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Design Procedure for Uncracked Reinforced Concrete Sections

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Abstract

Uncracked reinforced concrete sections are important elements for water structures. Classical approaches for designing uncracked sections depends on the concrete capacity for resisting all the stresses and neglect the presence of reinforcement in estimating the section capacity. These approaches calculate the reinforcement using the techniques of cracked sections, which leads to a great amount of reinforcement. This paper aims at finding a suitable design procedure and charts for the uncracked sections taking into account the presence of reinforcement in estimating the section dimension. In this paper, uncracked sections subjected to axial tension, pure bending moment, eccentric tensile force, and eccentric compressive force are used for the derivation of the design procedures. The obtained design procedures are presented also in different charts. The outputs of the proposed procedures are compared with that of classical ones to assess the procedures efficiency.

Keywords: *uncracked sections, eccentric forces, axial tension, reinforcing ratio, modular ratio.*

1. Introduction

The occurrence of cracks in concrete structures is inevitable due to the low tensile strength of concrete. In designing the most structural elements, the concrete tensile strength is neglected. For water structures, the concrete tensile strength is a major parameter in the design process. Classical approaches, for designing uncracked sections, estimate the dimensions of the uncracked section depending on the concrete tensile strength. These approaches define the section capacity according to the concrete section only, neglecting the presence of reinforcement. According to these approaches, the reinforcement content is determined using cracked sections concept, i.e., all the tensile stresses are resisted by steel only. This procedure means high reinforcement ratio and larger section dimension. For uncracked sections, considering the reinforcement in the design process leads to controlling the section dimension and the reinforcement ratio. In this paper, a design procedure for uncracked reinforced concrete section is presented taking into account the presence of reinforcement. Different reinforced concrete sections, subjected to axial tension, pure bending moment, eccentric tension and eccentric compression, are analyzed to derivate the suitable design equations and charts. The proposed procedures and charts are used for designing different cases to compare its outputs with the classical approaches ones.

2. Classical Approaches

2.1 Axial Tension

According to classical approaches, the uncracked reinforced concrete section subjected to axial tension can be designed as following:

$$A = T \,/\, f_{cto} \tag{1}$$

where T is the applied axial tension, A is the cross-sectional area ($A = A_c + n\, A_s$), A_c is the area of concrete, n is the modular ratio, A_s is the area of reinforcement and f_{cto} is the allowable tensile stresses in concrete under axial tension. The reinforcement is calculated to resist all the tensile force, as

$$A_s = T \,/\, f_s \tag{2}$$

where f_s is the allowable tensile stress of steel.

2.2 Pure Bending Moment

For uncracked reinforced concrete sections subjected to pure bending moments, the section dimensions are estimated using the following procedure:

$$f_t = M / Z \tag{3}$$

where M is the applied bending moment, f_t is the allowable tensile stresses in concrete under bending moments and Z is the section modulus. The required reinforcement is calculated according to the concept of cracked sections using the working stress design theory as

$$A_s = M / k_2 \, d \tag{4}$$

where d is the section depth and k_2 is a parameter depends on the concrete characteristic strength and the allowable tensile stresses of steel.

2.3 Eccentric Tension and Compression

According to classical approaches, the thickness t; of a section subjected to eccentric tension; is calculated according to bending moments and increased empirically to take into account the additional tensile stresses resulted from the tensile force, then the actual stresses are checked. The required reinforcement is calculated to resist all the tensile stresses according to the type of eccentricity as a cracked section. Similar to the case of eccentric tension, the section thickness t is calculated according to bending moments but decreased empirically to take into account the additional compressive stresses resulted from the compressive force, then the actual stresses are checked. Also, the required reinforcement is calculated as a cracked section.

3. The Proposed Design Procedures

The distension of the proposed design procedure is to improve the design process to take into account the presence of reinforcement in estimating the cross-sectional dimension, and to control the design outputs according to the reinforcing ratio. Different sections are analyzed under different loading conditions to drive the design procedure as following.

3.1 Sections under Axial Tension

The uncracked section, shown in Fig. 1, subjected to axial tensile force T can be analyzed as following:

$$A_v = b \times t + (n-1)\, A_s$$
$$A_v = b \times t + (n-1)\,(\mu \times b \times t)$$
$$T = f_{cto}\, A_v = f_{cto}\, b\, t\,(1 + (n-1)\,\mu)$$
$$t = \frac{T}{f_{cto}\, b\,(1 + (n-1)\mu)} \tag{5}$$

where A_v is the virtual cross-sectional area, n is the modular ratio, μ is the reinforcement ratio (=$A_s/(b\ t)$) and f_{cto} is the allowable tensile stress for concrete under axial tension. In this case the design process can be controlled according to the reinforcement ratio. Equation (5) can be rewritten as following:

$$t = k\, \frac{T}{f_{cto}\, b} \tag{6}$$

where

$$k = \frac{1}{(1 + (n-1)\mu)} \tag{7}$$

The factor k represents the effect of reinforcement on the dimensions of the uncracked section subjected to axial tension. The factor k can be represented in charts or tables for the design purposes.

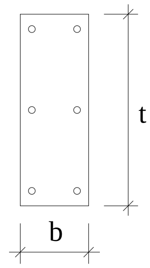

Fig. 1. Uncracked section subjected to axial tension

3.2 Sections under Pure Bending Moment

For the uncracked section, shown in Fig. 2, subjected to pure bending moment M, the analysis procedure is as following:

$$\frac{f_c}{f_t} = \frac{c}{t-c} \quad , \quad \frac{f_s/n}{f_t} = \frac{d-c}{t-c} = \frac{\beta t - c}{t-c} \quad and \quad \frac{f_s'/n}{f_c} = \frac{c-d'}{c} = \frac{c - \zeta t}{c} \tag{8}$$

where f_c, f_t and f_s are the compressive and tensile stress in concrete and the tensile stress in steel, β is the depth-thickness ratio (d/t) and ζ is the ratio of the compression steel depth to the section thickness (d'/t).

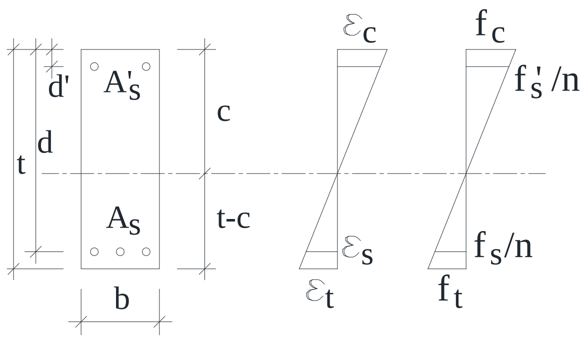

Fig. 2. Uncracked section subjected to pure bending moment

The statical moment, of the transformed section, about the $N.\ A.$ leads to:

$$0.5\, b\, c^2 + (n-1)\, A_s'\, (c - d') = 0.5\, b\, (t-c)^2 + (n-1)\, A_s\, (d-c)$$

Simplifying it, then:

$$0.5\, b\, c^2 = 0.5\, b\, (t-c)^2 + (n-1)\, A_s\, (\beta t - c) - (n-1)\, A_s'\, (c - \zeta t)$$
$$0.5\, b\, c^2 = 0.5\, b\, (t-c)^2 + (n-1)\, A_s\, (\beta t - c) - (n-1)\, \alpha\, A_s\, (c - \zeta t)$$

Dividing both sides by ($0.5\, b\, (t/2)^2$) and rearranging, then:

$$4\left(\frac{c}{t}\right)^2 = 4\left(\frac{t-c}{t}\right)^2 + 8\,(n-1)\,\frac{A_s}{b\,t^2}\,(\beta t - c - \alpha c + \alpha\zeta t)$$
$$\left(\frac{c}{t}\right)^2 = \left(1-\frac{c}{t}\right)^2 + 2\,(n-1)\,\mu\left(\beta - \frac{c}{t} - \alpha\frac{c}{t} + \alpha\zeta\right)$$

Assume that $c/t = r$, then:

$$r^2 = (1-r)^2 + 2\,(n-1)\,\mu\,(\beta - r - \alpha r + \alpha\zeta)$$

Simplifying it, then:

$$r = \frac{1 + 2\,(n-1)\,\mu\,(\beta+\alpha\zeta)}{2 + 2\,(n-1)\,\mu\,(1+\alpha)} \tag{9}$$

From Eq. (8), then

$$f_c = \frac{c}{t-c}\,f_t = \frac{r}{1-r}\,f_t \tag{10}$$

and

$$f_s'/n = \frac{c-d'}{t-c}\,f_t = \frac{c-\zeta t}{t-c}\,f_t = \frac{r-\zeta}{1-r}\,f_t \tag{11}$$

From equilibrium equations C = T, then:

$$0.5\,b\,c\,f_c + A_s'\,f_s' = 0.5\,b\,(t-c)\,f_t + A_s\,f_s$$

Dividing both sides by (*0.5 b t*), then:

$$\frac{c}{t}\,f_c + \frac{2A_s'\,f_s'}{b\,t} = \left(1-\frac{c}{t}\right)f_t + \frac{2A_s\,f_s}{b\,t}$$
$$r\,f_c + \frac{2\,\alpha\,A_s\,f_s'}{b\,t} = (1-r)\,f_t + \frac{2A_s\,f_s}{b\,t}$$
$$r\,f_c + 2\,\alpha\,\mu\,f_s' = (1-r)\,f_t + 2\mu\,f_s$$

From Eqs. (10) and (11), then:

$$r\,\frac{r}{1-r}\,f_t + 2\,\alpha\,\mu\,n\frac{r-\zeta}{1-r}\,f_t = (1-r)\,f_t + 2\mu\,f_s$$
$$\frac{r^2}{1-r}\,f_t + 2\,\alpha\,\mu\,n\frac{r-\zeta}{1-r}\,f_t - (1-r)\,f_t = 2\mu\,f_s$$
$$\left[\frac{r^2}{1-r} + 2\,\alpha\,\mu\,n\frac{r-\zeta}{1-r} - \frac{(1-r)^2}{1-r}\right]f_t = 2\mu\,f_s$$

Simplifying it, then

$$\left[\frac{2\,\alpha\,\mu\,n\,(r-\zeta) - 1 + 2r}{1-r}\right]f_t = 2\mu\,f_s$$

$$f_s = \frac{2\,\alpha\,\mu\,n\,(r - \zeta) - 1 + 2r}{2\,\mu\,(1 - r)} f_t \qquad (12)$$

Taking the moment about the reinforcement center, then:

$$M = 0.5\,f_c\,c\,b\left(d - \frac{c}{3}\right) + A_s'\,f_s'\,(d - d') - 0.5\,f_t\,(t - c)\,b\left(\frac{t - c}{3} - (t - d)\right)$$

$$M = 0.5 f_t\,\frac{c}{t - c}\,c\,b\left(d - \frac{c}{3}\right) + A_s'\,n\,\frac{r - \zeta}{1 - r}\,f_t\,(d - d') - 0.5\,f_t\,(t - c)\,b\left(\frac{t - c}{3} - (t - d)\right)$$

$$\frac{M}{b\,f_t} = 0.5\,\frac{r}{1 - r}\,rt\left(\beta t - \frac{rt}{3}\right) + \alpha\,n\,\frac{A_s}{b}\,\frac{r - \zeta}{1 - r}\,(\beta t - \zeta t) - 0.5\,(t - rt)\left(\frac{t - rt}{3} - (t - \beta t)\right)$$

$$\frac{M}{b\,f_t} = 0.5\,\frac{r^2}{1 - r}\left(\beta - \frac{r}{3}\right)t^2 + \alpha\,n\,\mu\,\frac{r - \zeta}{1 - r}\,(\beta - \zeta)\,t^2 - 0.5\,(1 - r)\left(\frac{1 - r}{3} - (1 - \beta)\right)t^2$$

then

$$t = k_1\sqrt{\frac{M}{b\,f_t}} \qquad (13)$$

where

$$k_1 = \frac{1}{\sqrt{\left(0.5\,\frac{r^2}{1 - r}\left(\beta - \frac{r}{3}\right) + \alpha\,n\,\mu\,\frac{r - \zeta}{1 - r}\,(\beta - \zeta) - 0.5\,(1 - r)\left(\frac{1 - r}{3} - (1 - \beta)\right)\right)}} \qquad (14)$$

Equations (9) and (14) can be represented by charts and tables. Figures 3 and 4 represent the design parameters r and k_1. Table 1 shows an example of these tables.

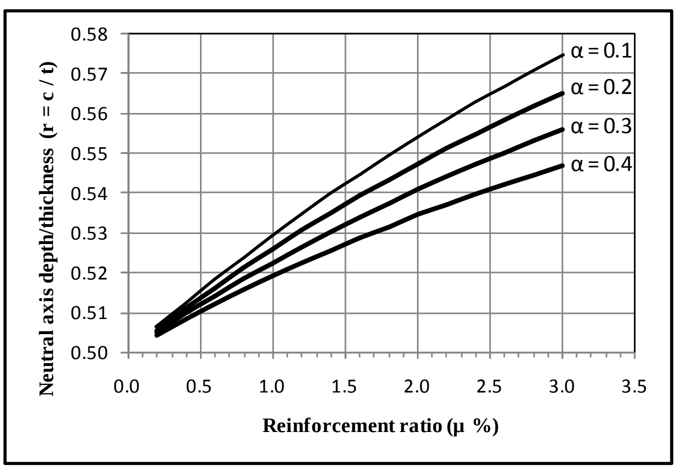

Fig. 3. Neutral axis depth – thickness ratio ($r=c/t$) ($\beta = 0.9$ *and* $\zeta = 0.1$).

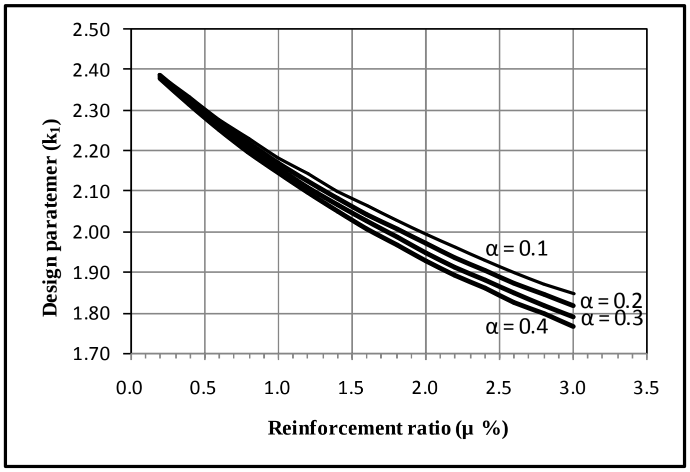

Fig. 4. Design parameter (k_1) (*β = 0.9 and ζ = 0.1*).

Table 1: Design parameters r and k_1 (β = 0.9 and ζ = 0.1)

µ %	α = 0.1		α = 0.2		α = 0.3		α = 0.4	
	r	***k₁***	***r***	***k₁***	***r***	***k₁***	***r***	***k₁***
0.2	0.506	2.389	0.506	2.385	0.505	2.382	0.504	2.379
0.4	0.512	2.332	0.511	2.326	0.510	2.320	0.508	2.314
0.6	0.518	2.280	0.516	2.271	0.514	2.262	0.512	2.254
0.8	0.524	2.231	0.521	2.219	0.518	2.208	0.516	2.198
1.0	0.529	2.185	0.526	2.171	0.523	2.158	0.519	2.146
1.2	0.535	2.142	0.531	2.126	0.527	2.111	0.523	2.097
1.4	0.540	2.102	0.535	2.084	0.530	2.067	0.526	2.052
1.6	0.545	2.063	0.539	2.044	0.534	2.026	0.529	2.009
1.8	0.549	2.027	0.543	2.006	0.537	1.986	0.532	1.968
2.0	0.554	1.993	0.547	1.970	0.541	1.949	0.535	1.930

3.3 Sections under Eccentric Tension

Analysis of uncracked sections subjected to eccentric tension depends on the eccentricity-thickness ratio (e/t). Since for axially tensioned section:

$$t = k\,\frac{T}{f_{t1}\,b} \qquad i.e. \qquad k = \frac{f_{t1}\,b\,t}{T} \tag{15}$$

where f_{t1} is the tensile stress due to the axial tension (T), and the reinforcement ratio μ^* used in calculating the parameter k_1 includes both A_s and A_s', i.e.

$$\mu^* = \frac{A_s + A_s'}{b\,t} = \frac{A_s + \alpha\,A_s}{b\,t} = (1+\alpha)\,\mu \tag{16}$$

For bending moment:

$$t^2 = k_1^2\,\frac{M}{b\,f_{t2}} \qquad k_1^2 = \frac{f_{t2}\,b\,t^2}{M} \tag{17}$$

where f_{t2} is the tensile stress due to the bending moment (M). Then, for eccentric tension, the total tensile stress is:

$$f_{ct} = f_{t1} + f_{t2}$$

$$f_{ct} = k \frac{T}{b\,t} + k_1^2 \frac{M}{b\,t^2}$$

$$1 = \frac{T}{f_{ct}\,b\,t}\left(k + k_1^2 \frac{e}{t}\right)$$

$$\frac{T}{f_{ct}\,b\,t} = \frac{1}{k + k_1^2 \frac{e}{t}} \quad (18)$$

Equation (18) can be represented in charts or tables as shown in Figure 5 and Table 2. Assuming the reinforcement ratio (μ) and the eccentricity-thickness ratio (e/t), then the factor ($T/(f_{ct}\ b\ t)$) can be determined using the design charts or tables. If the section dimensions are assumed first, the required reinforcement ratio can be obtained by locating the point (e/t, $T/(f_{ct}\ b\ t)$) in the chart, then determining the suitable reinforcing ratio, as shown in Figure 6.

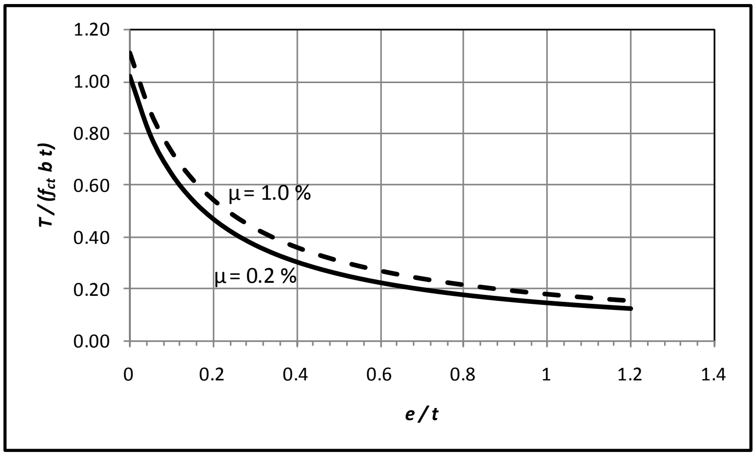

Fig. 5. Design parameter ($T/f_{ct}\ b\ t$) and eccentricity – thickness ratio (e/t) ($\alpha = 0.2$).

Table 2: Design parameter ($T/f_{ct}\ b\ t$) and eccentricity – thickness ratio (e/t) ($\alpha = 0.2$).

e/t	***T / (f_ct b t)***				
	μ = 0.2%	***μ = 0.4%***	***μ = 0.6%***	***μ = 0.8%***	***μ = 1.0%***
0.00	1.022	1.043	1.065	1.086	1.108
0.10	0.646	0.667	0.687	0.708	0.728
0.20	0.472	0.490	0.508	0.525	0.542
0.30	0.372	0.387	0.402	0.417	0.432
0.40	0.307	0.320	0.333	0.346	0.359
0.50	0.262	0.273	0.284	0.296	0.307
0.60	0.228	0.238	0.248	0.258	0.268
0.70	0.202	0.211	0.220	0.229	0.238
0.80	0.181	0.189	0.197	0.206	0.214
0.90	0.164	0.172	0.179	0.187	0.194
1.00	0.150	0.157	0.164	0.171	0.178

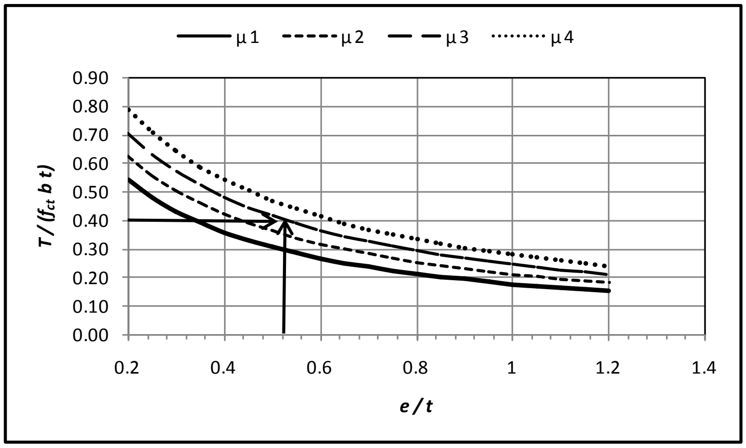

Fig. 6. Schematic diagram for parameter (T/f_{ct} b t) and eccentricity – thickness ratio (e/t).

Another technique for the sections subjected to eccentric tensile forces can be carried as following:

$$f_{ct} = k\,\frac{T}{b\,t} + k_1^2\,\frac{M}{b\,t^2}$$
$$b\,t^2\,f_{ct} = k\,T\,t + k_1^2\,M$$
$$t^2 - \frac{k\,T}{b\,f_{ct}}t - \frac{k_1^2\,M}{b\,f_{ct}} = 0$$
$$t^2 - \frac{k\,T}{b\,f_{ct}}t - \frac{f_{t2}\,t^2}{f_{ct}} = 0$$
$$t^2 - \frac{k\,T}{b\,f_{ct}}t - \frac{t^2}{\varrho} = 0$$

where

$$\varrho = \frac{f_{ct}}{f_{t2}} \quad and \quad f_{ct} > f_{t2} \tag{19}$$

then

$$\left(1 - \frac{1}{\varrho}\right) t^2 - \frac{k\,T}{b\,f_{ct}}t = 0$$
$$\left(1 - \frac{1}{\varrho}\right) t - \frac{k\,T}{b\,f_{ct}} = 0$$
$$\frac{T}{f_{ct}\;t\;b} = \frac{1}{k}\left(1 - \frac{1}{\varrho}\right) \tag{20}$$

Figure 7 and Table 3 are examples for representing Eq. (20). Assuming the reinforcement ratio (μ) and the stresses ratio (ϱ), then the design factor (T/(fct b t)) can be determined using the design charts or tables. Also, if the section dimensions are assumed first, the required reinforcement ratio can be obtained by locating the point (ϱ, T/(fct b t)) in the chart, then determining the suitable reinforcing ratio.

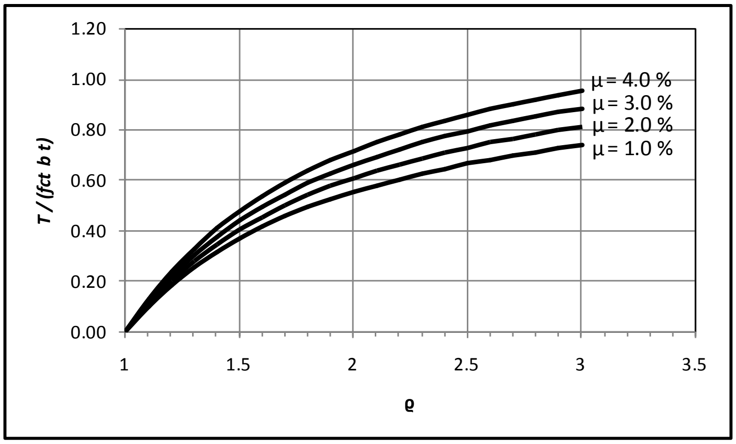

Fig. 7. Design parameter (T/f_{ct} b t) and stresses ratio (ϱ) ($\alpha = 0.2$).

Table 3: Design parameter (T/f_{ct} b t) and stresses ratio (ϱ) ($\alpha = 0.2$).

ϱ	$T / (f_{ct}\ b\ t)$				
	$\mu = 0.2\%$	$\mu = 0.4\%$	$\mu = 0.6\%$	$\mu = 0.8\%$	$\mu = 1.0\%$
1.0	0.000	0.000	0.000	0.000	0.000
1.2	0.170	0.174	0.177	0.181	0.185
1.4	0.292	0.298	0.304	0.310	0.317
1.6	0.383	0.391	0.399	0.407	0.416
1.8	0.454	0.464	0.473	0.483	0.492
2.0	0.511	0.522	0.532	0.543	0.554
2.2	0.557	0.569	0.581	0.593	0.604
2.4	0.596	0.609	0.621	0.634	0.646
2.6	0.629	0.642	0.655	0.669	0.682
2.8	0.657	0.671	0.685	0.698	0.712
3.0	0.681	0.695	0.710	0.724	0.739

3.4 Sections under Eccentric Compression

Similar to the case of sections under eccentric tension, the following procedure can be obtained:

$$f_{ct} = f_{t2} - f_{t1}$$
$$f_{ct} = k_1^2 \frac{M}{b\,t^2} - k\,\frac{P}{b\,t}$$
$$1 = \frac{k_1^2\,M}{f_{ct}\,b\,t^2} - \frac{k\,P}{f_{ct}\,b\,t}$$
$$1 = \frac{P}{f_{ct}\,b\,t}\left(k_1^2\frac{e}{t} - k\right)$$
$$\frac{P}{f_{ct}\,b\,t} = \frac{1}{k_1^2\frac{e}{t} - k} \quad (21)$$

Similarly, the other approach can be obtained as the case of eccentric tension, as following:

$$f_{ct} = \frac{k_1^2\, M}{b\, t^2} - \frac{k\, P}{b\, t}$$
$$b\, t^2\, f_{ct} = k_1^2\, M - k\, P\, t$$
$$b\, f_{ct}\, t^2 + k\, P\, t - k_1^2\, M = 0$$
$$t^2 + \frac{k\, P}{b\, f_{ct}} t - \frac{k_1^2\, M}{b\, f_{ct}} = 0$$
$$t^2 + \frac{k\, P}{b\, f_{ct}} t - \frac{k_1^2\, M}{b\, f_{t2}\, \varrho} = 0$$

where

$$\varrho = \frac{f_{ct}}{f_{t2}} \quad and \quad f_{ct} < f_{t2} \qquad (22)$$

then

$$t^2 + \frac{k\, P}{b\, f_{ct}} t - \frac{1}{\varrho} t^2 = 0$$
$$\left(1 - \frac{1}{\varrho}\right) t^2 + \frac{k\, P}{b\, f_{ct}} t = 0$$
$$\left(1 - \frac{1}{\varrho}\right) t + \frac{k\, P}{b\, f_{ct}} = 0$$
$$-\frac{P}{f_{ct}\ t\ b} = \frac{1}{k}\left(1 - \frac{1}{\varrho}\right) \qquad (23)$$

Equation (21) can be represented in charts or tables as shown in Figures 8 to 13 and Table 4. The behavior of the section for compressive force inside and outside the section core is represented in Figures 8 and 9 respectively, while Figure 10 represents the general behavior. Figures 8 to 10 show the behavior for two examples of reinforcement ratio (μ = 0.2 and 1%). Figures 11 to 13 show an example of the design charts for a wide range of reinforcement ratio. Assuming the reinforcement ratio (μ) and the eccentricity-thickness ratio (e/t), then the design factor (P/(fct b t)) can be determined using the design charts or tables. As in the case of tension force, if the section dimensions are assumed first, the required reinforcement ratio can be obtained by locating the point (e/t, P/(fct b t)) in the chart or in table, then determining the suitable reinforcing ratio.

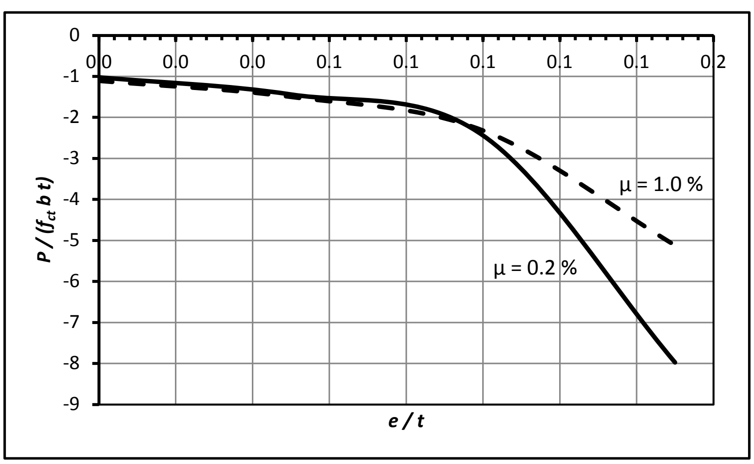

Fig. 8. Design parameter ($P/f_{ct}\ b\ t$) and eccentricity-thickness ratio (inside core & α = 0.2).

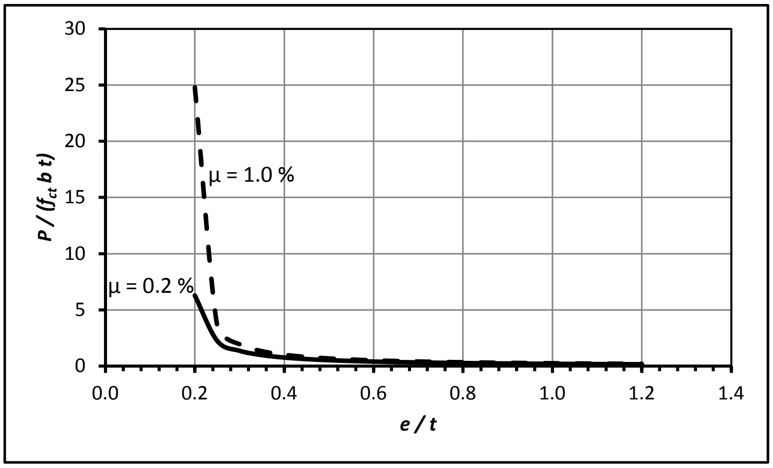

Fig. 9. Design parameter ($P/f_{ct}\ b\ t$) and eccentricity-thickness ratio (outside core & α = 0.2).

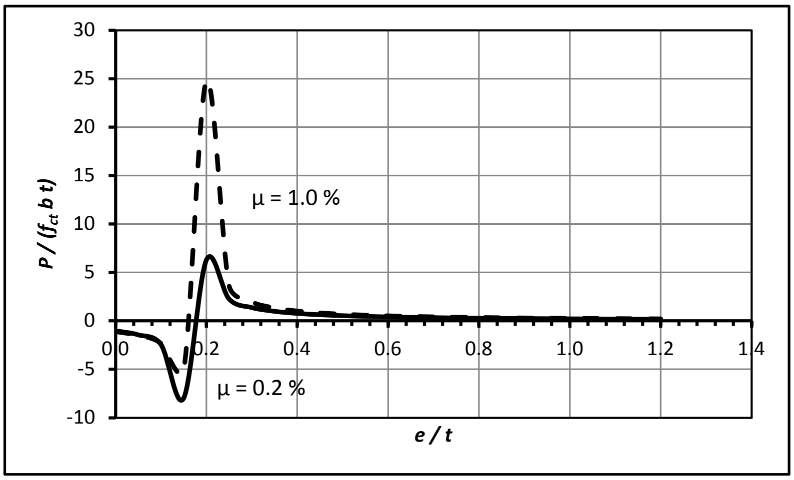

Fig. 10. Design parameter ($P/f_{ct}\ b\ t$) and eccentricity-thickness ratio (e/t) (α = 0.2).

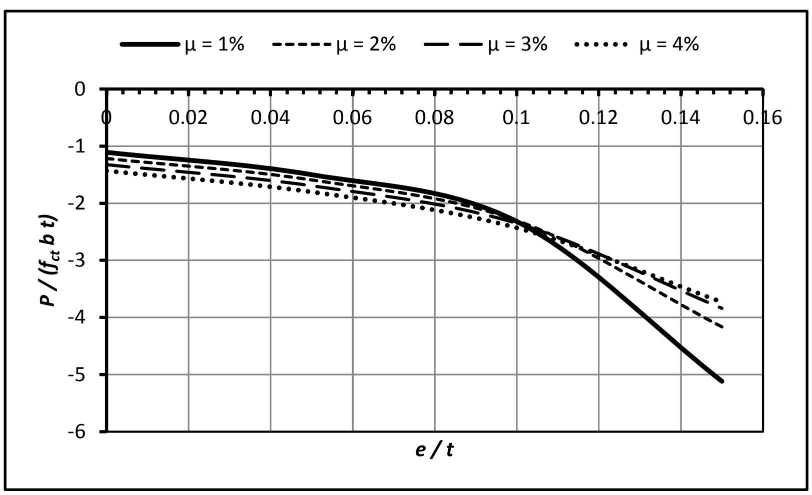

Fig. 11. Example for design chart for compressive force inside core (α = 0.2).

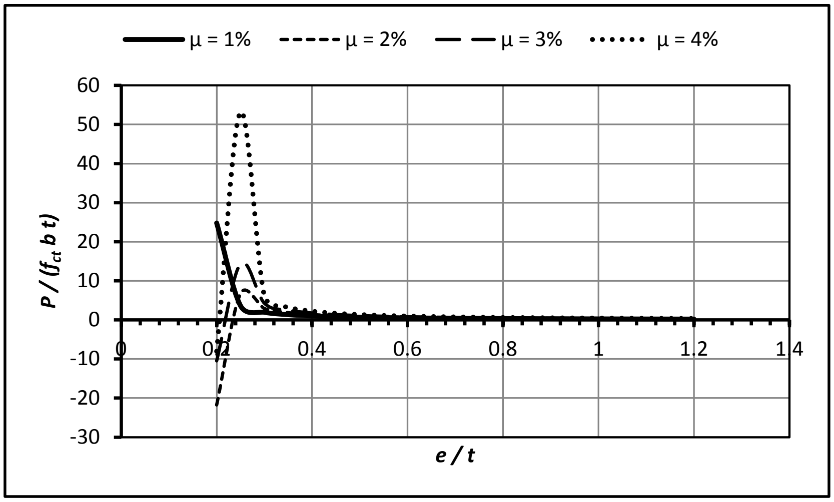

Fig. 12. Example for design chart for compressive force outside core (α = 0.2).

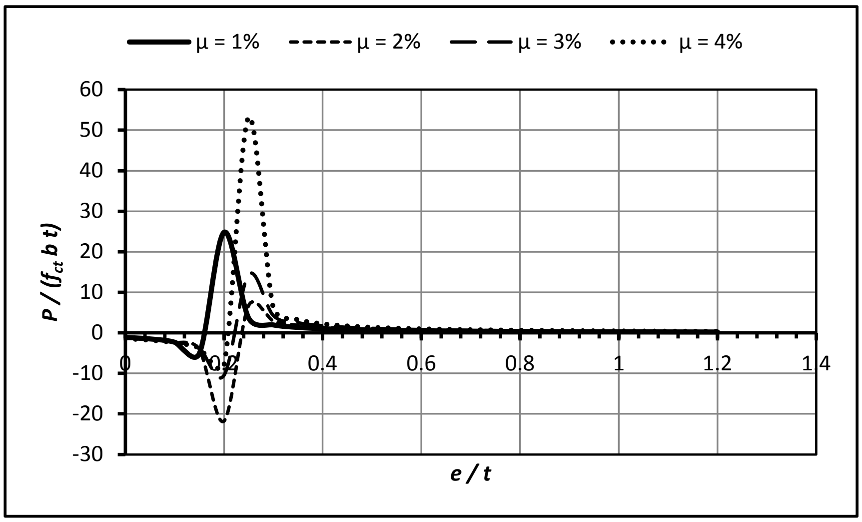

Fig. 13. Example for general design chart for compressive force (α = 0.2).

Table 4: Design parameter ($P/f_{ct}\ b\ t$) and eccentricity-thickness ratio (e/t) ($\alpha = 0.2$).

e/t	***P / (f_ct b t)***				
	μ = 0.2%	***μ = 0.4%***	***μ = 0.6%***	***μ = 0.8%***	***μ = 1.0%***
0.00	-1.022	-1.043	-1.065	-1.086	-1.108
0.10	-2.440	-2.395	-2.361	-2.337	-2.320
0.20	6.286	8.104	10.855	15.475	24.792
0.30	1.374	1.505	1.645	1.795	1.954
0.40	0.771	0.830	0.890	0.953	1.017
0.50	0.536	0.573	0.610	0.648	0.687
0.60	0.411	0.437	0.464	0.491	0.519
0.70	0.333	0.354	0.374	0.396	0.417
0.80	0.280	0.297	0.314	0.331	0.349
0.90	0.241	0.256	0.270	0.285	0.299
1.00	0.212	0.225	0.237	0.250	0.262
1.10	0.189	0.200	0.211	0.222	0.233
1.20	0.171	0.181	0.191	0.200	0.210

Figures 14 and 15 and Table 5 are examples for representing Eq. (23). Assuming the reinforcement ratio (μ) and the stresses ratio (ϱ), then the design factor (P/(fct b t)) can be determined using the design charts or tables. Also, if the section dimensions are assumed first, the required reinforcement ratio can be obtained by locating the point (ϱ, T/(fct b t)) in the chart or the table, then determining the suitable reinforcing ratio.

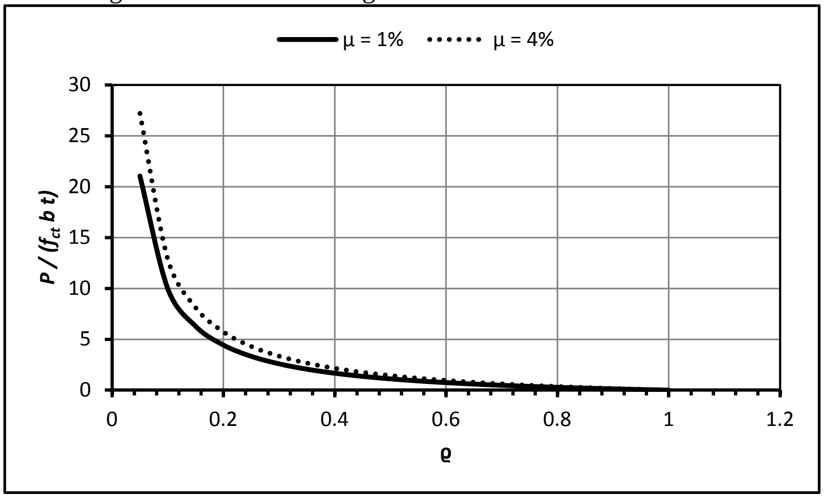

Fig. 14. Design parameter (P/f_{ct} b t) and stresses ratio (ϱ) (α = 0.2).

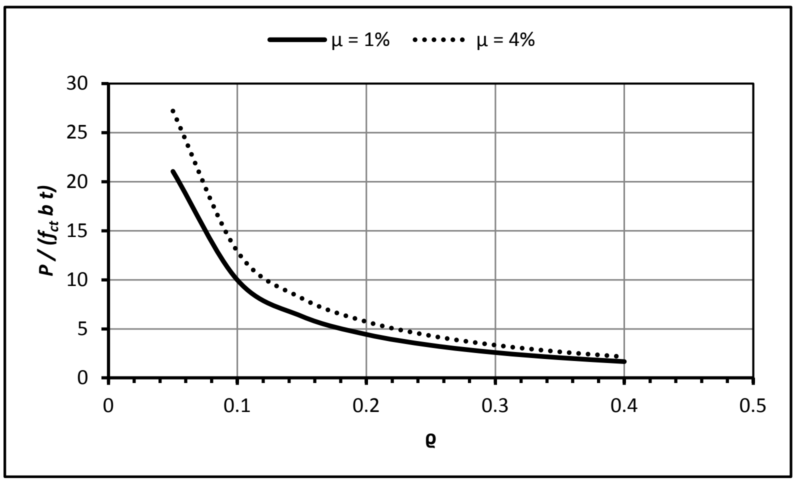

Fig. 15. Design parameter (P/f_{ct} b t) and stresses ratio (ϱ) (ϱ ≤0.4 & α = 0.2).

Table 5: Design parameter (P/f_{ct} b t) and stresses ratio (ϱ) ($\alpha = 0.2$).

ϱ	$P / (f_{ct}\ b\ t)$				
	μ = 0.2%	***μ = 0.4%***	***μ = 0.6%***	***μ = 0.8%***	***μ = 1.0%***
0.10	9.194	9.389	9.583	9.778	9.972
0.20	4.086	4.173	4.259	4.346	4.432
0.30	2.384	2.434	2.485	2.535	2.585
0.40	1.532	1.565	1.597	1.630	1.662
0.50	1.022	1.043	1.065	1.086	1.108
0.60	0.681	0.695	0.710	0.724	0.739
0.70	0.438	0.447	0.456	0.466	0.475
0.80	0.255	0.261	0.266	0.272	0.277
0.90	0.114	0.116	0.118	0.121	0.123
1.00	0.000	0.000	0.000	0.000	0.000

4. Assessment of the Proposed Procedures

To assess the effectiveness of the proposed procedures, the following cases can be studied:
For a reinforced concrete section 500 mm width subjected to 250 kN axial tension, 25 N/mm2 concrete characteristic strength and 360 N/mm2 steel yield stress, the allowable tensile stresses fcto is 1.02 N/mm2, then according to the classical approach the required thickness t is 490.2 mm, which can be used as 500 mm. The required reinforcement is calculated for the cracked section according to the working stresses design method using the allowable tensile stresses in steel fs as 200 N/mm2, and then the required area of steel is 1250 mm2, which represents about 0.51% of the cross-sectional area. Using the proposed procedure and using reinforcement ratio 0.2% and the modular ratio n as 10, it can be found that the required thickness is 481.5 mm which can be used as 500 mm, and the required area of steel is 500 mm2, which represents 0.4 of the classical one. For a reinforced concrete section 1000 mm width subjected to 15 kN.m moment, 25 N/mm2 concrete characteristic strength and 360 N/mm2 steel yield stress, the allowable tensile stresses ft is 2.3 N/mm2, then according to the classical approach the required thickness t is 197.8 mm, which can be used as 200 mm. The required reinforcement is calculated for the cracked section according to the working stresses design method using the allowable tensile stresses in steel fs as 200 N/mm2 and assuming compression reinforcement ratio α is 0.2 and the section depth 180 mm, then the required area of the main steel is 481.7 mm2, which represents about 0.24% of the cross-sectional area. Using the proposed procedure and using reinforcement ratio 0.2% and compression reinforcement ratio 0.2, then from Table 1 the factor k1 is 2.385, then the required thickness is 192.6 mm which can be used as 200 mm, and the required area of steel is 400 mm2, which represents 0.83 of the classical one.
For a reinforced concrete section 1000 mm width subjected to 40 kN.m moment and 40 kN tensile force, 25 N/mm2 concrete characteristic strength and 360 N/mm2 steel yield stress, the allowable tensile stresses fct is 1.7 N/mm2, then according to the classical approach the required thickness t is determined according to the applied moment, then increased empirically to take into account the effect of the tensile force. According to the classical approach, using 400 mm thickness leads to 1.6 N/mm2 actual stresses. The required reinforcement is calculated for the cracked section according to the working stresses design method using the allowable tensile stresses in steel fs as 200 N/mm2 and assuming compression reinforcement ratio α is 0.2 and the section depth 350 mm, then the required area of the main steel is 861 mm2, which represents about 0.22% of the cross-sectional area. Using the proposed procedure and assuming e/t as 2.5, (i.e. t is 400 mm), then from Eqs. (7), (14) and (18) it can be found that the required reinforcement ratio is 0.15%, which is about 0.68 of the classic one.
For a reinforced concrete section 250 mm width subjected to 30 kN.m moment and 20 kN compressive force, 25 N/mm2 concrete characteristic strength and 360 N/mm2 steel yield stress, the allowable tensile stresses fct is 1.7 N/mm2, then according to the classical approach using 650 mm thickness leads to 1.58 N/mm2 actual stresses. Assuming compression reinforcement ratio α is 0.2 and the section depth 600 mm, then the required area of the main steel is 190 mm2, which represents about 0.12% of the cross-sectional area. Using the proposed procedure and assuming ϱ as 0.93, then it can be found that the required reinforcement ratio is 0.10%, which is about 0.83 of the classic one.

5. Conclusions

The behavior of eccentrically loaded reinforced concrete sections was studied and major conclusions are as follows:

1. The proposed formulae take into account the effect of the reinforcement in determining the dimensions of the uncracked sections, which leads to minimum cross-section dimensions.
2. The proposed formulae minimize the reinforcement content in the section.
3. The proposed formula for the axially loaded sections has a great effect in minimizing the reinforcement content in the section than the formula for sections under pure bending, because the effect of reinforcement in the section capacity is small in the second case.
4. The proposed formula of the case of compression force has the smallest effect due to the effect of compression forces which reduces the role of the section reinforcement.
5. In the case of eccentric compression, the effect of the reinforcement ratio decreases by increasing the eccentricity-thickness ratio due to increasing the moment effect. The same conclusion can be noticed from decreasing the ratio of the total stresses to the moment stresses.
6. Controlling the reinforcement ratio in the design process of the uncracked sections is a good tool to get different design choices.

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