A New Class of Open Sets In Nano Topological Spaces

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ABSTRACT

The aim of this paper is to introduce a new class of function, namely β_N^* -open sets and β_N^* -closed sets in Nano topological spaces. Further we investigate fundamental properties are discussed. Additionally we relate with some other Nano topological spaces.

Keywords and phrases: Nano topological spaces, β_N^* -open sets, β_N^* -closed sets and β_N^* -continuous.

I INTRODUCTION

In 1983 M.E.Abd El-Monsef, S.N. El-Deeb, R.A. Mahmoud [3] introduced β-Open sets in Topological spaces. P. Anbarasi Rodrigo and K. Rajendra Suba [4] introduced β^*-closed sets in Topological spaces. M. Lellis Thivagar [1] introduced Nano topological space with respect to a subset X of a universe which is defined in terms of lower and upper approximations of X. He has also defined Nano closed sets, Nano-interior and Nano-closure of a set. He also introduced the weak forms of Nano open sets. In 2015 Revathy, A., llango, G. [5] introduced Nano β-open sets in Nano topological spaces. In 2013, M.Lellis Thivagar [

10]introduced A Nano continuous function in Nano topological spaces. In 2014, K.Bhuvaneswari et al., A.Ezhilarasi introduced the concept of Nano semi-generalized and Nano generalized-semi closed sets in Nano topological spaces. K.Bhuvaneswari and K.Mythili Gnanapriya [6] introduced Nano g-closed sets and obtained some of the basic results. In this paper, we define a study on new class of function is called **β_N^* -open sets** in Nano topological space and study the relationships with other Nano sets.

II PRELIMINARIES

Throughout this chapter (U, τ_R(X)) is a Nano topological space with respect to X where X⊆U, R is an equivalence relation on U, U/R denotes the family of equivalence classes of U by R. Here we recall the following known definitions and properties

Definition 2.1[7] Let U be a non empty finite set of objects called the ***universe*** and R be an equivalence relation on U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be discernible with one another. The pair (U,R) is said to be the approximation space. Let X⊆U

1. The lower approximation of X with respect to R is the set of all objects which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is $L_R(X) = \bigcup_{x\in U}\{R(x) / R(x) \subseteq X\}$ where R(x) denotes the equivalence class determined by X.
2. The upper approximation of X with respect to R is the set of all objects which can be possibly defined as X with respect to R and it is denoted by $U_R(X)$. That is $U_R(X)= \bigcup_{x\in U}\{R(x) / R(x) \cap X \neq \phi\}$
3. The boundary region of X with respect to R is the set of all objects which can be classified neither as X nor as not X with respect to R and is denoted by $B_R(X)$. That is $B_R(X) = U_R(X) – L_R(X)$.

Proposition 2.2[2] If (U,R) is an approximation space and X, Y ⊆ U, then

1. $L_R(X) \subseteq X \subseteq U_R(X)$
2. $L_R(\phi) = U_R(\phi) = \phi$ and $L_R(U) = U_R(U) = U$
3. $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$
4. $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$
5. $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$
6. $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$
7. $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$
8. $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$
9. $U_RU_R(X) = L_RU_R(X) = U_R(X)$
10. $L_RL_R(X) = U_RL_R(X) = L_R(X)$

Definition 2.3[1] Let U be the universe, R be an ***equivalence relation*** on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where X⊆U. Then by the proposition 2.2, R(X) satisfies the following axioms:

1. U and $\phi \in \tau_R(X)$
2. The union of the elements of any subcollection of $\tau_R(X)$ is in $\tau_R(X)$.
3.

The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

That is $\tau_R(X)$ is a topology on U called the Nano topology on U with respect to X. We call $(U, \tau_R(X))$ as the Nano topological space. The elements of $\tau_R(X)$ are called as Nano-open sets.

Remark 2.4[1] If $\tau_R(X)$ is the Nano topology on U with respect to X, then the set B = {U, ϕ, $L_R(X)$, $B_R(X)$} is the ***basis*** for $\tau_R(X)$.

Definition 2.5[1] If $(U,\tau_R(X))$ is a Nano topological space with respect to X and if A⊆ U, then the ***Nano interior*** of A is defined as the union of all Nano-open subsets of A and is denoted by Nint(A).That is, Nint(A) is the largest Nano-open subset of A.
The ***Nano closure*** of A is defined as the intersection of all Nano-closed sets containing A and it is denoted by Ncl(A). That is, Ncl(A) is the smallest Nano-closed set containing A.

Definition 2.6[1,5] Let $(U,\tau_R(X))$ be a Nano topological space and A ⊆ U. Then A is said to be

(i) ***Nano pre-open*** if A⊆Nint(Ncl(A))

(ii) ***Nano semi-open*** if A⊆Ncl(Nint(A))
(iii) ***Nano α-open*** if A⊆Nint(Ncl(Nint(A)))
(iv) ***Nano β-open*** if A⊆Ncl(Nint(Ncl(A)))
The complements of the above mentioned sets are called their respective ***Nano-closed*** sets.

Definition 2.7[6] Let (U, τ_R(X)) be a Nano topological space. A subset A of (U, τ_R(X)) is called ***Nano generalized-closed set*** (briefly Ng- closed) if Ncl(A) ⊆ V where A ⊆ V and V is Nano-open.
The complement of Nano generalized -closed set is called as ***Nano generalized-open set***.

Definition 2.8[8] For every set A ⊆ U, the ***Nano generalized closure of A*** is defined as the intersection of all Ng- closed sets containing A and is denoted by Ng-cl(A).
Definition 2.9[8] For every set A ⊆ U, the ***Nano generalized interior of A*** is defined as the union of all Ng- open sets contained in A and is denoted by Ng-int(A).

Proposition 2.10[8] For any A ⊆ U **,**
(i) NgCl(A) is the smallest Ng closed set containing A.
(ii) A is Ng- closed if and only if NgCl(A) = A.
(iii) A ⊆ NgCl(A) ⊆ Cl(A)

Proposition 2.11[8] For any two subsets A and B of U,
(i) If A ⊆ B, then NgCl(A) ⊆ NgCl(B)
(ii) NgCl(A∩B) ⊆ NgCl(A) ∩ NgCl(B)

Definition 2.12[4] A subset A of a topological space (X, τ) is called ***β*- open set*** A ⊆ cl(int*(cl(A))). (i.e)X\A is called β*-closed set.

Definition 2.13[9] A subset A of a Nano topological space $(U, \tau_R(X)$ is called a ***Nano semi*-open set*** if there is a Nano-open set V in U such that $V \subseteq A \subseteq Ngcl(V)$. The collection of all Nano semi*-open sets in $(U, \tau_R(X))$ is denoted by S_N^*O.

The complement of Nano semi*-open set is called a ***Nano semi*-closed set***. The set of all Nano semi*-closed sets $(U, \tau_R(X))$, is denoted by S_N^*C**.**

Definition 2.14[10] Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ be two Nano topological spaces. Then a mapping : $(U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is ***Nano-continuous*** function on U if the inverse image of every Nano-open set in $(V, \tau_{R'}(Y))$ is Nano-open in $(U, \tau_R(X))$.

Definition 2.15[13] Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ be two Nano topological spaces. Then a mapping : $(U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is ***Nano generalized-continuous*** function (shortly Ng-continuous) on U if the inverse image of every Nano-open set in $(V, \tau_{R'}(Y))$ is Nano generalized-open in $(U, \tau_R(X))$.

Definition 2.16[11] Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ be two Nano topological spaces. Then a mapping : $(U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is ***Nano semi-continuous*** function on U if the inverse image of every Nano-open set in $(V, \tau_{R'}(Y))$ is Nano semi-open in $(U, \tau_R(X))$.

Definition 2.17[11] Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ be two Nano topological spaces. Then a mapping f : $(U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is ***Nano α-continuous*** function on U if the inverse image of every Nano-open set in $(V, \tau_{R'}(Y))$ is Nano α-open in $(U, \tau_R(X))$.

Definition 2.18[12] Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ be two Nano topological spaces. Then a mapping : $(U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is ***Nano β-continuous*** function on U if the inverse image of every Nano-open set in $(V, \tau_{R'}(Y))$ is Nano β-open in $(U, \tau_R(X))$.

Definition 2.19[14] Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ be two Nano topological spaces. Then a mapping : $(U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is ***Nano pre-open continuous*** function on U if the inverse image of every Nano-open set in $(V, \tau_{R'}(Y))$ is Nano pre-open open in $(U, \tau_R(X))$.

III β_N^* −OPEN SETS

Definition 3.1 A subset A of a Nano topological space (U, τ_R(X)) is called $\boldsymbol{\beta_N^*}$ ***− open set*** if $A \subseteq Ncl(Ngint(Ncl(A)))$. The collection of all β_N^*-open sets is denoted by β_N^*O(U, τ_R(X)).

Example 3.2 Let U={a,b,c,d} with U/R = {{a},{c},{b,d}}. Let X= {a,b}. Then τ_R(X) = {U,ϕ,{a},{b,d},{a,b,d}}. The Nano-closed sets are { U, ϕ, {c}, {a, c}, {b, c, d}}. The Nano generalized -closed set are {U,ϕ,{c},{a,c},{b,c},{c,d},{a,b,c},{a,c,d},{b,c,d}. The Nano generalized-open sets are {U,ϕ,{a},{b},{d},{a,b},{a,d},{b,d},{a,b,d}}. β_N^*O(U, τ_R(X))={ U,ϕ, {a},{b},{c}, {a,b},{a,c},{a,d},{b,c},{b,d},{c,d},{a,b,c},{a,b,d},{a,c,d},{b,c,d}}.

Theorem 3.3 Let (U, τ_R(X)) be a topological spaces then every Nano-open set of (U, τ_R(X)), is β_N^*-open in (U, τ_R(X)).
Proof: Let A be any Nano-open set in U. Every Nano-open set is β_N^*-open set then, we have $A \subseteq Ncl(Nint(NclA)) \subseteq (Ncl(Ngint(NclA)))$. Hence A is β_N^*-open.

Remark 3.4 The converse of the above theorem is not true as can be seen from the following example.
Example 3.5 Let U = {a,b,c,d}, U/R = {{a},{b,c},{d}}. Let X= {a,b}. Then τ_R(X) = {U, ϕ,{a},{b,c},{a,b,c}}and β_N^*O(U, τ_R(X)) = {U,ϕ,{a},{b},{c},{a,b},{a,c},{a,d},{b,c},{b,d}, {c,d},{a,b,c},{a,b,d},{a,c,d},{b,c,d}}. Clearly the sets {b},{c},{a,b},{a,c}, {a,d}, {b,d}, {c,d},{a,b,d},{a,c,d},{b,c,d} are β_N^*-open but not Nano-open set.

Theorem 3.6 Every Nano α-open set is β_N^*-open.
Proof: Let A be a Nano α open set. Then $A \subseteq Nint\left(Ncl\left(Nint(A)\right)\right) \subseteq Ngint\left(Ncl\left(Ngint(A)\right)\right)$. Hence A is β_N^*-open.

Remark 3.7 The converse of the above theorem is not true as can be seen from the following example.

Example 3.8 Let U = {a,b,c,d}, U/R = {{a},{b,c},{d}}. Let X = {a,b}. Then τ_R(X) = {U,ϕ,{a}, {b,c},{a,b,c}}=N α O and β_N^*O(U, τ_R(X)) = {U,ϕ,{a},{b},{c},{a,b}, {a,c},{a,d}, {b,c},{b,d}, {c,d},{a,b,c},{a,b,d},{a,c,d}{b,c,d}}. Clearly the sets {b},{c},{a,b},{a,c}, {a,d}, {b,d},{c,d}, {a,b,d},{a,c,d}{b,c,d} are β_N^*-open but not Nano α-open set.

Theorem 3.9 If A is Nano generalized-open set in (U, τ_R(X)), then A is β_N^*-open in (U, τ_R(X)).

Proof: Since A is Nano generalized-open in (U, τ_R(X)), $Ngint(A) = A$. Then $Ncl(Ngint(A)) = Ncl(A) \supseteq A$. That is $A \subseteq Ncl(Ngint(A))$. Therefore $Ngint(A) \subseteq Ngint(Ncl(A) \subseteq NCl(Ngint(Ncl(A)))$. Thus A is β_N^* -open.

Remark 3.10 The converse of the above theorem is not true.

Example 3.11 Let U= {a,b,c,d}and U/R={{a},{b,c},{d}}. Let X = {a,b}. Then τ_R(X) = {U,ϕ,{a},{b,c},{a,b,c}}, β_N^*O(U, τ_R(X)) = {U,ϕ,{a},{b},{c}{a,b},{a,c},{a,d},{b,c},{b,d},{a,b,c},{a,b,d},{a,c,d},{b,c,d}} and Ng-O = { U,ϕ,{a},{b},{c},{a,b},{a,c}{b,c},{a,b,c}}. Clearly the sets {d},{c,d},{b,d},{a,d}{a,c,d},{b,c,d},{a,b,d}are β_N^* -open but not Nano generalized-open.

Theorem 3.12 Every Nano pre-open is β_N^*-open.
Proof: Let A be Nano pre-open in U. Then A $\subseteq Nint(Ncl(A)) \subseteq Ngint(Ncl(A)) \subseteq Ncl\left(Ngint(Ncl(A))\right)$. Therefore A is β_N^* -open.

Remark 3.13 The converse of the above theorem is not true

Example 3.14 Let U = {a,b,c,d}, U/R = {{a},{b,c},{d}}. Let X= {a,b}. Then τ_R(X) = {U,ϕ,{a},{b,c},{a,b,c}} and β_N^*O(U, τ_R(X)) = {U,ϕ,{a},{b},{c},{a,b},{a,c},{a,d},{b,c},{b,d},{c,d},{a,b,c},{a,b,d},{a,c,d}{b,c,d}}. Here Nano pre-open sets are {U,ϕ,{a},{b},{c},{a,b},{a,c},{a,d}, {b,c},{a,b,c},{a,b,d},{a,c,d}}. Clearly the sets {b,d},{c,d},{b,c,d} is β_N^* -open but not Nano pre-open.

Theorem 3.15 Every Nano semi-open is β_N^* -open.
Proof: Let A be Nano semi-open in U. Then A $\subseteq \left(Nint(Ncl(A)\right) \subseteq Ncl(Ngint(A)) \subseteq Ncl\left(Ngint(Ncl(A))\right)$. Therefore A is β_N^* -open.

Remark 3.16 The converse of the above theorem is not true
Example 3.17 Let U = {a,b,c,d}, U/R = {{a},{b,c},{d}}. Let X= {a,b}. Then τ_R(X) = {U,ϕ,{a}, {b,c},{a,b,c}} and β_N^*O(U, τ_R(X))={U,ϕ,{a},{b},{c},{a,b}, {a,c},{a,d}, {b,c},{b,d},{c,d},{a,b,c},{a,b,d},{a,c,d}{b,c,d}}. Here Nano semi-open sets are {U,ϕ,{a},{a,d},{b,c},{a,b,c},{b,c,d}}. Clearly the sets {b},{c},{a,b},{a,c},{b,d},{c,d},{a,c,d}, {a,b,d} is β_N^*-open but not Nano semi- open.

Theorem 3.18 Every Nano semi*-open is β_N^*-open.
Proof: Let A be Nano semi*-open in U. Then A $\subseteq \left(Ngcl(Nint(A)\right) \subseteq Ncl(Nint(A)) \subseteq Ncl\left(Ngint(Ncl(A))\right)$. Therefore A is β_N^*-open.

Remark 3.19 The converse of the above theorem is not true

Example 3.20 Let U = {a,b,c,d}, U/R = {{a},{b,c},{d}}. Let X= {a,b}. Then τ_R(X) = {U,ϕ,{a},{b,c},{a,b,c}}and β_N^*O(U, τ_R(X)) ={U,ϕ,{a},{b},{c},{a,b},{a,c},{a,d},{b,c},{b,d},{c,d},{a,b,c},{a,b,d},{a,c,d}{b,c,d}}. Here Nano semi*-open sets are {U,ϕ,{a},{a,d},{b,c},{a,b,c},{b,c,d}}. Clearly the sets {b},{c},{a,b},{a,c},{b,d},{c,d},{a,c,d}, {a,b,d} is β_N^*-open but not Nano semi-open.

Theorem 3.21 Every Nano β-open is β_N^*-open.
Proof: Let A be Nano β −open in U. Then A $\subseteq \left((Ncl\ (Nint(Ncl(A)))\right) \subseteq Ncl(Ngint(Ncl(A)))$. Therefore A is β_N^*-open.

Remark 3.22 The converse of the above theorem is not true.

Example 3.23 Let U = {a,b,c,d}, U/R = {{a},{b},{c,d}}. Let X= {b}. Then τ_R(X) = {U,ϕ,{b}} and $\beta_N^* O(U, \tau_R(X))$ = {U,ϕ,{a},{b},{c},{d},{a,b},{a,c},{a,d},{b,c},{b,d},{c,d},{a,b,c},{a,b,d},{a,c,d},{b,c,d}}. Here Nano β-open sets are { U,ϕ,{b},{a,b},{b,c},{b,d},{a,b,c},{a,b,d},{b,c,d}}. Clearly the sets{a},{c},{d},{a,c},{a,d},{c,d},{a,c,d} are β_N^*-open but not Nano β − open.

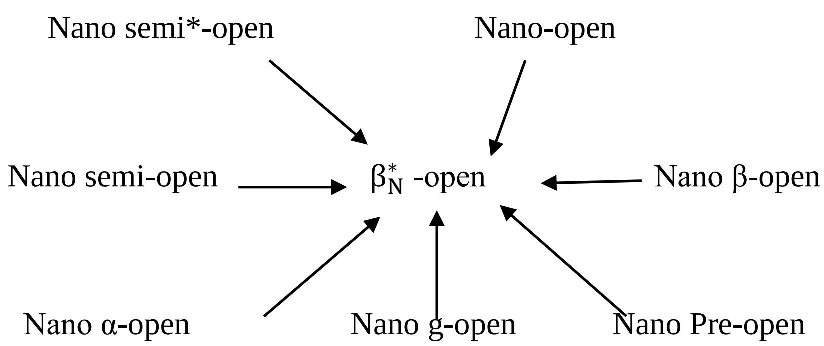
Theorem 3.24 Let $\{A_i\}$ be a collection of β_N^* -open in a Nano topological space U then $\cup A_i$ is β_N^* -open.
Proof: Since A_i is β_N^*-open for each *i*, then $A_i \subseteq Ncl\left(Ngint(Ncl(A_i))\right)$. This implies
$\cup A_i \subseteq \cup \left(Ncl\left(Ngint(Ncl(A_i))\right)\right) \subseteq \left(Ncl\left(\cup Ngint(Ncl(A_i))\right)\right)$
$\subseteq \left(Ncl\left(Ngint(\cup Ncl(A_i))\right)\right) \subseteq \left(Ncl\left(Ngint(Ncl(\cup A_i))\right)\right)$. Hence $\cup A_i$ is β_N^* -open.

Example 3.25 Let U = {a,b,c,d}, U\R = {{a},{b,c},{d}}, X = {a,b} then τ_R(X) = {U,ϕ,{a},{b,c},{a,b,c}}. Here {a,b} and {a,c} are β_N^*-open {a,b}∪{a,c} = {a,b,c} is also β_N^*-open.

Remark 3.26 The intersection of any two β_N^*-open set is not necessary β_N^*-open as can be seen from the following example.

Example 3.27 Let U = {a,b,c,d}, U/R = {{a},{b,c},{d}}. Let X= {a,b}. Then τ_R(X) = {U,ϕ,{a},{b,c},{a,b,c}} and β_N^*O(U, τ_R(X) }={U,ϕ,{a},{b},{c},{a,b},{a,c},{a,d},{b,c},{b,d},{c,d},{a,b,c},{a,b,d},{a,c,d}{b,c,d}}. The sets {a,d},{b,d} both are in β_N^*-open but their intersection {a,d}∩ {b,d} ={d} is not in β_N^*-open.
Remark 3.28 The collection of β_N^* O(U, τ_R(X)) does not form a topology.
Theorem 3.29 If a subset A is β_N^* -open and B is Nano open then A∪B is β_N^*-open.
Proof: Follows from theorem 3.3 and theorem 3.24.

Diagram 3.30 The following diagram shows the relationship between β_N^*-open sets and other Nano-open sets that are studied in this section.



IV β_N^* -CLOSED SETS

Definition 4.1 The complement of β_N^*-open set is called a ***β_N^* -closed set***. The collection of all β_N^* -closed sets is denoted by $\beta_N^*C(U, \tau_R(X))$.

Theorem 4.2 If A is Nano-closed set in (U, $\tau_R(X)$), then A is β_N^* -closed in (U, $\tau_R(X)$).
Proof: Since A is Nano-closed in (U, $\tau_R(X)$) , U\A is Nano-open by theorem 3.3 U\A is β_N^* -open. Therefore A is β_N^* -closed.

Remark 4.3 The converse of the above theorem is not true as can be seen from the following example.

Example 4.4 Let U = {a,b,c,d}, U/R = {{a},{b,c},{d}}. Let X= {a,b}. Then $\tau_R(X)$ = {U,ϕ,{a},{b,c},{a,b,c}}. The Nano-closed sets are {U,ϕ,{d},{a,d},{b,c,d}}, and $\beta_N^*C(U, \tau_R(X)$ } = {U,ϕ,{a},{b},{c},{d},{a,b},{a,c},{a,d},{b,c},{b,d},{c,d},{a,b,d},{a,c,d},{b,c,d}}. Clearly the sets {a},{b},{c},{a,b},{a,c},{b,c},{b,d},{c,d},{a,b,c},{a,b,d},{a,c,d} are β_N^*-closed but not Nano- closed set.

Theorem 4.5 Every Nano α-closed set is β_N^*-closed.
Proof: Let A be Nano α-closed Then U\A is Nano α-open by theorem 3.6, U\A is β_N^* -open which implies A is β_N^*-closed.

Remark 4.6 The converse of the above theorem is need not be true.

Example 4.7 Let U = {a,b,c,d}, U/R = {{a},{b,c},{d}}. Let X= {a,b}. Then $\tau_R(X)$ = {U,ϕ,{a}, {b,c},{a,b,c}}=NαO. The Nano-closed sets are {U,ϕ,{d},{a,d},{b,c,d}} and $\beta_N^*C(U\ \tau_R(X))$ = {U,ϕ,{a},{b},{c},{d},{a,b},{a,c},{a,d},{b,c},{b,d},{c,d},{a,b,d},{a,c,d},{b,c,d}}. Clearly the sets {a},{b},{c},{a,b},{a,c},{b,c},{b,d},{c,d},{a,b,c},{a,b,d},{a,c,d},{b,c,d} are $\beta_N^* -$ closed but not Nano α-closed.
Theorem 4.8 If A is Nano generalized-closed set in (U, $\tau_R(X)$), then A is β_N^*-closed in (U, $\tau_R(X)$).
Proof: Let A be a Nano generalized-closed set then U\A is Nano generalized-open by theorem 3.9, U\A is β_N^* -open. Hence A is β_N^* -closed.

Remark 4.9 The converse of the above theorem is not true as can be seen from the following example.

Example 4.10 Let U= {a,b,c,d}and U/R={{a},{b,c},{d}}. Let X = {a,b}. Then $\tau_R(X)$ = {U,ϕ,{a},{b,c},{a,b,c}}. The Nano-closed sets are {U,ϕ,{d},{a,d},{b,c,d}}, β_N^* C(U, $\tau_R(X)$) = {U,ϕ,{a},{b},{c},{d},{a,b},{a,c},{a,d},{b,c},{b,d},{a,b,d},{a,c,d}, {b,c,d}} and Ng-cl = {U,ϕ,{d},{a,d},{b,d},{c,d},{a,b,d},{a,c,d},{b,c,d}}. Clearly the sets {a},{b},{c},{a,b},{a,c},{b,c},{a,b,c} are $\beta_N^* - closed$ but not Nano generalized-closed

Theorem 4.11 Let $\{A_i\}$ be a collection of β_N^*-closed in a Nano topological space U then $\cap A_i$ is $\beta_N^* -$ closed.
Proof: Let A_i is β_N^* -closed for each *i* in U. This implies U/A_i is β_N^*-open in U. Then by theorem 3.24, $\cup\ (U/A_i)$ is β_N^* -open in U. This implies $(U/\cap A_i)$ is β_N^*-open in U. This implies $\cap A_i$ is β_N^*-closed in U.

Example 4.12 Let U = {a,b,c,d} U/R = {{a}, {b,c},{d}} Let X = {a,b}. Here {a,b} and {b,d} are $\beta_N^* -$ closed sets. Also {a,b}∩{b,d} = {b} is β_N^* -closed set.

Remark 4.13 The union of any two β_N^*-closed need not be β_N^*-closed as shown in the following example

Example 4.14 In example 3.27 {a.b} and {c} are β_N^*-closed but {a,b} ∪{c} = {a,b,c} is not β_N^*-closed.

Theorem 4.15 If A is β_N^* -closed and B is Nano closed , then A∩B is β_N^* -closed.
Proof: Follows from Theorem 4.2. and Theoerm 4.11.

Theorem 4.16 Every Nano β-closed set is β_N^* -closed.
Proof: Let A be Nano β-closed Then U\A is Nano β-open by theorem 3.21, U\A is β_N^* -open which implies A is β_N^*-closed.

Remark 4.17 The converse of the above theorem is not true as can be seen from the following example.

Example 4.18 Let U = {a,b,c,d}, U/R = {{a},{c,d},{b}}. Let X= {b}. Then $\tau_R(X)$ = {U,ϕ,{b}}.$\beta_N^*C(U, \tau_R(X))$ = {U,ϕ,{a},{b},{c},{d},{a,b},{a,c},{a,d},{b,c},{b,d},{c,d},{a,b,c},{a,b,d},{a,c,d},{b,c,d}}.$N\ \beta C$= {U,ϕ,{a},{c},{d},{a,c},{a,d},{b,c},{b,d},{c,d},{a,c,d}}. Clearly the sets {b},{a,b},{b,d},{a,b,c},{a,b,d},{b,c,d} are β_N^*-closed but not Nano β-closed.

Theorem 4.19 Every Nano semi-closed set is β_N^*-closed.
Proof: Let A be Nano semi-closed Then U\A is Nano semi-open by theorem 3.1.5, U\A is β_N^* - open which implies A is β_N^*-closed.
Remark 4.20 The converse of the above theorem is not true as can be seen from the following example.

Example 4.21 Let U = {a,b,c,d}, U/R = {{a},{c,d},{b}}. Let X= {b}. Then $\tau_R(X)$ = {U,ϕ,{b}}. $\beta_N^*C(U, \tau_R(X))$ = {U,ϕ,{a},{b},{c},{d},{a,b},{a,c},{a,d},{b,c},{b,d},{c,d},{a,b,c},{a,b,d},{a,c,d},{b,c,d}}.$N\ SC$={U,ϕ,{a},{c},{d},{a,c},{a,d},{b,c},{b,d},{c,d},{a,c,d}}. Clearly the sets {b},{a,b},{b,d},{a,b,c},{a,b,d},{b,c,d} are β_N^* −closed but not Nano semi-closed.

Theorem 4.22 Every Nano semi*-closed set is β_N^*-closed.
Proof: Let A be Nano semi*-closed Then U\A is Nano semi*-open by theorem3.18, U\A is β_N^*-open which implies A is β_N^*-closed.

Remark 4.23 The converse of the above theorem is need not be true.

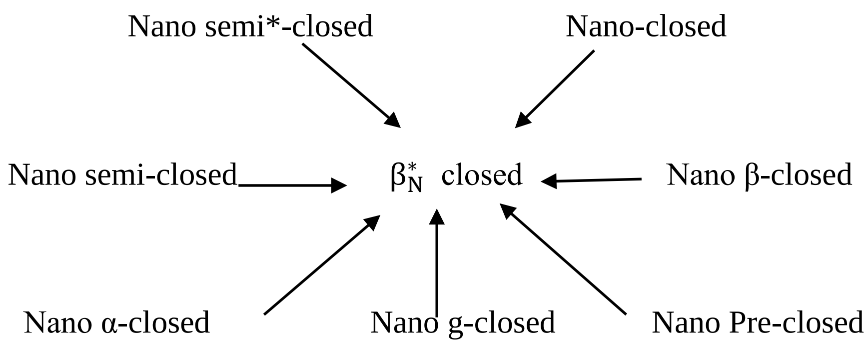
Example 4.24 Let U = {a,b,c,d}, U/R = {{a},{c,d},{b}}. Let X= {b}. Then $\tau_R(X)$ = {U,ϕ,{b}}. β_N^* C(U, $\tau_R(X)$) ={U,ϕ,{a},{b},{c},{d},{a,b},{a,c},{a,d},{b,c},{b,d},{c,d},{a,b,c},{a,b,d},{a,c,d}{b,c,d}. The Nano semi*-closed = {U,ϕ,{a},{c},{d},{a,c},{a,d},{b,c},{b,d},{c,d},{a,c,d}}. Clearly the sets {b},{a,b},{b,d},{a,b,c},{a,b,d},{b,c,d} are β_N^*-closed but not Nano semi*-closed.

Theorem 4.25 Every Nano pre-closed set is β_N^*-closed.
Proof: Let A be Nano pre-closed Then U\A is Nano pre-open by theorem 3.12, U\A is β_N^*-open which implies A is β_N^*-closed.

Remark 4.26 The converse of the above theorem is not true as can be seen from the following example.

Example 4.27 Let U = {a,b,c,d}, U/R = {{a},{b,c},{d}}. Let X= {a,b}. Then $\tau_R(X)$ = {U,ϕ,{a},{b,c},{a,b,c}}. The Nano pre-closed sets are {U,ϕ,{b},{c},{d},{a,d},{b,c},{b,d},{c,d},{a,b,d},{a,c,d},{b,c,d}} and $\beta_N^*C(U, \tau_R(X))$ = {U,ϕ,{a},{b},{c},{d},{a,b},{a,c},{a,d},{b,c},{b,d},{c,d},{a,b,d},{a,c,d}{b,c,d}}. Clearly the sets {a},{a,b}, {a,c} are β_N^* -closed but not Nano pre-closed.

Diagram 4.28 The following diagram shows the relationship between β_N^*-closed sets and other Nano-closed sets that are studied in this section.



V β_N^* -CONTINUOUS FUNCTIONS

Definition 5.1 Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ be two Nano topological spaces. Then a mapping $f: (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is ***β_N^* −continuous*** function on U if the inverse image of every Nano-open set in $(V, \tau_{R'}(Y))$ is β_N^* -open in $(U, \tau_R(X))$.

Example 5.2 Let U={a,b,c,d} with U/R={{a},{b,c},{d}} and X={a,b}. Then $\tau_R(X)$ = {U, ϕ,{a},{b,c},{a,b,c}}. Then β_N^*-open sets are {U, ϕ,{a},{b},{c},{a,b},{a,c},{a,d},{b,c},{b,d},{c,d},{a,b,c},{a,b,d},{a,c,d},{b,c,d}}. Let V={x.y.z,w} with V/R'={{x},{y,w},{z}} and Y={x,y}. Then $\tau_{R'}(Y)$)={V, ϕ,{x},{y,w},{x,y,w}}. Define $f: (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ as f(a)=y, f(b) =z, f(c)=x, f(d)=w. Then f^{-1}({x})={c}, f^{-1}({y,w})={a,d}, f^{-1}({x,y,w})={a,c,d} and f^{-1}(V)=U. That is, the inverse image of every Nano-open set in V is β_N^* −open set in U. Therefore f is β_N^*-continuous.

Theorem 5.3 Every Nano-continuous function is β_N^* −continuous.
Proof : Let $f: (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ be Nano-continuous on $(U, \tau_R(X))$. Since f is Nano- continuous of $(U, \tau_R(X))$, the inverse image of every Nano open set in $(V, \tau_{R'}(Y))$ is Nano- open in $(U, \tau_R(X))$. But every Nano-open set is β_N^* −open set. Hence the inverse image of every Nano open set in $(V, \tau_{R'}(Y))$ is β_N^* −open in $(U, \tau_R(X))$. Therefore f is β_N^*-continuous.

Remark 5.4 The converse of the above theorem is not true as seen from the following example

Example 5.5 Let U={a,b,c,d} with U/R={{a},{b,c},{d}} and X={a,b}. Then $\tau_R(X)$ = {U, ϕ,{a},{b,c},{a,b,c}}. Then β_N^*-open sets are {U, ϕ,{a},{b},{c},{a,b},{a,c}, {a,d},{b,c},{b,d},{c,d},{a,b,c},{a,b,d},{a,c,d},{b,c,d}}. Let V={x.y.z,w} with V/R'= {{x},{y,w},{z}} and Y={x,y}. Then $\tau_{R'}(Y)$)={V, ϕ ,{x},{y,w},{x,y,w}}. Define $f: (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ as f(a)=x, f(b)=y, f(c)=z, f(d)=w. Then f^{-1}({y,w})={b,d} which is not Nano-open set in $(U, \tau_R(X))$. Hence f is not Nano-continuous.

Theorem 5.6 Every Nano α −continuous function is Nano β_N^*-continuous.

Proof : Let $f: (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ be Nano α -continuous on $(U, \tau_R(X))$. Let C be Nano open in V. Since f is Nano α-continuous of $(U, \tau_R(X))$, the inverse image of every Nano-open set in $(V, \tau_{R'}(Y))$ is Nano α-open in $(U, \tau_R(X))$. Hence f^{-1}(C) is β_N^*-open in $(U, \tau_R(X))$. But every Nano α-open set is β_N^*-open set. Therefore f^{-1}(C) is β_N^*-open in $(U, \tau_R(X))$. Hence the inverse image of every Nano-open set in $(V, \tau_{R'}(Y))$ is β_N^* −open in $(U, \tau_R(X))$. Therefore f is β_N^*-continuous.

Remark 5.7 The converse of the above theorem is not true as seen from the following example

Example 5.8 Let U={a,b,c,d} with U/R={{a},{b,c},{d}} and X={a,b}. Then $\tau_R(X)$={U, ϕ,{a},{b,c},{a,b,c}}=N αO. Then β_N^* −open sets are {U, ϕ,{a},{b},{c},{a,b}, {a,c},{a,d},{b,c},{b,d},{c,d},{a,b,c},{a,b,d},{a,c,d},{b,c,d}}. Let V={x.y.z,w} with V/R' ={{x},{y,w},{z}} and Y={x,y}. Then $\tau_{R'}(Y)$)={V, ϕ ,{x},{y,w},{x,y,w}}. Define $f: (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ as f(a)=x, f(b)=y, fc)=z, fd)=w. Then f^{-1}({x,y,w})={a,b,d} which is not Nano α-open set in $(U, \tau_R(X))$. Hence f is not Nano α-continuous.

Theorem 5.9 Every Nano generalized−continuous function is β_N^*-continuous. .

Proof : Let $f: (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ be Nano generalized continuous on $(U, \tau_R(X))$. Since f is Nano generalized-continuous of $(U, \tau_R(X))$, the inverse image of every Nano generalized-open set in $(V, \tau_{R'}(Y))$ is Nano generalized-open set in $(U, \tau_R(X))$. But every Nano generalized-open set is β_N^* −open set. Hence the inverse image of every Nano-open set in $(V, \tau_{R'}(Y))$ is β_N^*-open in $(U, \tau_R(X))$. Therefore f is β_N^* -continuous.

Remark 5.10 The converse of the above theorem is not true as seen from the following example

Example 5.11 Let U={a,b,c,d} with U/R={{a},{b,c},{d}} and X={a,b}. Then $\tau_R(X)$ = {U , ϕ,{a},{b,c},{a,b,c}}. Then β_N^*-open sets are {U, ϕ,{a},{b},{c},{a,b},{a,c}, {a,d},{b,c},{b,d},{c,d},{a,b,c},{a,b,d},{a,c,d},{b,c,d}}. Let V={x.y.z,w} with V/R'= {{x},{y,w},{z}} and Y={x,y}. Then $\tau_{R'}(Y)$)={V, ϕ ,{x},{y,w},{x,y,w}}. Define $f: (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ as f(a)=y, f(b)=z, f(c)=x, f(d)=w. Then f^{-1}({x})={c}, f^{-1}({y,w})={a,d}, f^{-1}({x,y,w})={a,c,d} and f^{-1}(V)=U. Here f^{-1}({x,y,w})={a,c,d} which is not Nano generalized-open set in $(U, \tau_R(X))$. Hence f is not Nano generalized-continuous.

Theorem 5.12 Every Nano semi-continuous function is β_N^*-continuous.

Proof : Let $f: (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ be Nano semi-continuous on $(U, \tau_R(X))$. Since f is Nano semi-continuous of $(U, \tau_R(X))$, the inverse image of every Nano semi-open set in $(V, \tau_{R'}(Y))$ is Nano semi-open in $(U, \tau_R(X))$. But every Nano semi-open set is β_N^* -open set. Hence the inverse image of every Nano semi-open set in $(V, \tau_{R'}(Y))$ is β_N^* $-$open in $(U, \tau_R(X))$. Therefore f is β_N^* $-$continuous.

Remark 5.13 The converse of the above theorem is not true as seen from the following example

Example 5.14 Let U={a,b,c,d} with U/R={{a},{b,c},{d}} and X={a,b}. Then $\tau_R(X)$ = {U , ϕ,{a},{b,c},{a,b,c}}. Then β_N^*-open sets are {U, ϕ,{a},{b},{c},{a,b},{a,c},{a,d},{b,c},{b,d},{c,d},{a,b,c},{a,b,d},{a,c,d},{b,c,d}}. Here Nano semi-open sets are {U, ϕ, {a},{a,c},{b,d},{a,b,d}. Let V={x.y.z,w} with V/R'={{x},{y,w},{z}} and Y={x,y}. Then $\tau_{R'}(Y)$)={V, ϕ ,{x}, {y,w},{x,y,w}}. Define $f: (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ as f(a)=y, f(b)=z, f(c)=x, f(d)=w. Then f^{-1}({x,y,w})={a,c,d} which is not Nano semi-open set in $(U, \tau_R(X))$. Hence f is not Nano semi-continuous.

Theorem 5.15 Every Nano semi*-continuous function is β_N^* $-$continuous.
Proof : Let $f: (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ be Nano semi*-continuous on $(U, \tau_R(X))$. Since f is Nano semi*-continuous of $(U, \tau_R(X))$, the inverse image of every Nano semi*-open set in $(V, \tau_{R'}(Y))$ is Nano semi*-open in $(U, \tau_R(X))$. But every Nano semi*-open set is β_N^* $-$open set. Hence the inverse image of every Nano semi*-open set in $(V, \tau_{R'}(Y))$ is β_N^* $-$open in $(U, \tau_R(X))$. Therefore f is β_N^*-continuous.

Remark 5.16 The converse of the above theorem is not true as seen from the following example

Example 5.17 Let U={a,b,c,d} with U/R={{a},{b,c},{d}} and X={a,b}. Then $\tau_R(X)$={U, ϕ,{a}, {b,c},{a,b,c}}. Then β_N^*-open sets are { U, ϕ,{a},{b},{c},{a,b},{a,c},{a,d},{b,c},{b,d},{c,d}, {a,b,c},{a,b,d},{a,c,d},{b,c,d}}. Here Nano semi*-open sets are {U, ϕ, {a},{a,c},{b,d},{a,b,d}. Let V={x.y.z,w} with V/R'={{x},{y,w},{z}} and Y={x,y}. Then $\tau_{R'}(Y)$)={V, ϕ ,{x},{y,w}, {x,y,w}}. Define $f: (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ as f(a)=y, f(b)=z, f(c)=x, f(d)=w. Then f^{-1}({x}) = {c} which is not Nano semi*-open set in $(U, \tau_R(X))$. Hence f is not Nano semi*- continuous.

Theorem 5.18 Every Nano β-continuous function is β_N^* $-$continuous. .
Proof : Let $f: (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ be Nano β $-$continuous on $(U, \tau_R(X))$. Since f is Nano β-continuous of $(U, \tau_R(X))$, the inverse image of every Nano β-open set in $(V, \tau_{R'}(Y))$ is Nano β-open set in $(U, \tau_R(X))$. But every Nano β-open set is β_N^* $-$open set. Hence the inverse image of every Nano-open set in $(V, \tau_{R'}(Y))$ is β_N^* $-$open in $(U, \tau_R(X))$. Therefore f is β_N^* -continuous.

Remark 5.19 The converse of the above theorem is not true as seen from the following example

Example 5.20 Let U={a,b,c,d} with U/R={{a},{b,c,d}} and X={a,b}. Then $\tau_R(X)$={U, ϕ,{a},{b,c,d}}. Then β_N^* $-$open sets are { U, ϕ,{a},{b},{c},{d},{a,b},{a,c}, {a,d},{b,c},{b,d},{c,d},{a,b,c},{a,b,d},{a,c,d},{b,c,d}}. Here Nano β-open sets are {U, ϕ,{b},{b,c},{b,d},{a,b},{a,b,c},{a,b,d},{b,c,d}}. Let V={x.y.z,w}with V/R'= {{x},{w,z},{y} and Y={y}. Then $\tau_{R'}(Y)$) = {V, ϕ ,{y}}. Define $f: (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ as f(a)=y, f(b)=x, f(c)=z, f(d)=w. Then f^{-1}({y})={a}, and f^{-1}(V)=U. Here f^{-1}({y})={a} which is not Nano β-open set in $(U, \tau_R(X))$. Hence f is not Nano β-continuous.

Theorem 5.21 Every Nano pre-continuous function is β_N^* −continuous.

Proof : Let $f: (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ be Nano pre-continuous on $(U, \tau_R(X))$. Since f is Nano pre-continuous of $(U, \tau_R(X))$, the inverse image of every Nano pre-open set in $(V, \tau_{R'}(Y))$ is Nano pre-open in $(U, \tau_R(X))$. But every Nano preo-pen set is β_N^* −open set. Hence the inverse image of every Nano pre-open set in $(V, \tau_{R'}(Y))$ is β_N^* −open in $(U, \tau_R(X))$. Therefore f is β_N^* -continuous.

Remark 5.22 The converse of the above theorem is not true as seen from the following example

Example 5.23 Let U={a,b,c,d} with U/R={{a},{b,c},{d}} and X={a,b}. Then $\tau_R(X)$={U, ϕ,{a}, {b,c},{a,b,c}}. Then β_N^* -open sets are { U, ϕ,{a},{b},{c},{a,b},{a,c}, {a,d},{b,c},{b,d},{c,d},{a,b,c},{a,b,d},{a,c,d},{b,c,d}}. Here Nano pre-open sets are {U, ϕ, {a},{b},{d},{a,b},{a,d},{b,d},{a,b,c},{a,c,d},{a,b,d}. Let V={x.y.z,w} with V/R'= {{x},{y,w},{z}} and Y={x,y}. Then $\tau_{R'}(Y)$)={V, ϕ ,{x},{y,w},{x,y,w}}. Define $f: (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ as f(a)=y, f(b)=z, f(c)=x, f(d)=w. Then f^{-1}({x})={c} which is not Nano pre-open set in $(U, \tau_R(X))$. Hence f is not Nano pre-continuous.

Theorem 5.24 A function $f: (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ be β_N^* −continuous if and only if the inverse image of every Nano-closed set in V is β_N^*-closed in U.

Proof : Let $f: (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ be β_N^* −continuous function and F be Nano-closed in $(V, \tau_{R'}(Y))$. That is, V-F is Nano-open in $(V, \tau_{R'}(Y))$. Since f is β_N^*-continuous, the inverse image of every Nano-open set in $(V, \tau_{R'}(Y))$ is β_N^*-open in $(U, \tau_R(X))$. Hence f^{-1}(V-F) is β_N^* - open in $(U, \tau_R(X))$. That is, f^{-1}(V-F)= $f^{-1}(V) - f^{-1}(F) = U - f^{-1}(F)$ is β_N^* -open in $(U, \tau_R(X))$. Therefore, $f^{-1}(F)$ is β_N^* −closed in $(U, \tau_R(X))$. Conversely, let the inverse image of every Nano-closed set in $(V, \tau_{R'}(Y))$ is β_N^*-closed in $(U, \tau_R(X))$. Let G be Nano-open in $(V, \tau_{R'}(Y))$. Then V-G is Nano-closed in $(V, \tau_{R'}(Y))$. Hence f^{-1}(V-G) is β_N^* −closed in $(U, \tau_R(X))$. That is, $U - f^{-1}(G)$ is β_N^*-closed in $(U, \tau_R(X))$. Therefore, $f^{-1}(G)$ is β_N^* −open in $(U, \tau_R(X))$. Thus, the inverse image of every Nano-open set in $(V, \tau_{R'}(Y))$ is β_N^* -open in $(U, \tau_R(X))$. That is, f is β_N^* −continuous on $(U, \tau_R(X))$.

Theorem 5.25 A function $f: (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is β_N^*-continuous if and only if $f(\beta_N^*$Cl(A)) ⊆ NCl(f(A)) for every subset A of U.

Proof : Let f be a β_N^*-continuous and A ⊆ U. Then f(A) ⊆ V. Since f is β_N^* −continuous and NCl(f(A)) is Nano-closed in V, f^{-1}(NCl(f(A))) is β_N^*-closed in U. Since f(A) ⊆NCl(f(A)), $f^{-1}(f(A))$ ⊆ f^{-1}(NCl(f (A))), then $\beta_N^*\, Cl(A)$ ⊆ $\beta_N^*\, Cl[\, f^{-1}$(NCl(f (A)))] = f^{-1}(NCl(f(A))). Thus $\beta_N^*\, Cl(A)$ ⊆ f^{-1}(NCl(f(A))). Therefore $f(\beta_N^* Cl(A))$ ⊆ NCl(f(A)) for every subset A of U. Conversely, let $f(\beta_N^* Cl(A))$ ⊆ NCl(f(A)) for every subset A of U. If

F is Nano-closed in V, since $f^{-1}(F) \subseteq U$, $f(\beta_N^* Cl(f^{-1}(F))) \subseteq$ NCl($f(f^{-1}(F))$) = NCl(F). That is, $\beta_N^* Cl(f^{-1}(F)) \subseteq f^{-1}$(NCl(F)) = $f^{-1}(F)$, since F is Nano-closed. Thus $\beta_N^* Cl(f^{-1}(F)) \subseteq f^{-1}(F)$. But $f^{-1}(F) \subseteq \beta_N^* Cl(f^{-1}(F))$. Thus $\beta_N^* Cl(f^{-1}(F)) = f^{-1}(F)$. Therfore $f^{-1}(F)$ is β_N^* −closed in U for every Nano-closed set F in V. That is, f is β_N^* −continuous.

Remark 5.26 If $f: (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ be β_N^* −continuous then $f(\beta_N^*$ Cl(A)) is not necessarily equal to NCl(f(A)) for every subset A of U.

Example 5.27 Let U={a,b,c,d} with U/R={{a,b},{c},{d}} and X={a,c}. Then $\tau_R(X)$={U, ϕ,{a},{b},{a,b,c}}. Then β_N^*-open sets are { U, ϕ,{a},{b},{c},{a,b},{a,c}, {a,d},{b,c},{b,d},{c,d},{a,b,c},{a,b,d},{a,c,d},{b,c,d}}. Let V={x.y.z,w} with V/R'= {{x},{y,z,w}} and Y={x,y}. Then $\tau_{R'}(Y)$)={V,ϕ,{x},{y,z,w}}. Define $f: (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ as f(a)=y, f(b)=z, f(c)=x, f(d)=w. Then f^{-1}({x})={c}, f^{-1}({y,z,w})={a,b,d} and f^{-1}(V)=U. That is, the inverse image of every Nano-open set in V is the β_N^*-open set in U. Therefore f is β_N^* -continuous. Let A={b,c,d}⊆V. Then f (β_N^* Cl(A)) = f ({b,c,d}) = {x,z,w}. But NCl(f(A))=NCl({x,z,w})=V. Thus $f(\beta_N^*$ Cl(A))≠NCl(f(A)). That is, equality does not hold in the previous theorem when f is β_N^* −continuous.

Theorem 5.28 Let $(U, \tau_R(X))$ *and* $(V, \tau_{R'}(Y))$ be two Nano topological spaces where X ⊆ U and Y ⊆ V. Then $\tau_{R'}(Y) = \{V, \phi, L_{R'}(Y), U_{R'}(Y), B_{R'}$(Y)} and its basis is given by $B_{R'}$={V, $L_{R'}(Y), B_{R'}$(Y)}. A function $f: (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ be β_N^* −continuous if and only if the inverse image of every member of $B_{R'}$ is β_N^* in U.

Proof : Let f be a β_N^* −continuous on U. Let B ∈ $B_{R'}$. Then B is Nano-open in V. That is, B ∈ $\tau_{R'}(Y)$. Since β_N^* -continuous, f^{-1}(B) ∈ $\tau_R(X)$. That is, inverse image of every member of $B_{R'}$ is β_N^* in U. Conversely, let inverse image of every member of $B_{R'}$ is β_N^* in U. Let G be a Nano-open in V. Then G = ∪ $\{B: B \in B_1\}$, where $B_1 \subseteq B_{R'}$. Then f^{-1}(G) = $f^{-1}(\cup \{B: B \in B_1\}) = \cup \{f^{-1}(B): B \in B_1\}$, where $f^{-1}(B)$ is β_N^* in U and hence their union, which is f^{-1}(G) is β_N^* in U. Thus f is β_N^* -continuous on U.

Theorem 5.29 A function $f: (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is β_N^*-continuous if and only if $f^{-1}(NInt\ (B)) \subseteq \beta_N^* Int(f^{-1}$(B)) for every subset B of $(V, \tau_{R'}(Y)$.

Proof : Let $f: (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ be β_N^* −continuous. By the given hypothesis B ⊆ V. Then, NInt(B) is Nano-open in V. As f is β_N^* −continuous, f^{-1}(NInt(B)) is β_N^*-open in U. Hence it follows that β_N^* $Int(f^{-1}$(NInt(B)))= f^{-1}(NInt(B)). Also, for B ⊆ V, NInt(B) ⊆ B always. Then f^{-1}(NInt(B)) ⊆ f^{-1}(B). Since f is β_N^* −continuous, it follows that β_N^* $Int(f^{-1}$(NInt(B))) ⊆ $\beta_N^* Int\, f^{-1}$(B), hence $f^{-1}(NInt\ B) \subseteq \beta_N^* Int(f^{-1}$(B)). Conversely, let $f^{-1}(NInt\ B) \subseteq \beta_N^* Int(f^{-1}$(B)) for every subset B of V. Let B be Nano-open in V and hence NInt(B)=B, Given $f^{-1}(NInt\ (B)) \subseteq \beta_N^* Int(f^{-1}$(B)), that is f^{-1}(B) ⊆ $\beta_N^* Int(f^{-1}$(B)). Also $\beta_N^* Int(f^{-1}$(B)) ⊆ f^{-1}(B). Hence it follows that f^{-1}(B) = $\beta_N^* Int(f^{-1}$(B)) which implies that f^{-1}(B) is β_N^* −open in U for every subset B of V. Therefore, $f: (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is β_N^* − continuous.

Theorem 5.30 A function $f: (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is β_N^* −continuous if and only if $\beta_N^* Cl(f^{-1}$(B)) ⊆ $f^{-1}(NCl\ (B))$ for every subset B of $(V, \tau_{R'}(Y)$.

Proof : Let B⊆ V and $f: (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ be β_N^* -continuous. Then NCl(B) is Nano-closed in $(V, \tau_{R'}(Y)$ and hence $f^{-1}(NCl\ (B))$ is β_N^*-closed in $(U, \tau_R(X))$. Therefore,

$\beta_N^* \, Cl(f^{-1}$(NCl(B)))= f^{-1}(NCl(B)). Since B⊆ $NCl(B)$, then f^{-1}(B) ⊆ f^{-1}(NCl(B)), that is $\beta_N^* \, Cl(f^{-1}$(B)) ⊆ $\beta_N^* \, Cl(f^{-1}$(NCl(B)))= f^{-1}(NCl(B)). Hence $\beta_N^* \, Cl(f^{-1}$(B)) ⊆ f^{-1}(NCl(B)). Conversely, let $\beta_N^* \, Cl(f^{-1}$(B)) ⊆ $f^{-1}(NCl\,(B))$ for every subset B ⊆ V. Now, let B be a Nano-closed set in $(V, \tau_{R'}(Y),$ then NCl(B)=B. By the given hypothesis, $\beta_N^* \, Cl(f^{-1}$(B)) ⊆ $f^{-1}(NCl\,(B))$ and hence $\beta_N^* \, Cl(f^{-1}$(B)) ⊆ $f^{-1}(B)$. But we also have $f^{-1}(B)$ ⊆ $\beta_N^* \, Cl(f^{-1}$(B)) and hence $\beta_N^* \, Cl(f^{-1}$(B)) = $f^{-1}(B)$. Thus $f^{-1}(B)$ is β_N^* -closed set in $(U, \tau_R(X))$ for every Nano-closed set B in $(V, \tau_{R'}(Y)$. Hence $f: (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is β_N^* −continuous.

Example 5.31 Let U={a,b,c,d} with U/R={{a},{b,c},{d}} and X={a,b}. Then $\tau_R(X)$ = {U, ∅ ,{a},{b,c},{a,b,c}} and hence the Nano-closed sets in U are {U, ∅ ,{d},{a,d},{b,c,d}}. The β_N^* − open sets are {U,∅,{a},{b},{c},{a,b},{a,c},{a,d},{b,c},{b,d},{c,d},{a,b,c},{a,b.d},{a,c,d}, {b,c,d}}. Let V={x,y,z,w} with V/R′={{x},{y,z,w}} and Y={x,y}. Then $\tau_{R'}(Y)$ = {V, ∅, {x}, {y,z,w}} and hence Nano-closed sets in V are {V, ∅, {x}, {y,z,w}}. Define : $(U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ as f(a)=y, f(b)=z, f(c)=x, f(d)=w. Then f is β_N^* -continuous on U, since inverse image of every Nano-open set in V is β_N^* −open in U . Let B = {x,z}⊂ V. Then $\beta_N^* \, Cl(f^{-1}(B))$ = $\beta_N^* \, Cl(f^{-1}(x,z))$ = $\beta_N^* \, Cl(\{b,c\})$ = {b,c} and $f^{-1}(NCl(B)) = f^{-1}(NCl(\{x,z\})) = f^{-1}(V) = U$. Thus, $\beta_N^* \, Cl(f^{-1}(B)) \neq f^{-1}(NCl(B))$. Also $f^{-1}(NInt\,B) = f^{-1}(NInt\{x,z\}) = f^{-1}(z) = \{a\}$and $\beta_N^* \, Int(f^{-1}(B)) = \beta_N^* \, Int(f^{-1}(\{x,z\})) = \beta_N^* \, Int(\{a,b\}) = \{a,b\}$. That is, $f^{-1}(NInt\,B) \neq \beta_N^* \, Int(f^{-1}(B))$. Thus, equality does not hold in theorem 5.29 and theorem 5.30 when f is β_N^* -continuous.

Theorem 5.32 Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y)$ be two Nano Topological space with respect to X ⊆ U and Y ⊆ V respectively. Then for any function $f: (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$, the following are equivalent

(i) f is β_N^* −continuous.
(ii) The inverse image of every Nano-closed set in V is β_N^* −closed in $(U, \tau_R(X))$.
(iii) $f(\beta_N^*$ Cl(A)) ⊆ NCl(f(A)) for every subset A of $(U, \tau_R(X))$.
(iv) The inverse image of every member of $B_{R'}$ is β_N^* − open in $(U, \tau_R(X))$.
(v) $f^{-1}(NInt\,(B))$ ⊆ $\beta_N^* \, Int(f^{-1}$(B)) for every subset B of $(V, \tau_{R'}(Y)$.
(vi) $\beta_N^* \, Cl(f^{-1}$(B)) ⊆ $f^{-1}(NCl\,(B))$ for every subset B of $(V, \tau_{R'}(Y)$.

Proof : The proof of this theorem follows from 3.2.24 to 3.2.30.

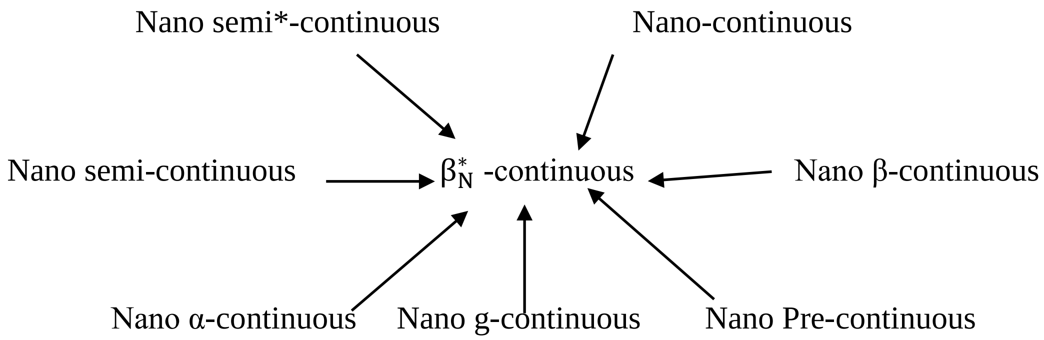
Theorem 5.33 If a map $f: (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ be β_N^* −continuous and $g: (V, \tau_{R'}(Y) \to (W, \tau_{R''}(Z))$ is Nano-continuous, then (g∘ f) is β_N^* -continuous.

Proof : Let G be Nano-open set in W. Since g is Nano-continuous g^{-1}(G) is Nano-open in V and we know that f is β_N^* -continuous then, $(g \circ f)^{-1}\,(G) = f^{-1}\,(g^{-1}(G))$ is β_N^* -open in U. Therefore, $(g \circ f)$ is β_N^* -continuous.

Remark 5.34 Composition of two β_N^* -continuous maps need not be β_N^* −continuous maps.

Example 5.35 Let U={a,b,c,d} with U/R={{a},{b,c},{d}} and X={a,b}. Then $\tau_R(X)$={U, ϕ,{a}, {b,c},{a,b,c}}. Then β_N^* -open sets are { U, ϕ,{a},{b},{c},{a,b},{a,c},{a,d},{b,c},{b,d},{c,d},{a,b,c},{a,b,d},{a,c,d},{b,c,d}}. Let V={x.y.z,w} with V/R'= {{x},{y,w,z}} and Y={x,y}. Then $\tau_{R'}(Y))$ ={V, ϕ ,{x},{y},{z},{w},{x,y},{x.z},{x,w}, {y,z},{y,w},{z,w}, {x,y,z},{x,y,w},{x,z,w},{y,z,w}}. Define $f\colon \big(U,\tau_R(X)\big) \to (V,\tau_{R'}(Y))$ as f(a)=y, f(b)=z, f(c)=x, f(d)=w. Then f^{-1}({x})={c}, f^{-1}({y,z,w})={a,b,d} and f^{-1}(V)=U. That is, the inverse image of every Nano-open set in V is the β_N^*-open set in U. Therefore f is β_N^*-continuous. Let W={a,b,c,d} with W/R''={{a},{b,d},{c}} and Z={a,b}. Then $\tau_{R''}(Z)$={W, ϕ,{a}, {b,d},{a,b,d}}. Define $g\colon \big(V,\tau_{R'}(Y)\big) \to (W,\tau_{R''}(Z))$ as g(x)=b, g(y)=c, g(z)=d, g(w)=a. Then g^{-1}({b,d})={x,z} g^{-1}({a}) = {w}, g^{-1}({a,b,d})={x,z,w} and f^{-1}(W)=V. That is, the inverse image of every Nano-open set in W is the β_N^*-open set in V. Therefore f is β_N^*-continuous. But $(g\circ f)^{-1}(\{a\}) = f^{-1}\big(g^{-1}(\{a\})\big) = f^{-1}(\{w\}) = \{d\}$ is not β_N^* −open in U. Therefore, Composition of two β_N^* −continuous maps need not be β_N^*-continuous maps.

Diagram 5.36 The following diagram shows the relationship between β_N^*-continuous and other Nano- continuous that are studied in this section.



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