y^*(x,t)$ problem (17) met a condition

$$y^*(x,\tau_0)=\varphi(x) \tag{19}$$

where τ_0 - bottom face of value τ, for which the condition is met

$$y(x,\tau)=\varphi(x)$$

for some solution of the problem (17) and some $\tau\in(0,T)$.

Theorem 1. Let there be control $u(t)\in U_\partial$ such that the appropriate decision $y(x,t)$ problem (17) meets a condition $y(x,\tau)=\varphi(x)$ for some $\tau\in(0,T)$. Then there is control $u^*(t)\in U_\partial$, optimal in terms of speed:

$$y^*(x,\tau_0)=\varphi(x),\ \tau_0=\inf\tau .$$

The following shows the relayability of optimal control in the sense of speed.

Theorem 2. Let $u^*(t)\in U_\partial$ - optimal control in terms of responsiveness.

Then

$$\left|u^*(t)=1\ in\ everywhere\ at\ (0,\tau_0)\right|.$$

In addition, the first order optimization conditions are entered in the paragraph. The conjugate state is defined as the solution to the problem:

$$\left.\begin{array}{l}\dfrac{\partial z}{\partial t}+\dfrac{\partial^2 z}{\partial x^2}=0\ in\ D_T,\\ z(x,\tau_0)=h(x),\\ z_x(1,t)=0,\ z(0,t)=z(1,t)\end{array}\right\} \tag{20}$$

The following theorem is proved below:

Theorem 3. Let $u^*(t)\in U_\partial$ optimal control in the sense of speed. Then there is a solution $z(x,t)$ a conjugate problem (20) such that for anyone $u\in[0,1]$ inequality is performed

$$\left(\int_0^1 z(x,t)p(x)dx\right)(u-u^*(t))\le 0\ in\ everywhere\ at\ (0,\tau_0).$$

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