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On Contra (τ^*-G , G) – Open Maps in Topological Space

Maysaa Zaki Salman, Dunya Mohamed Hameed

Department of Mathematics − College of Education − Mustansiriyah University
dunya_mahamed@uomustansiriyah.edu.iq

Abstract :

This artic is to submit and study some new types of contra open maps it is called (contra(τ^*- G , G) – open and (G , τ^*- G) – open)maps in topological spaces Furthermore , we will discuss the relation between these maps and prove some of their properties .

1-Introduction :

In (1997), Baker. C .W [1] defined and introduced the concepts contra open and contra closed maps .Pushpalatha .A. Eswaran .S and Rajarubi , [8] they introduced and studied the concepts τ^*- g-closed sets and investigated some of it is properties . The goal in this purpose is to give some types of contra open maps which are(contra(τ^*-G , G) – open and (G , τ^*-G) – open)maps and introduce the relationships between these contra maps.

2-Preliminaries:

Definition (2-1): [3]

For a sub set C of (X,$\mathcal{T}$), then the generalized closure operate $CL^*(C)$ is defined by the intersection of all g-closed sets containing C.

Definition (2-2): [8]

For the subset C of (X,) the topology $\tau^*$is defined by $\tau^* = \{G : cL^*(G^c) = G^c\}$.

Definition (2-3):

A subset A of a space (X, $\mathcal{T}$) is called

1- g-closed in X [5]if CL(C) $\subseteq$ D whenever C $\subseteq$ D and G is open in X ,a subset C is called generalized open (briefly g-open) in X if its complement C^c is g-closed.

2- τ^*-g-closed [8]if $CL^*(C)$ $\subseteq$ D whenever C $\subseteq$ D and G is τ^*-open in X. The complement of τ^*-generalized closed set is called the τ^*-g-open.

Remark (2-4), [5],[8]:

(i) every closed set is τ^*-g-closed and

(ii) every g-closed set is τ^*-g-closed.

Definition (2-5): A topological space

1- (X ,) is called $\tau_{1/2}$- space [6]if every g-closed set in X is closed.

2- (X, τ^*) is called τ^*-Tg space [8] if every τ^*-g-closed set in X is g-closed in X.

Definition (2-6): a mapping map ḟ: Ẋ → Ẏ is said to be:

1- g-closed (resp . g –open) [6] if the image of every closed (open)set in X is g-closed (g-open)set in Y.
2- τ^*-generalized closed map (briefly τ^*-G-closed) [4] if for every g-closed subset H of X, then f(H) is τ^*-g-closed subset of Y.
3- Strongly τ^*-generalized closed map (briefly τ^{**}-G-closed) [7] if foe every τ^*-g-closed subset H of X, then f(H) is g-closed subset of Y.
4- τ^*-Gc-closed [8] if every τ^*-g-closed subset H of X, then f(H) is a τ^*-g-closed set in Y.
5- Contra open [1] if f(H) is closed set in Y , for each open set H in X .
6- Contra –g- open [2] if f(H) is g-closed set in Y , for each open set H in

3-On Contra (τ^*-G , G) – Open Maps:

In this section we will introduce some contra (τ^*-G , G) – open maps types called (contra(τ^*-G , G) − open and (G , τ^*-G)- open)maps and we discuss and prove some of it is properties .

Definition (3-1): A map ḟ: (Ẋ,τ) → (Ẏ ,ξ) is called contra(τ^*- G , G) −open if ḟ(C) is g-closed in Ẏ for all τ^*-g- open in Ẋ .

Example (3-2): Let Ẋ= Ẏ = { m , n , o} with the topologies τ ={ Ẋ, ϕ ,{m},{m ,o}} and ξ ={Ẏ,ϕ,{m}}.Let ḟ: (Ẋ,τ) → (Ẏ,ξ) is define by ḟ(m)= n , f(n)= m and f(o) = o. It is clear that ḟ is contra (τ^*-G, G)−open map .

Proposition (3-3): Each contra (τ^*-G, G)−open map is contra g−open .

Proof : Let ḟ: (Ẋ,τ) → (Ẏ ,ξ)is contra (τ^*-G, G)−open and C is open set in Ẋ , since (every open set is τ^*-g-open) . Thus , C is τ^*-g-open in Ẋ . ḟ is contra (τ^*-G, G)−open , then ḟ(C) is g-closed in Ẏ . Hence , ḟ is contra g-open .

The next example show that the converse is not true of proposition (3-3).

Example (3-4):

Let Ẋ={ m , n , o} with the topology τ ={ Ẋ, ϕ ,{m ,n}}.Let ḟ: (Ẋ,τ) → (Ẋ,τ) is define by ḟ(m)= n , f(n)= o and f(o) = m. clearly ḟ is contra g−open but is not contra (τ^*-G, G)−open map , since for τ^*-g-open C={m}in Ẋ , ḟ({m})={n} is not g-closed in Ẏ .

The next proposition introduce the condition that make proposition (3-3) true :

Proposition (3-5): If ḟ: (Ẋ , τ) → (Ẏ,ξ) is contra g-open map and Ẋ is discrete space , then ḟ is contra (τ^*-G, G)−open .

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Proof : Let C be τ^*-g-open in Ẋ , since Ẋ is discrete space .Thus , C is open set in Ẋ , ḟ is contra g-open map, then , ḟ(C) is g- closed in Ẏ. Therefore ,ḟ is contra (τ^*-G, G)−open .

Remark (3-6):

The concepts of contra (τ^*-G, G)−open and contra open maps are independents . the following examples show that .

Example (3-7):

Let Ẋ= Ẏ = { m , n , o} with the topologies τ ={Ẋ, ϕ ,{m},{m ,o}} and ξ ={Ẏ,ϕ,{n},{n ,o}}.Let ḟ: (Ẋ,τ) → (Ẏ,ξ) is define by ḟ(m)= n , f(n)= m and f(o) = o. It is clear that ḟ is contra open but is not contra (τ^*-G, G)−open map , since for τ^*-g-open C={o}in Ẋ , ḟ({o})={o} is not g-closed in Ẏ .

Example (3-8): Let Ẋ= Ẏ = { m , n , o} with the topologies τ ={Ẋ, ϕ ,{n ,o}} and ξ ={Ẏ,ϕ,{o},{m ,n}}.Let ḟ: (Ẋ,τ) → (Ẏ,ξ) is define by ḟ(m)= m , f(n)= n and f(o) = o. It is clear that ḟ contra (τ^*-G, G)−open map , but is not contra open , since for open C={n ,o}in Ẋ , ḟ({n, o})={n ,o} is not closed in Ẏ .

The next proposition introduce the condition that make Remark(3-6) true :

Proposition (3-9): If ḟ: (Ẋ , τ) ⟶ (Ẏ,ξ) is contra open map and Ẋ is discrete space , then ḟ is contra (τ^*-G, G)−open .

Proof :

Let C be τ^*-g-open in Ẋ , since Ẋ is discrete space .Thus , C is open set in Ẋ , ḟ is contra open map, then , ḟ(C) is closed in Ẏ. Since each closed is g-closed , then ḟ(C) is g- closed in Ẏ Therefore ,ḟ is contra (τ^*-G, G)−open .

Proposition (3-10): If ḟ: (Ẋ , τ) ⟶ (Ẏ,ξ) is contra (τ^*-G, G)−open map and Ẏ is $\tau_{1/2}$- space , then ḟ is contra−open .

Proof : Let C be open in Ẋ , since every open is τ^*-g- open , then C is τ^*-g- open in Ẋ , ḟ is contra (τ^*-G, G)−open, then , ḟ(C) is g- closed in Ẏ. Since Ẏ is $\tau_{1/2}$- space , then ḟ(C) is closed in Ẏ Therefore ,ḟ is contra −open .

Proposition (3-11):

If ḟ: (Ẋ,τ) → (Ẏ,ξ) is contra (τ^*-G, G)−open and Ẏ;ξ) →(Ẑ ,$\mathcal{E}$) is τ^{**}-G-closed , then ọḟ: (Ẋ,τ) →(Ẑ ,$\mathcal{E}$) is contra (τ^*-G, G)−open .

Proof :

Let C be τ^*-g-open in Ẋ , ḟ is contra (τ^*-G, G)−open . Thus , ḟ(C) is g-closed in Ẏ and since (every g- closed set is τ^*-g-closed) , then ḟ(C) is τ^*-g-closed in Ẏ. Also , sinc

τ^{**}-G-closed , so we have ḟ(C)) is g-closed in Ẑ(B)ut,Therefore , ∘ḟ(ḟ (C)) = : (Ẋ,τ) →(Ẑ ,ℰ) is contra (τ^*-G, G)−open .

Proposition (3-12):

If ḟ: (Ẋ,τ) → (Ẏ,ξ) is contra (τ^*-G, G)−open , Ẏ,ξ) →(Ẑ ,ℰ) is g-closed and Ẏ is $\tau_{1/2}$-space , then ∘ḟ : (Ẋ,τ) →(Ẑ ,ℰ) is contra (τ^*-G, G)−open .

Proof :

Let C be τ^*-g-open in Ẋ , ḟ is contra (τ^*-G, G)−open . Thus , ḟ(C) is g-closed in Ẏ and since Ẏ is $\tau_{1/2}$- space , then ḟ(C) is closed in Ẏ. Also ,since g-closed , so we have ḟ((C)) is g-closed in Ẑ(B)ut,Therefore , ∘ḟ: (Ẋ,τ) →(Ẑ ,ℰ) is contra (τ^*-G, G)−open .

Some way we prove the following corollaries :

Corollary(3-13):

If ḟ: (Ẋ,τ) → (Ẏ,ξ) is contra (τ^*-G, G)−open , Ẏ,ξ) →(Ẑ ,ℰ) is closed and Ẏ is $\tau_{1/2}$-space , then ∘ḟ : (Ẋ,τ) →(Ẑ ,ℰ) is contra (τ^*-G, G)−open .

Corollary(3-14):

If ḟ: (Ẋ,τ) → (Ẏ,ξ) is contra (τ^*-G, G)−open , Ẏ,ξ) →(Ẑ ,ℰ) is any map and Ẑ is τ^*-Tg-space , then ∘ḟ : (Ẋ,τ) →(Ẑ ,ℰ) is contra (τ^*-G, G)−open .If a map is

- **(i)** τ^*-G-closed
- **(ii)** τ^*-Gc-closed

Proposition (3-15): If ḟ: (Ẋ,τ) → (Ẏ,ξ) is any map , :(Ẏ,ξ) →(Ẑ ,ℰ) is contra (τ^*-G, G)−open , then ∘ḟ : (Ẋ,τ) →(Ẑ ,ℰ) is contra (τ^*-G, G)−open . If ḟ is

- **(i)** open map
- **(ii)** g-open map
- **(iii)** τ^*-G- open .

Proof(i) : Let C be τ^*-g-open in Ẋ , ḟ is open . Thus , ḟ(C) is open in Ẏ and since ḟ is open map then ḟ(C) is open in Ẏ also since(every open set is τ^*-g-open) , so ḟ(C) is τ^*-g-open in Ẏ, is contra (τ^*-G,G)- open , so we have ḟ((C)) is g-closed in Ẑ. But , ∘ḟ(C) Therefore , ∘ḟ: (Ẋ,τ) →(Ẑ ,ℰ) is contra (τ^*-G, G)−open .

Similarly , we prove steps (ii) and (iii) .

Now , we will define another types of contra open map is called contra (G , τ^*-G)- open .

Definition (3-16): A map ḟ: (Ẋ,τ) → (Ẏ ,ξ)is called contra(G ,τ^*- G) –open if ḟ(C) is τ^*-g-closed in Ẏ for all g- open in Ẋ .

Example (3-17): Let Ẋ={ m , n , o , r} with the topologies τ ={Ẋ, ϕ ,{m ,n},{o, r}}.Let ḟ: (Ẋ,τ) → (Ẋ,τ))is define by ḟ(m)= m , f(n)= n, f(o) = o and ḟ®=r. It is clear that ḟ is contra(G ,τ^*- G) –open .

Proposition (3-18): Each contra (τ^*-G, G) –open map is contra(G ,τ^*- G) –open .

Proof: Let ḟ: (Ẋ,τ) → (Ẏ ,) is contra (τ^*-G, G)–open and C isg- open set in Ẋ , since (everyg- open set is τ^*-g-open) . Thus, C is τ^*-g-open in Ẋ . ḟ is contra (τ^*-G, G)–open , then ḟ(C) is g-closed in Ẏ . Hence , ḟ is (G ,τ^*- G) –open .

The next example show that the converse is not true of proposition (3-18).

Example(3-19):

Let Ẋ= Ẏ ={ m , n , o} with the topologies τ ={ Ẋ, ϕ ,{m}, {n} ,{m ,n}} and ξ={Ẏ,ϕ,{m}.Let ḟ: (Ẋ,τ) → (Ẏ,ξ) is define by ḟ(m)= m , f(n)= n and f(o) = o. It is clear that ḟ contra (G ,τ^*-G)–open map , but is not contra(τ^*-G, G)–open , since for τ^*-g-open C={m}in Ẋ , ḟ({m})={m} is not g-closed in Ẏ .

Proposition (3-20): If ḟ: (Ẋ , τ) → (Ẏ,ξ) is contra (G ,τ^*- G) –open and Ẋ , Ẏ are τ^*-Tg space , then ḟ is contra (τ^*-G, G)–open .

Proof :

Let C be τ^*-g-open in Ẋ , since Ẋ is τ^*-Tg space .Thus , C is g- open set in Ẋ , ḟ is contra (G ,τ^*- G) –open map, then , ḟ(C) is τ^*-g- closed in Ẏ. Since Ẏ is τ^*-Tg closed space so we obtain , ḟ(C) is g-closed , then ḟ(C) is g- closed in Ẏ Therefore ,ḟ is contra (τ^*-G, G)–open .

Remark(3-21): the concepts of contra-open and contra g-open are independent of concepts contra (G ,τ^*- G) –open. The next examples show that:

Example(3-22):

Let Ẋ= Ẏ ={ m , n , o ,r} with the topologies τ ={ Ẋ, ϕ ,{m}, {n} ,{m ,n},{m ,n ,o}} and ξ={Ẏ,ϕ,{m},{n},{m ,n},{m ,r},{m ,n ,o},{ m ,n ,r}}Let ḟ: (Ẋ,τ) → (Ẏ,ξ) is define by ḟ(m)= o , f(n)= r , f(o) = n and ḟ (r)= m. It is clear that ḟ contra open and contra g-open but is not contra (G ,τ^*-G)–open map, since for g-open C={m}in Ẋ , ḟ({o})={n} is not τ^*-g-closed in Ẏ .

Example(3-23):

Let Ẋ= Ẏ ={ m , n , o} with the topologies τ ={ Ẋ, ϕ ,{m},{n ,o}} and ξ={Ẏ,ϕ,{m}.Let ḟ: (Ẋ,τ) → (Ẏ,ξ) is define by ḟ(m)= m , f(n)= n and f(o) = o. It is clear that ḟ contra (G ,τ^*-

G)−open map , but is not contra –open and contra g-open , since for open C={m} in Ẋ , ḟ({m})={m} is not closed (g-closed) in Ẏ .

The next proposition introduce the condition that make Remark(3-6) true :

Proposition (3-24): A map ḟ: (Ẋ , τ) → (Ẏ,ξ) is contra(G , ,τ*-G)−open map. If ḟ is

- **(i)** Contra –open and Ẋ is $\tau_{1/2}$- space.
- **(ii)** Contra g-open and Ẋ is $\tau_{1/2}$- space

Proof(i) : Let C be g-open in Ẋ , since Ẋ is $\tau_{1/2}$- space .Thus , C is open set in Ẋ , ḟ is contra open , then , ḟ(C) is closed in Ẏ. Since each closed is τ*-g-closed , then ḟ(C) is τ*- g- closed in Ẏ Therefore ,ḟ is contra (G , ,τ*-G)−open .

Same way we prove step-ii- .

Proposition (3-25): If ḟ: (Ẋ , τ) → (Ẏ,ξ) is contra (G , ,τ*-G)−open map and Ẏ is discrete space , then ḟ is contra−open (resp. contra -g-open) .

Proof : Let C be open in Ẋ , since every open is g- open , then C is g- open in Ẋ , ḟ is contra (G , ,τ*-G)−open, then , ḟ(C) is τ*- g- closed in Ẏ. Since Ẏ is discrete space , then ḟ(C) is closed (resp. g- closed) in Ẏ Therefore ,ḟ is contra −open (resp. g-open) map.

Proposition (3-26):

If ḟ: (Ẋ,τ) → (Ẏ,ξ) is contra (G , ,τ*-G)−open and Ẏ,ξ) →(Ẑ ,$\mathcal{E}$) is any map , then □∘ḟ : (Ẋ,τ) →(Ẑ ,$\mathcal{E}$) is contra (G , ,τ*-G)−open .If a map □ is

- (i) τ**-G-closed
- (ii) τ*-Gc-closed

Proof(i) : Let C be g- open in Ẋ , ḟ is contra (G , ,τ*-G)−open .Thus , ḟ(C) is τ*- g- closed in Ẏ .Also , since □**-G-closed , so we have □(ḟ(C)) is g-closed in Ẑ and (each g-closed is τ*-g-closed set) , then □(ḟ(C)) is τ*-g-closed in Ẑ But , □∘ḟ(C) = □(ḟ(C)) Therefore , □∘ḟ : (Ẋ,τ) →(Ẑ ,$\mathcal{E}$) is contra (G , ,τ*-G)−open .

Similarly , we prove steps(ii)

Proposition (3-27):

If ḟ: (Ẋ,τ) → (Ẏ,ξ) is contra (G , ,τ*-G)−open and Ẏ,ξ) →(Ẑ ,$\mathcal{E}$) is τ*-G-closed if Ẏ is τ*-Tg space, then □∘ḟ : (Ẋ,τ) →(Ẑ ,$\mathcal{E}$) is contra (G , ,τ*-G)−open .

Proof : Let C be g- open in Ẋ , ḟ is contra (G , ,τ*-G)−open .Thus , ḟ(C) is τ*- g- closed in Ẏ . Also , since Ẏ is τ*-Tg space , then ḟ(C) is g-closed in Ẏ, by hypotheses □ is τ*-G-closed , then □(ḟ(C)) is τ*-g-closed in Ẑ. But □∘ḟ(C) = □(ḟ(C)), Therefore □∘ḟ : (Ẋ,τ) →(Ẑ ,$\mathcal{E}$) is contra (G , ,τ*-G)−open .

Proposition (3-28):

If ʄ: (Ẋ ,τ) →(Ẏ ,ξ) is τ^*- G-open , □ : (Ẏ,ξ) → (Ẑ ,ℰ) is contra(G ,τ^*-G)−open , Ẋ is $\tau_{1/2}$-space and Ẏ is τ^*-Tg space, then □∘ʄ : (Ẋ,τ) →(Ẑ ,ℰ) is contra (G , ,τ^*-G)−open .If a map ʄ is

(i) open

(ii) g- open

Proof(i) :

Let C be g- open in Ẋ , since Ẋ is $\tau_{1/2}$- space , then C is open set in Ẋ . Also , since ʄ is τ^*-G-open .Thus , ʄ(C) is τ^*- g-open set in Ẏ and since Ẏ is τ^*-Tg space , so we get ʄ(C) is g-open in Ẏ. Also □ is contra(G, ,τ^*-G)−open , so we have □(ʄ(C)) is τ^*-closed in Ẑ. But , □(ʄ (C)) = □∘ʄ(C) Therefore , □∘ʄ: (Ẋ,τ) →(Ẑ ,ℰ) is contra (G , ,τ^*-G)−open .

Similarly , we prove the next proposition

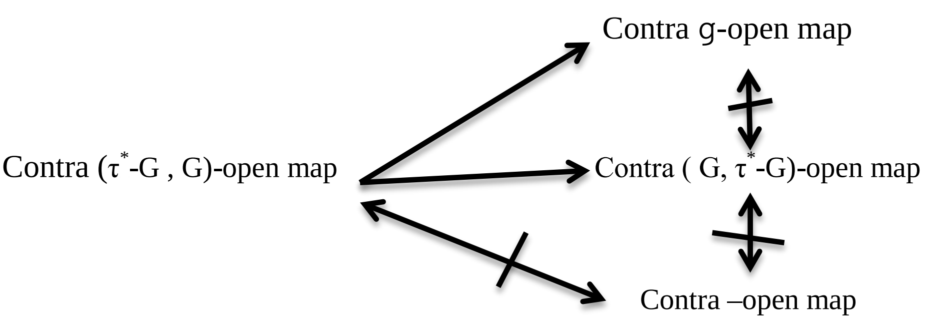
Proposition (3-29):

If ʄ: (Ẋ ,τ) →(Ẏ ,ξ) is any map , □ : (Ẏ,ξ) → (Ẑ ,ℰ) is contra(G ,τ^*-G)−open and Ẋ is $\tau_{1/2}$-space , then □∘ʄ : (Ẋ,τ) →(Ẑ ,ℰ) is contra (G , ,τ^*-G)−open .If a map ʄ is

(i) open

(ii) g- open

Remark(3-30): in the following diagram illustrates the relation between the contra (τ^*-G- ,G) – open maps types (without using condition), where the converse is not necessarily true.



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