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α_{Ng}-Irresolute Function in Nano Topological Spaces

J. Arul Jesti*1 , P. Suganya*2

1.Assistant Professor, Department of Mathematics, St. Mary's College(Autonomous),

(Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli)

Thoothukudi-1,TamilNadu, India

2.Research Scholar, Reg.No.19222212092015, Department of Mathematics, St. Mary's College(Autonomous),

(Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli)

Thoothukudi-1,TamilNadu, India

[1]jestiarul@gmail.com

[2]suganyaprince20@gmail.com

Abstract

The aim of this paper is to initiate the new concept of α_{Ng}-irresolute function, α_{Ng}-open map α_{Ng}-closed map in Nano Topological Spaces. Further, some of their basic properties and condition for a function to be α_{Ng}-open are investigated.

Keywords : α_{Ng}-irresolute, α_{Ng}-open, α_{Ng}-closed

1.Introduction

Levine[2] derived the concept of generalized closed set in topological space. Pious Missier and Anbarasi Rodrigo[7] studied α^*-open set in topological space. The notion of Nano Topology was introduced by Lellis Thivagar[3] defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it. He also defined Nano-interior, Nano-closure and Nano-continuity. Bhuvaneswari and Mythili Gnanapriya[1] introduced Nano generalized closed set and Nano generalized continuous functions and studied their properties. Arul Jesti and Suganya[9,10] define α_{Ng}-open set and α_{Ng}-continuous function and discussed some of their properties. In this paper, we introduce a new function called α_{Ng}-irresolute function, α_{Ng}-open map α_{Ng}-closed map in Nano Topological Spaces and its properties are discussed.

2.Preliminaries

Definition 2.1:[7] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be discernible with one another. The pair (U, R) is said to be the ***approximation space***.

Let $X \subseteq U$

1. The lower approximation of X with respect to R is the set of all objects which can be certain classified as X with respect to R and it is denoted by $L_R(X)$. That is $L_R(X) = \bigcup_{x\in U}\{R(x) / R(x) \subseteq X\}$ where $R(x)$ denotes the equivalence class determined by X.
2. The upper approximation of X with respect to R is the set of all objects which can be possibly defined as X with respect to R and it is denoted by $U_R(X)$. That is $U_R(X) = \bigcup_{x\in U}\{R(x) / R(x) \cap X \neq \phi\}$
3. The boundary region of X with respect to R is the set of all objects which can be classified neither as X nor as not X with respect to R and is denoted by $B_R(X)$. That is $B_R(X) = U_R(X) - L_R(X)$.

Proposition 2.2:[4] If (U, R) is an approximation space and $X, Y \subseteq U$, then

1. $L_R(X) \subseteq X \subseteq U_R(X)$
2. $L_R(\phi) = U_R(\phi) = \phi$ and $L_R(U) = U_R(U) = U$
3. $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$
4. $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$
5. $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$
6. $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$
7. $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$

8. $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$

9. $U_R U_R(X) = L_R U_R(X) = U_R(X)$

10. $L_R L_R(X) = U_R L_R(X) = L_R(X)$

Definition 2.3:[4] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$ and $\tau_R(X)$ satisfies the following axioms:

1. U and $\phi \in \tau_R(X)$
2. The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.
3. The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ is a topology on U called the Nano topology on U with respect to X. We call $(U, \tau_R(X))$ as the Nano topological space. The elements of $\tau_R(X)$ are called as Nano-open sets.

Remark 2.4:[4] If $\tau_R(X)$ is the Nano topology on U with respect to X, then the set $B = \{U, \phi, L_R(X), B_R(X)\}$ is the ***basis*** for $\tau_R(X)$.

Definition 2.5:[5] If $(U, \tau_R(X))$ is a Nano topological space with respect to X and if $A \subseteq U$, then the ***Nano-interior*** of A is defined as the union of all Nano-open subsets of A and is denoted by $Nint(A)$. That is, $Nint(A)$ is the largest Nano-open subset of A.

The ***Nano-closure*** of A is defined as the intersection of all Nano-closed sets containing A and it is denoted by $Ncl(A)$.That is, $Ncl(A)$ is the smallest Nano-closed set containing A.

Definition 2.6:[1] For every set $A \subseteq U$, the ***Nano generalized closure*** of A is defined as the intersection of all Ng-closed sets containing A and is denoted by $Ngcl(A)$.

Definition 2.7:[1] For every set $A \subseteq U$, the ***Nano generalized interior*** of A is defined as the union of all Ng-open sets contained in A and is denoted by $Ngint(A)$.

Definition 2.8:[6] Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ be two Nano topological spaces. Then a mapping $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is ***Nano-continuous function*** on U if the inverse image of every Nano-open set in $(V, \tau_{R'}(Y))$ is Nano-open in $(U, \tau_R(X))$.

Definition 2.9:[2] A function $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is ***Nano-irresolute*** if the inverse image of every Nano-open set in V is Nano-open in U.

Definition 2.10:[6] A function $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is ***Nano-open map*** if the image of every Nano-open set in U is Nano-open in V.

Definition 2.11:[6] A function $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is ***Nano-closed map*** if the image of every Nano-closed set in U is Nano-closed in V.

Definition 2.12:[1] A function $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is ***Nano g-closed map*** if the image of every Nano-closed set in U is Nano g-closed in V.

Definition 2.13:[1] A function $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is ***Nano g-open map*** if the image of every Nano-open set in U is Nano g-open in V.

3. α_{Ng}-Irresolute Function

Definition 3.1: A function $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is said to be ***α_{Ng}-irresolute*** if $f^{-1}(O)$ is a α_{Ng}-open in $(U, \tau_R(X))$ for every α_{Ng}-open set O in $(V, \tau_{R'}(Y))$.

Example 3.2: Let $U = \{a, b, c, d\}$ with $U/R = \{\{a, b\}, \{c\}, \{d\}\}$ and $X = \{a, c\}$. Then $\tau_R(X) = \{U, \emptyset, \{c\}, \{a, b, c\}, \{a, b\}\}$. The α_{Ng}-open sets of U are $\{U, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}\}$. Let $V = \{x, y, z, w\}$ with $V/R' = \{\{x, z\}, \{y\}, \{w\}\}$ and $Y = \{x, y\}$. Then $\tau_{R'}(Y) = \{V, \emptyset, \{y\}, \{x, z\}, \{x, y, z\}\}$. The α_{Ng}-open sets of V are $\{V, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}\}$. Define $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ as $f(a) = x, f(b) = z, f(c) = y, f(d) = w$. Then, f is α_{Ng}-irresolute.

Theorem 3.3: A map $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is α_{Ng}-irresolute if and only if the inverse image of every α_{Ng}-closed set in $(V, \tau_{R'}(Y))$ is α_{Ng}-closed in $(U, \tau_R(X))$.

Proof: Assume that f is α_{Ng}-irresolute map. Let S be any α_{Ng}-closed set in $(V, \tau_{R'}(Y))$.Then S^c is α_{Ng}-open in $(V, \tau_{R'}(Y))$. Since, f is α_{Ng}-irresolute, $f^{-1}(S^c)$ is α_{Ng}-open in $(U, \tau_R(X))$. But $f^{-1}(S^c)$ = U / $f^{-1}(S)$ and so, $f^{-1}(S)$ is α_{Ng}-closed set in $(U, \tau_R(X))$. Hence, the inverse image of every α_{Ng}-closed set in $(V, \tau_{R'}(Y))$ is α_{Ng}-closed in $(U, \tau_R(X))$.

Conversely, assume that the inverse image of every α_{Ng}-closed set in $(V, \tau_{R'}(Y))$ is α_{Ng}- closed in $(U, \tau_R(X))$. Let C be any α_{Ng}-open in $(V, \tau_{R'}(Y))$. Then C^c is α_{Ng}-closed set in $(V, \tau_{R'}(Y))$. By assumption, $f^{-1}(C^c)$ is α_{Ng}-closed set in $(U, \tau_R(X))$. But $f^{-1}(C^c)$= U / $f^{-1}(C)$ and so, $f^{-1}(C)$ is α_{Ng} open set in $(U, \tau_R(X))$. Therefore, f is α_{Ng}-irresolute.

Theorem 3.4: Every α_{Ng}-irresolute map is α_{Ng}-continuous.

Proof: Let $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ be a α_{Ng}-irresolute map. Let N be a Nano-open set in V. Then N is α_{Ng}-open in V. Since f is α_{Ng}-irresolute map, $f^{-1}(N)$ is α_{Ng}-open in U. Therefore, f is α_{Ng}-continuous.

Remark 3.5: The converse of the above theorem need not be true as shown in the following example.

Example 3.6: Let $U = \{a, b, c\}$, $U/R = \{\{a\}, \{b, c\}\}$ and $X = \{b, c\}$. Then $\tau_R(X) = \{U, \emptyset, \{b, c\}\}$. Let $V = \{x, y, z\}, V/R' = \{\{x, y\}, \{z\}\}$ and $Y = \{z\}$. Then $\tau_{R'}(Y) = \{V, \emptyset, \{z\}\}$. The α_{Ng}-open sets of V are $P(V)$. Define $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ as $f(a) = x, f(b) = y, f(c) = z$. Then $f^{-1}(z) = c$ which is α_{Ng}-open in U. Then f is α_{Ng}-continuous. But, $f^{-1}(x) = a$ which is not α_{Ng}-open in U. Hence, f is not α_{Ng}-irresolute.

Definition 3.7: A Nano topological space $(U, \tau_R(X))$ is said to be a ***α_{Ng}-$T_{1/2}$ space*** if every α_{Ng}-open set of U is Nano-open in U.

Theorem 3.8: Let $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ be a α_{Ng}-continuous map and $(V, \tau_{R'}(Y))$ is α_{Ng}-$T_{½}$ space. Then f is α_{Ng}-irresolute.

Proof: Let A be a α_{Ng}-open set in $(V, \tau_{R'}(Y))$. Since $(V, \tau_{R'}(Y))$ is α_{Ng}-$T_{½}$ space, A is a Nano-open set in $(V, \tau_{R'}(Y))$. Since f is α_{Ng}-continuous, $f^{-1}(A)$ is α_{Ng}-open in $(U, \tau_R(X))$. Hence f is a α_{Ng}-irresolute function.

Theorem 3.9: Let U, V and W be a Nano topological spaces. If $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is α_{Ng}-irresolute and $g : (V, \tau_{R'}(Y)) \to (W, \tau_{R''}(Z))$ is α_{Ng}-continuous then, the composition $g \circ f : (U, \tau_R(X)) \to (W, \tau_{R''}(Z))$ is α_{Ng}-continuous.

Proof: Let H be a Nano-open set in W. Since g is α_{Ng}-continuous $g^{-1}(H)$ is α_{Ng}-open in V. Since f is α_{Ng}-irresolute , $f^{-1}(g^{-1}(H))$ is α_{Ng} open in U. But $f^{-1}(g^{-1}(H)) = (g \circ f)^{-1}(H)$. Therefore, $g \circ f : (U, \tau_R(X)) \to (W, \tau_{R''}(Z))$ is α_{Ng}-continuous.

Theorem 3.10: Let U, V and W be a Nano topological spaces. If $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ and $g : (V, \tau_{R'}(Y)) \to (W, \tau_{R''}(Z))$ are α_{Ng}-irresolute then , the composition $g \circ f : (U, \tau_R(X)) \to (W, \tau_{R''}(Z))$ is α_{Ng}-irresolute.

Proof: Let K be a α_{Ng}-open set in W. Since g is α_{Ng}-irresolute, $g^{-1}(K)$ is α_{Ng}-open in V. Since f is α_{Ng}-irresolute, $f^{-1}(g^{-1}(H))$ is α_{Ng}-open in U. But $f^{-1}(g^{-1}(H)) = (g \circ f)^{-1}(H)$. Therefore, $g \circ f : (U, \tau_R(X)) \to (W, \tau_{R''}(Z))$ is α_{Ng}-irresolute.

Theorem 3.11: If $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is α_{Ng}-irresolute and $g : (V, \tau_{R'}(Y)) \to (W, \tau_{R''}(Z))$ is Nano-continuous then , the composition $g \circ f : (U, \tau_R(X)) \to (W, \tau_{R''}(Z))$ is α_{Ng}-continuous.

Proof: Let O be a Nano-open set in W. Since g is Nano-continuous, $g^{-1}(O)$ is Nano-open in U. Since every Nano-open is α_{Ng}-open, $g^{-1}(O)$ is α_{Ng}-open in V. Since f is α_{Ng}-irresolute, $f^{-1}(g^{-1}(O))$ is α_{Ng}-open in U. But $f^{-1}(g^{-1}(O)) = (g \circ f)^{-1}(O)$. Therefore, $g \circ f : (U, \tau_R(X)) \to (W, \tau_{R''}(Z))$ is α_{Ng}-continuous.

Theorem 3.12: If $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is α_{Ng}-irresolute and $g : (V, \tau_{R'}(Y)) \to (W, \tau_{R''}(Z))$ is Nano α-continuous then , the composition $g \circ f : (U, \tau_R(X)) \to (W, \tau_{R''}(Z))$ is α_{Ng}-continuous.

Proof: Let J be a Nano-open set in W. Since g is Nano α-continuous and every Nano α-open is α_{Ng}-open, $g^{-1}(J)$ is α_{Ng}-open in V. Since f is α_{Ng}-irresolute, $f^{-1}(g^{-1}(J))$ is α_{Ng}-open in U. But $f^{-1}(g^{-1}(J)) = (g \circ f)^{-1}(J)$. Therefore, $g \circ f : (U, \tau_R(X)) \to (W, \tau_{R''}(Z))$ is α_{Ng}-continuous.

Theorem 3.13: If $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is α_{Ng}-irresolute and $g : (V, \tau_{R'}(Y)) \to (W, \tau_{R''}(Z))$ is Nano g-continuous then , the composition $g \circ f : (U, \tau_R(X)) \to (W, \tau_{R''}(Z))$ is α_{Ng}-continuous.

Proof: Let Q be a Nano-open set in W. Since g is Nano g-continuous, $g^{-1}(Q)$ is Nano g-open in U. Since every Nano g-open is α_{Ng}-open, $g^{-1}(Q)$ is α_{Ng}-open in V. Since f is α_{Ng}-irresolute, $f^{-1}(g^{-1}(Q))$ is α_{Ng}-open in U. But $f^{-1}(g^{-1}(Q)) = (g \circ f)^{-1}(Q)$. Therefore, $g \circ f : (U, \tau_R(X)) \to (W, \tau_{R''}(Z))$ is α_{Ng}-continuous.

4. α_{Ng}-Open Maps and α_{Ng}-Closed Maps

Definition 4.1: Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ be two Nano topological spaces. Then a mapping $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is ***α_{Ng}-open map*** if the image of every Nano-open set in $(U, \tau_R(X))$ is α_{Ng}-open in $(V, \tau_{R'}(Y))$. The mapping f is said to be ***α_{Ng}-closed map*** if the image of every Nano-closed set in $(U, \tau_R(X))$ is α_{Ng}-closed in $(V, \tau_{R'}(Y))$.

Example 4.2: Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{c\}, \{b, d\}\}$ and $X = \{a, qb\}$. Then $\tau_R(X) = \{U, \emptyset, \{a\}, \{a, b, d\}, \{b, d\}\}$. Let $V = \{x, y, z, w\}$ with $V/R' = \{\{x, y\}, \{z\}, \{w\}\}$ and $Y = \{x, z\}$. Then $\tau_{R'}(Y) = \{V, \emptyset, \{z\}, \{x, y\}, \{x, y, z\}\}$. The α_{Ng}-open sets of V are $\{V, \emptyset, \{x\}, \{y\}, \{z\}, \{x, y\}, \{x, z\}, \{y, z\}, \{x, y, z\}, \{x, z, w\}, \{y, z, w\}\}$. Define $f : (U, \tau_R(X)) \to$ `$(V, \tau_{R'}(Y))$ as $f(a) = y, f(b) = x, f(c) = w, f(d) = z$. Then $f(a) = \{y\}$, $f(\{b, d\}) = \{x, z\}$, $f(\{a, b, d\}) = \{x, y, z\}$ and $f(U) = V$. That is, the image of every Nano-open set in U is α_{Ng}-open set in V. Therefore, f is α_{Ng}-open map.

Example 4.3: Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{c\}, \{b, d\}\}$ and $X = \{a, b\}$. Then $\tau_R(X) = \{U, \emptyset, \{a\}, \{a, b, d\}, \{b, d\}\}$. The Nano-closed set of U are $\{U, \emptyset, \{b, c, d\}, \{c\}, \{a, c\}\}$. Let $V = \{x, y, z, w\}$ with $V/R' = \{\{x, y\}, \{z\}, \{w\}\}$ and $Y = \{x, z\}$. Then $\tau_{R'}(Y) = \{V, \emptyset, \{z\}, \{x, y\}, \{x, y, z\}\}$. The α_{Ng}-closed sets of V are $\{V, \emptyset, \{x\}, \{y\}, \{w\}, \{x, w\}, \{y, w\}, \{z, w\}, \{x, y, w\}, \{x, z, w\}, \{y, z, w\}\}$. Define $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ as $f(a) = x, f(b) = y, f(c) = w, f(d) = z$. Then $f(c) = \{w\}$, $f(\{a, c\}) = \{x, w\}$, $f(\{b, c, d\}) = \{y, z, w\}$ and $f(U) = V$. That is, the image of every Nano-closed set in U is α_{Ng}-closed set in V. Therefore, f is α_{Ng}-closed map.

Theorem 4.4: If $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is Nano-open(closed) map, then f is α_{Ng} open(closed) map.

Proof: Let $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ be a Nano-open(closed) map and G be a Nano-open(closed) in U. Then $f(G)$ is Nano-open(closed). Since every Nano-open(closed) is α_{Ng}-open(closed), $f(G)$ is α_{Ng}-open(closed). Hence the image of every Nano-open(closed) set in U is α_{Ng}-open(closed) in V. Therefore, f is α_{Ng}-open(closed) map.

Remark 4.5: The converse of the theorem may not be true as seen from the following example.

Example 4.6: Let $U = \{a, b, c, d\}$, $U/R = \{\{a, b\}, \{c\}, \{d\}\}$ and $X = \{b, c\}$. Then $\tau_R(X) = \{U, \emptyset, \{c\}, \{a, b, c\}, \{a, b\}\}$. Let $V = \{x, y, z, w\}$, $V/R' = \{\{x, z\}, \{y\}, \{w\}\}$ and $Y = \{x, y\}$. Then $\tau_{R'}(Y) = \{V, \emptyset, \{y\}, \{x, z\}, \{x, y, z\}\}$. The α_{Ng} open sets of V are $\{V, \emptyset, \{x\}, \{y\}, \{z\}, \{x, y\}, \{x, z\}, \{y, z\}, \{x, y, z\}, \{x, z, w\}, \{y, z, w\}\}$. Define $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ as $f(a) = x, f(b) = y, f(c) = z, f(d) = w$. Then $f(c) = \{z\}$, $f(\{a, b\}) = \{x, y\}$, $f(\{a, b, c\}) = \{x, y, z\}$ and $f(U) = V$. That is, the image of every Nano-open set in U is α_{Ng}-open set in V. Therefore, f is α_{Ng}-open map. But $f(c) = z, f(\{a, b\}) = \{x, y\}$ which are not Nano-open in V. Hence, f is not Nano-open map.

Example 4.7: Let $U = \{a, b, c, d\}$, $U/R = \{\{a, b\}, \{c\}, \{d\}\}$ and $X = \{b, c\}$. Then $\tau_R(X) = \{U, \emptyset, \{c\}, \{a, b, c\}, \{a, b\}\}$. The Nano-closed sets of U are $\{U, \emptyset, \{d\}, \{c, d\}, \{a, b, d\}\}$. Let $V = \{x, y, z, w\}$, $V/R' = \{\{x, z\}, \{y\}, \{w\}\}$ and $Y = \{x, y\}$. Then $\tau_{R'}(Y) = \{V, \emptyset, \{y\}, \{x, z\}, \{x, y, z\}\}$. The Nano-closed sets of V are $\{V, \emptyset, \{w\}, \{y, w\}, \{x, z, w\}\}$. The α_{Ng}-closed sets of V are $\{V, \emptyset, \{x\}, \{y\}, \{w\}, \{x, w\}, \{y, w\}, \{z, w\}, \{x, z, w\}, \{x, y, w\}, \{y, z, w\}\}$. Define $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ as $f(a) = x, f(b) = y, f(c) = z, f(d) = w$. Here, f is α_{Ng}-closed map. But, $f(\{c, d\}) = \{z, w\}, f(\{a, b, d\}) = \{x, y, w\}$ which are not Nano-closed in V. Hence, f is not Nano-closed map.

Theorem 4.8: Every Nano α-open(closed) map is α_{Ng}-open(closed) map.

Proof: Let $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ be a Nano α-open(closed) map. Let J be Nano-open(closed) in U. Then, $f(J)$ is Nano α-open(closed) in V. Since every Nano α-open(closed) is α_{Ng}-open(closed), $f(J)$ is α_{Ng}-open(closed). Hence, f is α_{Ng} open(closed) map.

Remark 4.9: The converse of the above theorem is not true as seen from the following example.

Example 4.10: Let $U = \{a, b, c, d\}$, $U/R = \{\{a, d\}, \{b\}, \{c\}\}$ and $X = \{c, d\}$. Then $\tau_R(X) = \{U, \emptyset, \{c\}, \{a, c, d\}, \{a, d\}\} = \tau_R{}^{\alpha}(X)$. Let $V = \{x, y, z, w\}, V/R' = \{\{x\}, \{z\}, \{y, w\}\}$ and $Y = \{x, w\}$. Then $\tau_{R'}(Y) = \{V, \emptyset, \{x\}, \{y, w\}, \{x, y, w\}\} = \tau_R{}^{\alpha}(Y)$. The α_{Ng}-open sets of V are $\{V, \emptyset, \{x\}, \{y\}, \{w\}, \{x, y\}, \{x, w\}, \{y, w\}, \{x, y, z\}, \{x, y, w\}, \{x, z, w\}\}$. Define $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ as $f(a) = x, f(b) = z, f(c) = y, f(d) = w$. Then, $f(c) = \{y\}$, $f(\{a, d\}) = \{x, w\}$, $f(\{a, c, d\}) = \{x, y, w\}$ and $f(U) = V$. That is, the image of every Nano-open set in

U is α_{Ng}-open set in V. Therefore, f is α_{Ng}-open map. But $f(c) = y$, $f(\{a,d\}) = \{x,w\}$ which are not Nano α-open in V. Hence, f is not Nano α-open map.

Example 4.11: Let $U = \{a,b,c,d\}$, $U/R = \{\{a,d\},\{b\},\{c\}\}$ and $X = \{c,d\}$. Then $\tau_R(X) = \{U, \emptyset, \{c\}, \{a,c,d\}, \{a,d\}\}$. The Nano-closed sets of U are $\{U, \emptyset, \{b\}, \{b,c\}, \{a,b,d\}\}$. Let $V = \{x,y,z,w\}$, $V/R' = \{\{x\},\{z\},\{y,w\}\}$ and $Y = \{x,w\}$. Then $\tau_{R'}(Y) = \{V, \emptyset, \{x\}, \{y,w\}, \{x,y,w\}\}$. The Nano α-closed sets of V are $\{V, \emptyset, \{z\}, \{x,z\}, \{y,z,w\}\}$. The α_{Ng}-closed sets of V are $\{V, \emptyset, \{y\}, \{z\}, \{w\}, \{x,z\}, \{y,z\}, \{z,w\}, \{x,y,z\}, \{x,z,w\}, \{y,z,w\}\}$. Define $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ as $f(a) = w, f(b) = z, f(c) = y, f(d) = x$. Then, f is α_{Ng}-closed map. But, $f(\{b,c\}) = \{y,z\}$, $f(\{a,b,d\}) = \{x,w,z\}$ which are not Nano α-closed in V. Hence, f is not Nano α-closed map.

Theorem 4.12: If $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is a Nano generalized open(closed) map, then f is α_{Ng}-open(closed) map.

Proof: Let S be Nano-open in U. Since f is Nano generalized open(closed), $f(S)$ is Nano generalized open(closed) in V. Every Nano generalized open(closed) is α_{Ng}-open(closed). Therefore, $f(S)$ is α_{Ng}-open(closed) in V. Hence, f is α_{Ng}-open(closed) map.

Remark 4.13: The converse of the above theorem is not true as seen from the following example.

Example 4.14: Let $U = \{a,b,c\}$, $U/R = \{\{a\},\{b,c\}\}$ and $X = \{b,c\}$. Then $\tau_R(X) = \{U, \emptyset, \{bc\}\}$. Let $V = \{x,y,z\}$, $V/R' = \{\{x\},\{y,z\}\}$ and $Y = \{x\}$. Then $\tau_{R'}(Y) = \{V, \emptyset, \{x\}\}$. The α_{Ng} open sets of V are $P(V)$. Define $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ as $f(a) = x$, $f(b) = y$, $f(c) = z$. Then, $f(\{b,c\}) = \{y,z\}$ and $f(U) = V$. Hence, f is α_{Ng}-open map. But, $f(\{b,c\}) = \{y,z\}$ which is not Nano generalized open in V. Hence, f is not Nano generalized open map.

Example 4.15: Let $U = \{a,b,c\}$, $U/R = \{\{a\},\{b,c\}\}$ and $X = \{b,c\}$. Then $\tau_R(X) = \{U, \emptyset, \{bc\}\}$. $\{U, \emptyset, \{a\}\}$ are the Nano-closed sets of U. Let $V = \{x,y,z\}$, $V/R' = \{\{x\},\{y,z\}\}$ and $Y = \{x\}$. Then $\tau_{R'}(Y) = \{V, \emptyset, \{x\}\}$. The α_{Ng}-closed sets of V are $P(V)$. The Nano generalized closed sets of V are $\{V, \emptyset, \{y\}, \{z\}, \{x,y\}, \{x,z\}, \{y,z\}\}$. Define $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ as $f(a) = x, f(b) = y, f(c) = z$. Then, $f(a) = \{x\}$ and $f(U) = V$. Here, f is α_{Ng}-closed map. But $f(a) = \{x\}$ which is not Nano generalized closed in V.

Hence, f is not Nano generalized closed map.

Theorem 4.16: A map $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is α_{Ng}-closed map if and only if $\alpha_{Ng}cl(f(A)) \subseteq f(Ncl(A))$, for every subset A of U.

Proof: Suppose f is a α_{Ng}-closed map. Since for every subset A of U, $Ncl(A)$ is Nano-closed in U then $f(Ncl(A))$ is α_{Ng}-closed in V. Since $A \subseteq Ncl(A)$, $f(A) \subseteq f(Ncl(A))$ implies $\alpha_{Ng}cl(f(A)) \subseteq \alpha_{Ng}cl(f(Ncl(A)))$ which implies $\alpha_{Ng}cl(f(A)) \subseteq f(Ncl(A))$. Conversely, let A be a Nano-closed in U. Since $\alpha_{Ng}cl(f(A))$ is the smallest α_{Ng}-closed set containing $f(A)$, $f(A) \subseteq \alpha_{Ng}cl(f(A)) \subseteq f(Ncl(A)) = f(A)$. Thus, $f(A) = \alpha_{Ng}cl(f(A))$. Hence, $f(A)$ is a α_{Ng}-closed set in V. Therefore, f is a α_{Ng}-closed map.

Theorem 4.17: A map $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is α_{Ng}-open map if and only if $f(int(A)) \subseteq \alpha_{Ng}int(f(A))$ for each set A of U.

Proof: Let f be a α_{Ng}-open map. Since $Nint(A) \subseteq A$, then $f(Nint(A)) \subseteq f(A)$. By hypothesis, $f(Nint(A))$ is α_{Ng}-open and $\alpha_{Ng}int(f(A))$ is the largest α_{Ng}-open set contained in $f(A)$. Hence $f(Nint(A)) \subseteq \alpha_{Ng}int(f(A))$. Conversely, Let A be a Nano-open set in U and $f(Nint(A)) \subseteq \alpha_{Ng}int(f(A))$. Since $Nint(A) = A$, then $f(A) \subseteq \alpha_{Ng}int(f(A))$. Thus, $f(A) = \alpha_{Ng}int(f(A))$. Therefore, $f(A)$ is α_{Ng}-open set in V and f is α_{Ng}-open map.

Theorem 4.18: A map $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is α_{Ng}-open if and only if for any subset G of $(V, \tau_{R'}(Y))$ and any Nano-closed set F of $(U, \tau_R(X))$ containing $f^{-1}(G)$, there exists a α_{Ng}-closed set A of $(V, \tau_{R'}(Y))$ containing G such that $f^{-1}(A) \subseteq F$.

Proof: Suppose f is α_{Ng}-open. Let $G \subseteq V$ and F be a Nano-closed set of $(U, \tau_R(X))$ such that $f^{-1}(G) \subseteq F$. Now $U - F$ is a Nano-open set in $(U, \tau_R(X))$. Since f is α_{Ng}-open map, $f(U - F)$ is α_{Ng}-open set in $(V, \tau_{R'}(Y))$. Then, $A = V - f(U - F)$ is a α_{Ng}-closed set in $(V, \tau_{R'}(Y))$. Note that $f^{-1}(G) \subseteq F$ implies $G \subseteq A$ and $f^{-1}(A) = U - f^{-1}(f(U-F)) \subseteq U - (U - F) = F$. That is, $f^{-1}(A) \subseteq F$. Conversely, let B be a Nano-open set of $(U, \tau_R(X))$. Then, $f^{-1}(V - f(B)) \subseteq U - B$ and $U - B$ is a Nano-closed set in $(U, \tau_R(X))$. By hypothesis, there exists a α_{Ng}-closed set A of $(V, \tau_{R'}(Y))$ such that $V - f(B) \subseteq A$ and $f^{-1}(A) \subseteq U - B$ and so $B \subseteq (f^{-1}(A))^c$. Hence, $V - A \subseteq f(B) \subseteq f((f^{-1}(A))^c)$ which implies $f(B) = V - A$. Since $V - A$ is a α_{Ng}-open, $f(B)$ is α_{Ng}-open in $(V, \tau_{R'}(Y))$ and Therefore, f is α_{Ng}-open map.

Theorem 4.19: Let $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ be a bijective map. Then the following are equivalent:

(i) f is a α_{Ng}-open map.

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(ii) f is a α_{Ng}-closed map.

(iii) f^{-1} is a α_{Ng}-continuous map.

Proof:(i) ⇒ (ii) Let H be a Nano-closed set in U. Then $U - H$ is a Nano-open set in U and by (i) $f(U - H)$ is α_{Ng}-open in V. Since, f is bijective, then $f(U - H) = V - f(H)$. Hence, $f(H)$ is a α_{Ng}-closed in V. Therefore, f is a α_{Ng}-closed map.

(ii) ⇒ (iii) Let f be a α_{Ng}-closed map and G be a Nano-closed in U. Since f is bijective, $(f^{-1})^{-1}(G) = f(G)$ which is a α_{Ng}-closed set in V. Therefore, f is a α_{Ng}-continuous map.

(iii) ⇒ (i) Let B be a Nano-open set in U. Since f^{-1} is a α_{Ng}-continuous map, $(f^{-1})^{-1}(B) = f(B)$ is a α_{Ng}-open set in V. Hence, f is α_{Ng}-open map.

Remark 4.20: The composition of two α_{Ng}-open(closed) maps need not be α_{Ng}- open(closed) as shown in the following example.

Example 4.21: Let $U = V = W = \{a, b, c\}$, $U/R = \{\{a\}, \{b, c\}\}$ and $X = \{a\}$. Then $\tau_R(X) = \{U, \emptyset, \{a\}\}$. Let $V/R' = \{\{a, b\}, \{c\}\}$ and $Y = \{c\}$. Then $\tau_{R'}(Y) = \{V, \emptyset, \{c\}\}$. The α_{Ng}-open sets of V is $P(V)$. Define $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ as $f(a) = b, f(b) = a, f(c) = c$. Therefore, f is α_{Ng}-open map. Let $W/R'' = \{\{a\}, \{b, c\}\}$ and $Z = \{b\}$. Then $\tau_{R''}(Z) = \{W, \emptyset, \{bc\}\}$. The α_{Ng}-open sets of W are $\{W, \emptyset, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$. Let $g : (V, \tau_{R'}(Y)) \to (W, \tau_{R''}(Z))$ defined by $g(a) = c$, $g(b) = a$, $g(c) = b$. Therefore, g is α_{Ng}-open map. Now $g \circ f : (U, \tau_R(X)) \to (W, \tau_{R''}(Z))$, then $g(f(a)) = g(b) = a$ which is not α_{Ng}-open in W. Hence, $g \circ f$ is not α_{Ng}-open map.

Example 4.22: By example 4.21, the Nano-closed sets of U are $\{U, \emptyset, \{b, c\}\}$. The α_{Ng}-closed sets of V is $P(V)$. Define $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ as $f(a) = a, f(b) = b, f(c) = c$. Hence $f(\{b, c\}) = \{b, c\}$ which is α_{Ng}-closed in V. Therefore, f is α_{Ng}- closed map. The Nano-closed sets of V are $\{V, \emptyset, \{a, b\}\}$. The α_{Ng}-closed sets of W is $P(W)$. Let $g : (V, \tau_{R'}(Y)) \to (W, \tau_{R''}(Z))$ defined by $g(a) = a, g(b) = c, g(c) = b$. Here $g(\{a, b\}) = \{a, c\}$ which is α_{Ng}-closed set in W. Therefore, g is α_{Ng}-closed map. Now $g \circ f : (U, \tau_R(X)) \to (W, \tau_{R''}(Z))$ then $g(f(\{b, c\})) = g(\{b, c\}) = \{b, c\}$ which is not α_{Ng}-closed in W. Hence, $g \circ f$ is not α_{Ng}-closed map.

Theorem 4.23: If $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is Nano-open map and $g : (V, \tau_{R'}(Y)) \to (W, \tau_{R''}(Z))$ is α_{Ng}-open, then the composition $g \circ f : (U, \tau_R(X)) \to (W, \tau_{R''}(Z))$ is α_{Ng}- open map.

Proof: Suppose R is Nano-open set in U. Since f is Nano-open map, $f(R)$ is Nano-open in V. Since, g is α_{Ng}-open map, $g(f(R))$ is α_{Ng}-open in W which implies $g \circ f(\{R\}) = g(f\{R\})$ is α_{Ng}-open and hence, $g \circ f$ is α_{Ng}-open.

Remark 4.24: If $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is α_{Ng}-open map and $g : (V, \tau_{R'}(Y)) \to (W, \tau_{R''}(Z))$ is Nano-open, then the composition $g \circ f : (U, \tau_R(X)) \to (W, \tau_{R''}(Z))$ is not α_{Ng}-open map as shown in the following example.

Example 4.25: Let $U = V = W = \{a, b, c\}$, $U/R = \{\{a\}, \{b, c\}\}$ and $X = \{b, c\}$. Then, $\tau_R(X) = \{U, \emptyset, \{a\}, \{b, c\}\}$. Let $V/R' = \{\{a\}, \{b, c\}\}$ and $Y = \{b, c\}$. Then $\tau_{R'}(Y) = \{V, \emptyset, \{b, c\}\}$. The α_{Ng} open sets of V are $\{V, \emptyset, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$. Define $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ as $f(a) = a, f(b) = b$ and $f(c) = c$. Then $f(a) = a$, $f(\{b, c\}) = \{b, c\}$ and $f(U) = V$. Therefore, f is α_{Ng}-open map. Let $W/R'' = \{\{a, b\}, \{c\}\}$ and $Z = \{a, b\}$. Then $\tau_{R'}(Z) = \{W, \emptyset, \{a, b\}\}$. The α_{Ng}-open sets of W are $\{W, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$. Define $g : (V, \tau_{R'}(Y)) \to (W, \tau_{R''}(Z))$ as $g(a) = c$, $g(b) = a$ and $g(c) = b$. Then $g(\{b, c\}) = \{a, b\}$ and $g(V) = W$. Hence g is Nano-open map. But $g \circ f(a) = g(f(a)) = g(a) = c$ which is not α_{Ng}-open in W. Therefore, $g \circ f$ is not α_{Ng}-open map.

Theorem 4.26: Let $(U, \tau_R(X))$, $(W, \tau_{R''}(Z))$ be Nano topological spaces and $(V, \tau_{R'}(Y))$ is Nano topological space where every α_{Ng}-closed set is Nano-closed. Let $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y)$ and $g : (V, \tau_{R'}(Y)) \to (W, \tau_{R''}(Z))$ are α_{Ng}-closed maps, then the composition $g \circ f : (U, \tau_R(X)) \to (W, \tau_{R''}(Z))$ is α_{Ng}-closed.

Proof: Suppose P is a Nano-closed set in U. Since, f is α_{Ng}-closed, $f(P)$ is α_{Ng}-closed in V. By hypothesis, $f(P)$ is Nano-closed. Since g is α_{Ng}-closed, $g(f\{P\})$ is α_{Ng}-closed in W and $g(f\{P\}) = (g \circ f)(P)$. Therefore, $g \circ f$ is α_{Ng}-closed.

Theorem 4.27: If $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is Nano generalized open map and $g : (V, \tau_{R'}(Y)) \to (W, \tau_{R''}(Z))$ is α_{Ng}-open and $(V, \tau_{R'}(Y))$ is Nano $T_{1/2}$ space , then the composition $g \circ f : (U, \tau_R(X)) \to (W, \tau_{R''}(Z))$ is α_{Ng}-open map.

Proof: Let A be a Nano-open set in $(U, \tau_R(X))$. Since f is Nano generalized open, $f(A)$ is Nano generalized open in $(V, \tau_{R'}(Y))$. By hypothesis, $f(A)$ is Nano-open and g is α_{Ng}-open which implies $g(f(A))$ is α_{Ng}-open in W and $g(f(A)) = g \circ f\,(A)$. Hence, $g \circ f$ is α_{Ng}-open.

Theorem 4.28: Let $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ and $g : (V, \tau_{R'}(Y)) \to (W, \tau_{R''}(Z))$ be two mappings such that their composition $g \circ f : (U, \tau_R(X)) \to (W, \tau_{R''}(Z))$ is a α_{Ng}- closed mapping. Then the following statements are true.

(i) If f is Nano-continuous and surjective, then g is α_{Ng}-closed.

(ii) If f is Nano generalized continuous , surjective and $(U, \tau_R(X))$ is a α_{Ng}-$T_{1/2}$space, then g is α_{Ng}-closed.

Proof: (i) Let O be a Nano-closed set in $\left(V, \tau_{R'}(Y)\right)$. Since, f is Nano continuous, $f^{-1}(O)$ is Nano-closed in $\left(U, \tau_R(X)\right)$. Since, $g \circ f$ is α_{Ng} closed which implies $g \circ f\ (f^{-1}(O))$ is α_{Ng}-closed in $\left(W, \tau_{R''}(Z)\right)$. That is $g(O)$ is α_{Ng}-closed in $(W, \tau_{R''}(Z))$, since f is surjective. Therefore, g is α_{Ng}-closed.

(ii) Let P be a Nano-closed set in $(V, \tau_{R'}(Y))$. Since, f is Ng- continuous , $f^{-1}(P)$ is Ng – closed in $(U, \tau_R(X))$ and $(U, \tau_R(X))$ is a Nano $T_{1/2}$space, $f^{-1}(P)$ is Nano-closed in $(U, \tau_R(X))$. Since $g \circ f$ is α_{Ng}-closed, which implies $g \circ f\ (f^{-1}(P))$ is α_{Ng}-closed in $(W, \tau_{R''}(Z))$. That is $g(P)$ is α_{Ng}-closed in $(W, \tau_{R''}(Z))$, since f is surjective. Therefore, g is α_{Ng}-closed.

Theorem 4.29: Let $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ and $g : (V, \tau_{R'}(Y)) \to (W, \tau_{R''}(Z))$ be two maps. Then

(i) If $(g \circ f)$ is α_{Ng}-open and f is Nano-continuous, then g is α_{Ng}-open.

(ii) If $(g \circ f)$ is Nano-open and g is α_{Ng}-continuous, then f is α_{Ng}-open map.

Proof: (i) Let C be a Nano-open set in V. Then, $f^{-1}(C)$ is a Nano-open set in U. Since $(g \circ f)$ is α_{Ng}-open map, then $(g \circ f)\,(f^{-1}(C)) = g(f(f^{-1}(C))) = g(C)$ is α_{Ng}-open set in W. Hence, g is a α_{Ng}-open map.

(ii) Suppose A is a Nano-open set in U. Then, $g(f(A))$ is a Nano-open set in W. Therefore, $g^{-1}\left(g\left(f(A)\right)\right) = f(A)$ is a α_{Ng}-open set in V. Therefore, f is a α_{Ng}-open map.

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J. Arul Jesti M.Sc.,B.Ed.,M.Phil.,Ph.D, is presently working as Assistant Professor in the PG and Research Department of Mathematics in St.Mary’s College(Autonomous), Thoothukudi, Tamilnadu, India. She has been teaching Mathematics from 2010 onwards. 4 scholars are currently doing Ph.D under her guidance in Topology. She has published 15 research articles in reputed National and International Journals. She has attended many State and National Seminars.

P. Suganya M.Sc.,M.Phil.,SET., is presently doing research under the guidance of Dr.J.Arul Jesti in PG and Research Department of Mathematics, St.Mary's College (Autonomous), Thoothukudi, Tamilnadu, India. I have published 2 research articles in the reputed journals and attended state and national level seminar.