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Prioritized Continuous Dynamic Contraflow on Multi-Terminal Network

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Abstract

This paper introduces the prioritized maximum continuous dynamic contraflow model with intermediate storage, i.e., the inflow into the intermediate node may be greater than outflow violating the conservations constraints in continuous time. We present a polynomial time algorithm to solve the prioritized maximum dynamic contraflow problem allowing intermediate storage in continuous time setting.

Keywords: *Continuous dynamic flow, prioritized sources and sinks, contraflow, intermediate storage.*

1. Introduction

Due to different disasters such as earthquake, volcano, flooding, tsunami, a large number of infrastructures may be damaged and lots of people may be lost. Hence, the evacuation planning is necessary to save lives by shifting the evacuees from the disastrous place (sources) to safe destination (sinks) as quickly and efficiently as possible. To shift the large number of evacuees quickly, one of the best techniques is contraflow which is widely accepted for emergency planning. By reversing the direction of arcs towards the safe areas, the capacity of the arcs can be increased. To deal the evacuation planning problem, many mathematical models in both discrete and continuous times have been developed, [2,13]. The continuous dynamic contraflow was introduced by Pyakurel and Dhamala [12] and presented efficient algorithms to solve the maximum dynamic, earliest arrival and quickest flow problems using natural transformation of [4]. After that, there are number of efficient algorithms developed to solve the continuous dynamic flow problems, [11,13]. All the continuous flow models with and without contraflow satisfy the flow conservation constraints at each intermediate nodes.

Recently, the flow model with intermediate storage has been introduced and solved maximum static, maximum dynamic and maximum dynamic contraflow problems in polynomial time complexities on the two terminal networks by Pyakurel and Dempe [9,10]. The intermediate storage is possible if total capacity of the outgoing arcs from the sources is greater than the minimum cut capacities of the network. They have also introduced the earliest arrival flow and earliest arrival contraflow problems and solved in strongly polynomial time complexities on the two terminal series parallel network. However, the possibility of intermediate storage has been suggested for the first time by Pyakurel and Dhamala [1]. Extending the result of [9,10], the prioritized maximum static and dynamic flow problems have been introduced and presented polynomial time algorithms to solve them on a single source and multiple sinks networks, [8]. The problems with contraflow approach have been introduced and solved with same time complexities by Pyakurel et al. [14]. Their solution technique follows the algorithm of [5,6].

In this paper, we investigate the continuous dynamic flow model with intermediate storage. In Section 2, we describe notations and flow models with intermediate storage allowing the contraflow. Then the prioritized maximum continuous dynamic contraflow (PMCDCF) problem with intermediate storage is introduced and solved by a polynomial time algorithm in Section 3. The paper concludes in Section 4.

2. Preliminaries

Let us define a network by the set G of n nodes with, $S = \{s_k : k = 1,2, \ldots, p\}$ be the set of sources with priority order $s_1 > s_2 > \cdots > s_p$, $D = \{d_l : l = 1,2, \ldots, q\}$ be the set of sinks with priority order $d_1 > d_2 > \cdots > d_q$ and $I = \{i_x : x = 1,2, \ldots, r\}$ be the intermediate nodes. Let A be the set of arcs with $|A| = m$. If V be the set of nodes then $V = S \cup I \cup D$. Let us assume that the reverse of the arc $a = (v, w) \in A$ be $a' = (w, v) \in A$. Let $b(a) : A \to \mathbb{Z}^+$ and $b(v) : I \to \mathbb{Z}^+$ be the capacities of the arcs $(a, a') \in A$ and holdover capacity of nodes $v \in I$, respectively. Let $\psi(a)$ be the rate of flow i.e. the amount of flow entering the arc a per unit time and $\tau(a)$ be the transit time to travel the arc a. Let $A_v = \{a \in A : a = (v, w)\}$ and $B_v = \{a \in A : a = (w, v)\}$ represents the set of incoming and outgoing arcs of $v \in V$, respectively. Then with predetermined time T, the dynamic network is represented by $G = (V, A, b(a), b(v), \tau, S, I, D, \boldsymbol{T})$**.** The domain of the time in continuous $\boldsymbol{T} = [0, T]$.

2.1 Natural transformation

Let $\phi : \boldsymbol{A} \times \boldsymbol{T} \to \mathbb{R}^+$ and $\psi : \boldsymbol{A} \times \boldsymbol{T} \to \mathbb{R}^+$ be the amount of flow (load) and rate of flow on the arc a, respectively. Then, by Kotnyek [7] the value of ϕ in continuous and discrete models respectively, given by

$$\phi(a, t) = \int_0^{\tau(a)} \psi(a, t - \theta) d\theta \text{ and } \phi(a, t) = \sum_{\theta=0}^{\tau(a)-1} \psi(a, t - \theta) \qquad (1)$$

Note that, in continuous model, the flow entering the arc a at time $t - \tau(a)$ has already arrived at its head by time t or is still on the arc at that moment. Whereas in discrete model, we assume that such flow is already arrived at the head node at time t.

Let, $\psi(a, t)$ be a feasible discrete flow entering the arc $a \in A$ for $t = \{0,1, \ldots, T - 1\}$ and set the continuous flow rate $\xi(a, \theta)$ to $\psi(a, t)$ for $\theta \in [t, t + 1)$ continuous $\boldsymbol{T} = [0, T]$.

2.2 Continuous dynamic contraflow model

For contraflow configuration of network G with symmetric transit time, the auxiliary network is denoted by $\bar{G} = (V, E, b(\bar{a}), b(v), \tau, S, I, D, T)$. This is obtained by reversing the arcs towards the sinks and modifying the arc capacities and transit times as follows

$$b(\bar{a}) = b(a) + b(a') \text{ and } \tau(a) = \begin{cases} \tau(a) & if\ a \in A \\ \tau(a') & otherwise \end{cases}$$

Where, $\bar{a} \in E$ if $a \cup a' \in A$ in G, and other parameters do not change.

Based on the natural transformation as described in Subsection 2.1, we transform the discrete dynamic flow model with intermediate storage of [14] in to continuous flow model in which flow is maximized according to the given priority ordering as follows.

$$val(\phi, T) = \sum_{\bar{a} \in A_S} \int_{\sigma=0}^{T} \psi(\bar{a}, \sigma) d\sigma = \sum_{\bar{a} \in B_D} \int_{\sigma=\tau(\bar{a})}^{T} \psi\big(\bar{a}, \sigma - \tau(\bar{a})\big) d\sigma + \phi(v, T) \qquad (2)$$

$$\sum_{\bar{a} \in B_v} \int_{\sigma=\tau(\bar{a})}^{\theta} \psi\big(\bar{a}, \sigma - \tau(\bar{a})\big) d\sigma - \sum_{\bar{a} \in A_v} \int_{\sigma=0}^{\theta} \psi(\bar{a}, \sigma) d\sigma \ \geq 0, \forall\ v \in I, \theta \in \boldsymbol{T} \qquad (3)$$

$$0 \leq \psi(\bar{a}, \theta) \leq b(\bar{a}, \theta) = b(a) + b(\bar{a}) \ \ \forall a, a' \in A, \forall\ v \in I, \theta \in \boldsymbol{T} \qquad (4)$$

$$0 \leq \psi(v, \theta) \leq b(v) \qquad (5)$$

Where, $\psi(\bar{a},t)$ is the continuous flow entering the arc $\bar{a} \in E$. Also, $val(\phi, \boldsymbol{T})$ and $\phi(v, \boldsymbol{T}) = \sum_{v \in I, b(v)>0} \int_0^T \psi(v, \sigma) d\sigma$ be the maximum amount of flow that reaches to the sink and store flow at the intermediate nodes, respectively, in time T with contraflow configuration. The maximum continuous dynamic contraflow problem with intermediate storage is to maximize the objective function in Equation (2) with respect to the constraints (3) – (5)

3. PMCDCF with intermediate storage

In this section, we introduce the prioritized maximum continuous dynamic contraflow (PMCDCF) problem with intermediate storage on G. This problem on discrete dynamic network has been introduced in Pyakurel et al. [14] and solved by polynomial time algorithm. The maximum continuous dynamic contraflow (PCDCF) problem on two terminal network without intermediate storage has been solved by presenting a polynomial time algorithm [11,12]

Problem 1 *Given prioritized network $G = (V, A, b(a), b(v), \tau, S, I, D, T)$. The PMCDCF problem with intermediate storage is to maximize the amount of flow out of the sources which is equal to the total flow reached to the sinks plus stored at intermediate nodes within time T if the direction of the arcs can be reversed.*

Authors in [14] solved the PMDCF problem with intermediate storage. They constructed auxiliary network $\bar{G}$ of the prioritized given network, fixed first the source with first priority. To send flow, they gave first priority to the sinks and then to the intermediate nodes. For intermediate nodes, first priority is given to the node which is at a longest distance from the source and so on. They modified $\bar{G}$ into G' by creating the dummy sinks $v' \in I', \forall\, v \in I$ with $b(v) = b(v')$ and with their priority ordering. On G' the PMDF is computed and obtained flow at dummy sinks is transformed into $\bar{G}$. This process is repeated to all sources with priority. Then, the updated flow on $\bar{G}$ is equal to the PMDCF with intermediate storage on G

Algorithm 1 *PMCDCF with intermediate storage.*

1. ***Input:*** *Given prioritized network $G = (V, A, b(a), b(v), \tau, S, I, D, \boldsymbol{T})$.*
2. *Construct the auxiliary network $G' = (V, E, b(\bar{a}), b(v), \tau(\bar{a}), S, I, D, \boldsymbol{T})$.*
3. *For prioritized source $\{s_k : k = 1,2, \dots, p\}$, fix s_1 and solve the PMDF as in [8].*
   - i. *Determine the shortest distance $d(s_1, v), \forall v \in I$ using algorithm in [3] and give the first priority after sinks to the node v with longest $d(s_1, v)$.*
   - ii. *Create the dummy sinks $v' \in I', \forall\, v \in I$ with the same priority order.*
   - iii. *Obtain the transformed network $G' = (V', E', b'(\bar{a}), b'(v), \tau(\bar{a}), s_1, D', \boldsymbol{T})$ considering $v' \in I'$ as sinks in which the priority ordering of given sinks is followed by the dummy sinks.*
   - iv. *Compute PMDF without intermediate storage in G' according to [5,6].*
   - v. *Transform the solution of G' to $\bar{G}$ by removing the dummy sinks.*
4. *Continue this process (i) - (v) for $s_2, s_3, \dots, s_p$.*
5. *Using natural transformation of [4], the obtained discrete flow is transformed into continuous flow.*
6. *Decompose the flow into the paths and removable cycle.*
7. *An arc $(w, v) \in A$ is reversed iff $\phi(v, w) > b(v, w)$ or if there is a non negative flow along the arc $(v, w) \notin A$.*
8. ***Output:*** *PMCDCF with intermediate storage.*

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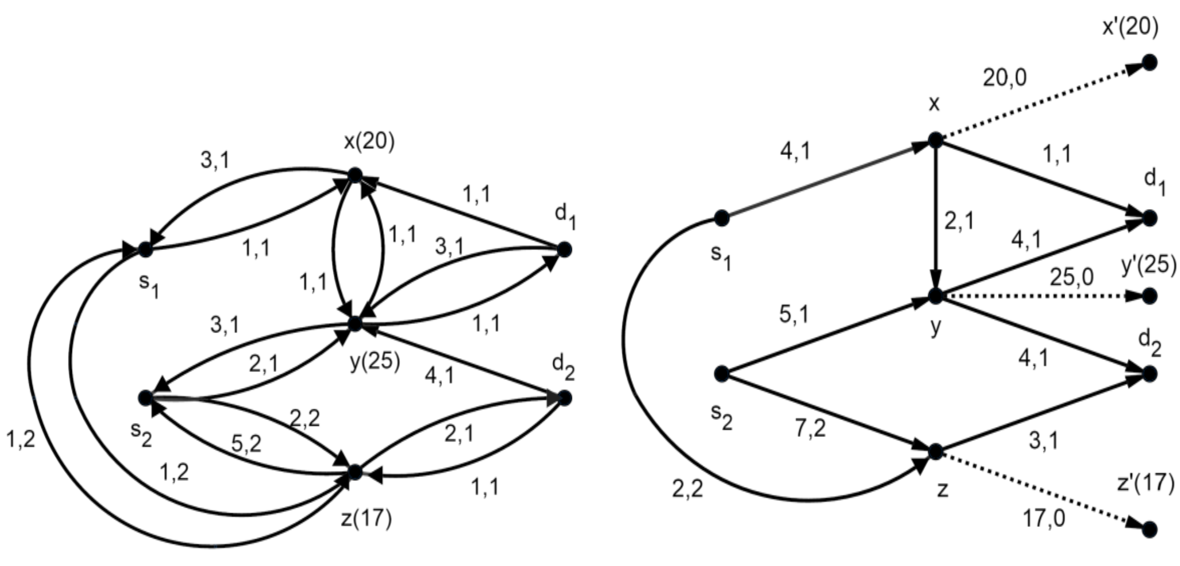
Theorem 1 *Algorithm 1 computes an optimal solution for PMCDCF problem with intermediate storage.*

Proof: According to [14], the steps $1-4,6,7$ are feasible. The natural transformation of [4] is also feasible; this implies that Step 5 is also feasible. So, our algorithm is feasible. The PMDCF problem with intermediate storage is solved in discrete time optimally as in [14]. Using natural transformation of [4] to the optimal solution on discrete dynamic network, as in Pyakurel and Dhamala [9,10] without intermediate storage, the optimal solution to the PMCDCF problem with intermediate storage can be obtained in continuous dynamic network. So, Algorithm 1 computes the optimal solution to the problem PMCDCF with intermediate storage in the original network. □

Corollary 1 *The PMCDCF problem with intermediate storage can be solved in O(k(MCF)), where k be the number of terminals and O(MCF) be the complexity to perform the minimum cost circulation.*

Example 1 *Consider a multi-terminal two ways network containing two sources with priority order* s_1, s_2 *and two sinks with priority order* d_1, d_2 *as shown in Fig. 1(i). The intermediate nodes* x, y *and* z *have storage capacities 20, 25 and 17, respectively. Each arc has capacity and transit time. First auxiliary network is constructed and we fix* s_1*. Then we construct the modified network by using Algorithm 1, Fig 1 (ii) is obtained. Using Algorithm, first we send flow from* s_1*, then from* s_2 *and Table 1 gives the final solution.*



i. (Arc: capacity, time; node: capacity)   ii.

Fig. 1: i. Given network, ii. Modified network

From Table 1, finally the sinks d_1 and d_2 receive 8 and 9 units of flow, respectively. Similarly, 6,7, and 15 units of flow are stored at the intermediate nodes x, y and z, respectively.

Table 1: PMCDCF with intermediate storage

S.N.	Path	T = [0,1)	T = [1,2)	T = [2,3)	T= [3,4)	Total
1	$s_1 - x - d_1$	--	--	1	1	2
2	$s_1 - x - y - d_1$	--	--	--	2	2
3	$s_1 - z - d_2$	--	--	--	2	2
4	$s_1 - x - y - y'$	--	--	--	2	2
5	$s_1 - z - z'$	--	--	--	2	2
6	$s_1 - x - x'$	--	1	1	4	6
7	$s_2 - y - d_1$	--	--	2	2	4
8	$s_2 - y - d_2$	--	--	3	3	6
9	$s_2 - z - d_2$	--	--	--	1	1
10	$s_2 - z - z'$	--	--	6	7	13
11	$s_2 - y - y'$	--	--	--	5	5

4. Conclusions

We have studied the network flow models without and with intermediate storage. Using the natural transformation, we have developed the prioritized maximum continuous dynamic contraflow model with intermediate storage on multi-terminal network and solved it in polynomial time.
To the best of our knowledge, we have introduced and solved this problem for the first time. Further we are interested in the flow management stored at the intermediate node.

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