IJISET - International Journal of Innovative Science, Engineering & Technology, Vol. 8 Issue 2, February 2021
ISSN (Online) 2348 – 7968 | Impact Factor (2020) – 6.72
www.ijiset.com

A Fixed Point Theorem for Generalized *F*-Suzuki-type Multivalued Contraction in Metric Spaces

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Abstract

In this paper, we prove a new fixed point theorem for generalized *F*-Suzuki-rational-type multivalued contraction in the setting of complete metric space. The result in this paper is extension of the Banach contraction principle, Suzuki contraction theorem and Wordowski fixed point theorem.

MSC: 47H10; 54H25.

Keywords: *F*-contraction, *F*-Suzuki contraction, generalized *F*-Suzuki contraction.

1. Introduction and Preliminaries.

In 1922, Banach [10] proved a very important result regarding a contraction mapping, known as the Banach contraction principle. It is one of the fundamental result in fixed point theory. Due to its importance and simplicity, several authors have obtained many interesting extensions of the Banach contraction principle (see [1,2,4,8,14,17]).

The fixed point theory of multivalued contraction mappings using the Hausdorff metric was initiated by Nadler [20], who extended the Banach contraction principle to multivalued mappings. Since then many authors have studied fixed points for multivalued mappings (see [3,9,13,18]).

In 2012, Wardowski [25] introduced a new concept named *F*-contraction and proved a fixed point theorem which generalizes the Banach contraction principle. Afterwards, Abbas et al. [2] extended the concept of *F*-contraction mapping and obtain fixed point of a generalized nonexpansive mappings on star shaped subsets of normed linear spaces. Minak et al. [19] proved some fixed point results for Ciric-type generalized *F*-contractions on complete metric spaces. Recently, Cosentino and Vetro [15] followed the approach of *F*-contraction and obtained some fixed point theorems of Hardy-Rogers-type for self-mappings in complete metric spaces and complete ordered metric space. Then Sgroi and Vetro [23] extended this Hardy-Rogers-type fixed point result for multivalued mappings. The reader can see [5,6,7,11,12,16,21,22,26] for recent results in this direction.

Now we recall some basic known definitions and results which will be used in the sequel. Throughout this paper, $\mathbb{N}, \mathbb{R}^+, \mathbb{R}, CB(X)$ denote the set of natural numbers, the set of positive real numbers, the set of real numbers and the family of non-empty closed bounded subsets of X respectively, where X be a non-empty set.

To be consistent with Wardowski [25], let $\mathcal{F}$ be the collection of all mappings $F: \mathbb{R}^+ \to \mathbb{R}$ which satisfy the following conditions:

(F1) F is strictly increasing, that is, for all $\alpha, \beta \in \mathbb{R}^+$ such that $\alpha < \beta \Rightarrow F(\alpha) < F(\beta)$;
(F2) For every sequence $\{\alpha_n\}_{n\in\mathbb{N}}$ of positive numbers $\lim_{n\to\infty} \alpha_n = 0$ if and only if $\lim_{n\to\infty} F(\alpha_n) = -\infty$;
(F3) There exists $k \in (0,1)$ such that $\lim_{n\to 0^+} \alpha^k F(\alpha) = 0$.

Wardowski [25] introduced the following concept of F-contraction mappings.

Definition 1.1[25]. Let (X, d) be a metric space. A self map T on X is said to be an F-contraction on X if there exists $\tau > 0$ such that

$$d(Tx, Ty) > 0 \Rightarrow \tau + F\big(d(Tx, Ty)\big) \leq F\big(d(x, y)\big) \qquad (1.1)$$

for all $x, y \in X$, where $F \in \mathcal{F}$.

Remark 1.2 [25]. Every F-contraction mapping is continuous.
Example 1.3 [25]. Let $F: \mathbb{R}^+ \to \mathbb{R}$ be defined by $F(\alpha) = \ln \alpha$. It is clear that F satisfies (F1)-(F3) for any $k \in (0,1)$. Each mapping $T: X \to X$ satisfying (1.1) is an F-contraction such that

$$d(Tx, Ty) \leq e^{-\tau} d(x, y) \text{ for all } x, y \in X,\ Tx \neq Ty.$$

It is clear that for any $x, y \in X$ such that $Tx = Ty$, the inequality $d(Tx, Ty) \leq e^{-\tau} d(x, y)$ also holds, i.e., T is a Banach contraction.

Wardowski [25] stated a modified version of the Banach contraction principle as follows.
Theorem 1.4[25]. Let (X, d) be a complete metric space and let $T: X \to X$ be an F-contraction. Then T has a unique fixed point $z \in X$ and for every $x \in X$ the sequence $\{T^n x\}_{n\in\mathbb{N}}$ converges to z.

Cosintino and Vetro [15] proved the following Hardy-Rogers-type fixed point theorem for F-contractive condition in the setting of complete metric spaces.

Theorem 1.5[15]. Let (X, d) be a complete metric space and let $T: X \to X$ be a self mapping. If there exists $\tau > 0$ and reals $\alpha, \beta, \gamma, \delta, L \geq 0$ such that for all $x, y \in X$,

$$d(Tx, Ty) > 0 \Rightarrow \tau + F\big(\alpha d(x, y) + \beta d(x, Tx) + \gamma d(y, Ty) + \delta d(x, Ty) + L d(y, Tx)\big) \quad (1.2)$$

where $F \in \mathcal{F}$ and $\alpha + \beta + \gamma + 2\delta = 1, \gamma \neq 1, and\ L \geq 0$, then T has a unique fixed point.

Sgroi and Vetro [23] proved the following result to obtain fixed point of multivalued mappings as a generalization of Nadler's Theorem [20].

Theorem 1.6[23]. Let (X, d) be a complete metric space and $T: X \to CB(X)$ be a multivalued mapping. Assume that there exists an $F \in \mathcal{F}$ and $\tau \in \mathbb{R}^+$ such that

$$2\tau + F\big(H(Tx, Ty)\big) \leq F\big(\alpha d(x, y) + \beta d(x, Tx) + \gamma d(y, Ty) + \delta d(x, Ty) + L d(y, Tx)\big)$$

for all $x, y \in X$, $Tx \neq Ty$, where $\alpha, \beta, \gamma, \delta, L \geq 0$, $\alpha + \beta + \gamma + 2\delta = 1, \gamma \neq 1$. Then T has a fixed point.

In [21], Piri and Kumam introduced the notion of F-Suzuki contraction as follows.
Definition 1.7[21]. Let (X, d) be a complete metric space. A mapping $T: X \to X$ is said to be an F-Suzuki contraction if there exists $\tau > 0$ such that for all $, y \in X$, $Tx \neq Ty$,

$$\tfrac{1}{2} d(x, Tx) < d(x, y) \Rightarrow \tau + F\big(d(Tx, Ty)\big) \leq F\big(d(x, y)\big) \qquad (1.3)$$

where $F \in \mathcal{F}$.

They also proved the following theorem which is a modified version of the Suzuki fixed point theorem [24].

Theorem 1.8[21]. Let (X, d) be a complete metric space and let $T: X \to X$ be an F-Suzuki contraction. Then T has a unique fixed point x^* in X and for each $x \in X$, the sequence $\{T^n x\}_{n\in\mathbb{N}}$ converges to x^*.

The purpose of this paper is to study the existence problem of fixed points for generalized F-Suzuki-rational-type contraction for multivalued mappings in the setting of complete metric spaces. The result presented in the paper extend and improve the Banach contraction principle [10], Suzuki contraction theorem [24], and the corresponding main results in Wardowski [25] and Piri and Kumam [21] for multivalued mapping.

2. Main Result

Theorem 2.1. Let (X, d) be a metric space and $T: X \to CB(X)$ be a multivalued mapping. Assume that there exists a function $F \in \mathcal{F}$ which is continuous from right and $\tau \in \mathbb{R}^+$ such that

$$\lambda d(x, Tx) \leq d(x, y) \tag{2.1}$$

implies

$$2\tau + F\big(H(Tx, Ty)\big) \leq F\left(\alpha d(x, y) + \beta \frac{d(y,Ty)[1+d(x,Tx)]}{1+d(x,y)} + \gamma \frac{d(y,Tx)[1+d(x,Ty)]}{1+d(x,y)}\right) \tag{2.2}$$

for all $x, y \in X$, $Tx \neq Ty$ where α, β, γ are non-negative numbers and $\alpha + \beta = 1, \beta \neq 1$. Here $\frac{1-\gamma}{1+\alpha-\gamma} = \lambda < 1$. Then T has a fixed point.

Proof. Let $x_0 \in X$ be an arbitrary point of X and choose $x_1 \in Tx_0$. If $x_1 \in Tx_1$, then x_1 is a fixed point of T and the proof is complete.
Assume that $x_1 \notin Tx_1$, then $Tx_0 \neq Tx_1$.
Now, $\lambda d(x_0, Tx_0) \leq \lambda d(x_0, x_1) < d(x_0, x_1)$.
From the assumption, we have

$$\begin{aligned} 2\tau + F\big(H(Tx_0, Tx_1)\big) &\leq F\left(\alpha d(x_0, x_1) + \beta \frac{d(x_1,Tx_1)[1+d(x_0,Tx_0)]}{1+d(x_0,x_1)} + \gamma \frac{d(x_1,Tx_0)[1+d(x_0,Tx_1)]}{1+d(x_0,x_1)}\right) \\ &\leq F\left(\alpha d(x_0, x_1) + \beta \frac{d(x_1,Tx_1)[1+d(x_0,x_1)]}{1+d(x_0,x_1)} + \gamma \frac{d(x_1,x_1)[1+d(x_0,Tx_1)]}{1+d(x_0,x_1)}\right) \\ &= F\big(\alpha d(x_0, x_1) + \beta d(x_1, Tx_1)\big) \end{aligned}$$

As F is continuous from right, there exists a real number $h \geq 1$ such that

$$F\big(hH(Tx_0, Tx_1)\big) \leq F\big(H(Tx_0, Tx_1)\big) + \tau$$

Now from

$$d(x_1, Tx_1) \leq H(Tx_0, Tx_1) < hH(Tx_0, Tx_1)$$

we deduce that there exists $x_2 \in Tx_1$ such that

$$d(x_1, x_2) \leq H(Tx_0, Tx_1)$$

Consequently, we have

$$F\big(d(x_1, x_2)\big) \leq F\big(hH(Tx_0, Tx_1)\big) < F\big(H(Tx_0, Tx_1)\big) + \tau$$

which implies that

$$2\tau + F\big(d(x_1,x_2)\big) \le 2\tau + F\big(H(Tx_0,Tx_1)\big) + \tau$$

$$\le F\left(\alpha d(x_0,x_1) + \beta\frac{d(x_1,Tx_1)[1+d(x_0,Tx_0)]}{1+d(x_0,x_1)} + \gamma\frac{d(x_1,Tx_0)[1+d(x_0,Tx_1)]}{1+d(x_0,x_1)}\right) + \tau$$

thus

$$\tau + F\big(d(x_1,x_2)\big) \le F\big(\alpha d(x_0,x_1) + \beta d(x_1,x_2)\big) \tag{2.3}$$

Since F is strictly increasing, we deduce

$$d(x_1,x_2) < \alpha d(x_0,x_1) + \beta d(x_1,x_2)$$

and hence

$$d(x_1,x_2) < \frac{\alpha}{1-\beta} d(x_0,x_1) = d(x_0,x_1).$$

Consequently from (2.3), we have

$$\tau + F\big(d(x_1,x_2)\big) < F\big(d(x_0,x_1)\big)$$

Continuing in this way, we can define a sequence $\{x_n\} \subset X$ such that $x_n \notin Tx_n, x_{n+1} \in Tx_n$ and

$$\tau + F\big(d(Tx_{n-1},Tx_n)\big) < F\big(d(x_{n-1},x_n)\big) \tag{2.4}$$

for all $n \in \mathbb{N} \cup \{0\}$. Therefore,

$$F\big(d(x_n,x_{n+1})\big) < F\big(d(x_{n-1},x_n)\big) - \tau < F\big(d(x_{n-2},x_{n-1})\big) - 2\tau < \cdots \le F\big(d(x_0,x_1)\big) - n\tau \tag{2.5}$$

for all $n \in \mathbb{N}$. Since $F \in \mathcal{F}$, so by taking limit as $n \to \infty$ in (2.5), we have

$$\lim_{n\to\infty}\left[F\big(d(x_n,x_{n+1})\big)\right]^{\lambda} = -\infty \Leftrightarrow \lim_{n\to\infty} d(x_n,x_{n+1}) = 0 \tag{2.6}$$

Now from (F3), there exists $0 < \lambda < 1$ such that

$$\lim_{n\to\infty}[d(x_n,x_{n+1})]^{\lambda}\ F\big(d(x_n,x_{n+1})\big) = 0 \tag{2.7}$$

By (2.5), we have

$$[d(x_n,x_{n+1})]^{\lambda}F\big(d(x_n,x_{n+1})\big) - [d(x_n,x_{n+1})]^{\lambda}F\big(d(x_0,x_1)\big)$$
$$< [d(x_n,x_{n+1})]^{\lambda}\left[F(d(x_0,x_1) - n\tau) - F\big(d(x_0,x_1)\big)\right]$$
$$= -n\tau[d(x_n,x_{n+1})]^{\lambda} \le 0 \tag{2.8}$$

By taking limit as $n \to \infty$ in (2.8) and applying (2.6) and (2.7), we have

$$\lim_{n\to\infty} n[d(x_n,x_{n+1})]^{\lambda} = 0 \tag{2.9}$$

It follows from (2.9) that there exists $n_1 \in \mathbb{N}$ such that

$$n[d(x_n,x_{n+1})]^{\lambda} \le 1 \tag{2.10}$$

for all $n > n_1$. This implies

$$d(x_n,x_{n+1}) \le \frac{1}{n^{1/\lambda}} \tag{2.11}$$

for all $n > n_1$.

Now we prove that $\{x_n\}$ is a Cauchy sequence.

For $> n > n_1$, we have

$$d(x_n, x_m) \leq \sum_{i=n}^{m-1} d(x_i, x_{i+1}) \leq \sum_{i=n}^{m-1} \frac{1}{i^{1/\lambda}} \tag{2.12}$$

Since $0 < \lambda < 1$, then $\sum_{i=1}^{\infty} \frac{1}{i^{1/\lambda}}$ converges. Therefore, $d(x_n, x_m) \to 0$ as $m, n \to \infty$. Thus $\{x_n\}$ is a Cauchy sequence of X ensures that there exists $z \in X$ such that $x_n \to z$ as $n \to \infty$. If there exists an increasing sequence $\{n_k\} \subset \mathbb{N}$ such that $x_{n_k} \in Tz$ for all $k \in \mathbb{N}$. Since Tz is closed and $x_{n_k} \to z$, we get $z \in Tz$ and the proof is completed. So we may assume that there exists $n_0 \in \mathbb{N}$ such that $x_n \notin Tz$ for all $n_0 \in \mathbb{N}$ with $n \geq n_0$. Then we assume that $Tx_{n-1} \neq Tz$ for all $n \geq n_0$.

Now we show that

$$\lambda(z, Tx) \leq d(z, x)$$

for all $x \in X/\{z\}$.

Since $x_n \to z$, so there exists $n_0 \in \mathbb{N}$ such that

$$d(z, x_n) \leq \frac{1}{3} d(z, x)$$

for all $n \in \mathbb{N}$ with $n \geq n_0$. Then we have

$$\begin{aligned} \lambda d(x_n, Tx_n) < d(x_n, Tx_n) &\leq d(x_n, x_{n+1}) \\ &\leq d(x_n, z) + d(z, x_{n+1}) \\ &\leq \tfrac{2}{3} d(x, z) = d(x, z) - \tfrac{1}{3} d(x, z) \\ &\leq d(x, z) - d(z, x_n) \leq d(x, x_n). \end{aligned}$$

Thus, by assumption, we get

$$\begin{aligned} 2\tau + F\big(H(Tx, Tx_n)\big) &\leq F\left(\alpha d(x, x_n) + \beta \frac{d(x_n, Tx_n)[1 + d(x, Tx)]}{1 + d(x_n, x)} + \gamma \frac{d(x_n, Tx)[1 + d(x, Tx_n)]}{1 + d(x_n, x)}\right) \\ &\leq F\left(\alpha d(x, x_n) + \beta \frac{d(x_n, x_{n+1})[1 + d(x, Tx)]}{1 + d(x_n, x)} + \gamma \frac{d(x_n, Tx)[1 + d(x, x_{n+1})]}{1 + d(x_n, x)}\right) \end{aligned}$$

As F is continuous from right, so there exists a real number $h > 1$ such that

$$F\big(hH(Tx, Tx_n)\big) < F\big(H(Tx, Tx_n)\big) + \tau$$

Now from

$$d(Tx, Tx_{n+1}) \leq H(Tx, Tx_n) < hH(Tx, Tx_n)$$

We get

$$F\big(d(Tx, Tx_{n+1})\big) \leq F\big(hH(Tx, Tx_n)\big) < F\big(H(Tx, Tx_n)\big) + \tau$$

Thus, we have

$$\begin{aligned} 2\tau + F\big(d(Tx, Tx_{n+1})\big) &\leq 2\tau + F\big(H(Tx, Tx_n)\big) + \tau \\ &\leq F\left(\alpha d(x, x_n) + \beta \frac{d(x_n, x_{n+1})[1 + d(x, Tx)]}{1 + d(x_n, x)} + \gamma \frac{d(x_n, Tx)[1 + d(x, x_{n+1})]}{1 + d(x_n, x)}\right) + \tau \end{aligned}$$

IJISET - International Journal of Innovative Science, Engineering & Technology, Vol. 8 Issue 2, February 2021
ISSN (Online) 2348 – 7968 | Impact Factor (2020) – 6.72
www.ijiset.com

Since F is strictly increasing, we have

$$d(Tx, Tx_{n+1}) < \alpha d(x, x_n) + \beta \frac{d(x_n, x_{n+1})[1 + d(x, Tx)]}{1 + d(x_n, x)} + \gamma \frac{d(x_n, Tx)[1 + d(x, x_{n+1})]}{1 + d(x_n, x)}$$

Letting $n \to \infty$, we obtain

$$d(Tx, z) \leq \alpha d(x, z) + \gamma d(z, Tx) \leq \frac{\alpha}{1-\gamma} d(z, x)$$

for all $x \in X\{z\}$.

We prove that

$$2\tau + F(H(Tz, Tx)) \leq F\left(\alpha d(x, z) + \beta \frac{d(z,Tz)[1+d(x,Tx)]}{1+d(x,z)} + \gamma \frac{d(z,Tx)[1+d(x,Tz)]}{1+d(x,z)}\right)$$

for all $x \in X$.

Then for every $n \in \mathbb{N}$ there exists $y_n \in Tx$ such that

$$d(z, y_n) \leq d(z, Tx) + \frac{1}{n} d(z, x)$$

So, we have the following

$$\begin{aligned} d(x, Tx) &\leq d(x, y_n) \\ &\leq d(x, z) + d(z, y_n) \\ &\leq d(x, z) + d(z, Tx) + \frac{1}{n} d(z, x) \\ &\leq d(x, z) + \frac{\alpha}{1-\gamma} d(z, x) + \frac{1}{n} d(z, x) \\ &= \left(1 + \frac{\alpha}{1-\gamma} + \frac{1}{n}\right) d(x, z) \end{aligned}$$

for all $n \in \mathbb{N}$ and hence $\lambda d(x, Tx) \leq d(x, z)$.

Thus, by assumption, we get

$$2\tau + F(H(Tz, Tx)) \leq F\left(\alpha d(x, z) + \beta \frac{d(z, Tz)[1 + d(x, Tx)]}{1 + d(x, z)} + \gamma \frac{d(z, Tx)[1 + d(x, Tz)]}{1 + d(x, z)}\right)$$

Taking $x = x_n$, we have

$$\begin{aligned} 2\tau + F\big(d(x_{n+1}, Tz)\big) &\leq 2\tau + F\big(H(Tx_n, Tz)\big) \\ &\leq F\left(\alpha d(x_n, z) + \beta \frac{d(z,Tz)[1+d(x_n,Tx_n)]}{1+d(x_n,z)} + \gamma \frac{d(z,Tx_n)[1+d(x_n,Tz)]}{1+d(x_n,z)}\right) \end{aligned}$$

Since F is strictly increasing, we have

$$d(x_{n+1}, Tz) < \alpha d(x_n, z) + \beta \frac{d(z,Tz)[1+d(x_n,Tx_n)]}{1+d(x_n,z)} + \gamma \frac{d(z,Tx_n)[1+d(x_n,Tz)]}{1+d(x_n,z)}$$

Letting $n \to \infty$, we obtain

$$d(z, Tz) \leq \beta d(z, Tz)$$

as $\beta < 1$. Thus, we get $d(z, Tz) = 0$. Since Tz is closed, we obtain $z \in Tz$. Thus z is a fixed point of T.

References

[1] Abbas, M. and Nazir, T., *Fixed point of generalized weakly contractive mappings in ordered partial metric spaces,* Fixed point Theory Appl., **Vol.2012** Article No 1, 19 pages.

[2] Abbas, M., Ali, B and Romaguera, S., *Fixed and periodic points of generalized contractions in metric spaces*, Fixed Point Theory & Appl., **2013** (2013), 1-11.

[3] Acar, O., and Altun, I., *Fixed point theorem for multi-valued mappings with δ −distance*, Abstract and Appl. Anal., **2014**(2014), Article ID 497092.

[4] Agarwal, R. P. , O'Regan, D. and Shahzad, N., *Fixed point theorems for generalized contractive maps of Meir-Keeler type*, Math. Nachriten**, 276**(2004), 3-12.

[5] Altun, I., G. Minak and H. Dag, *Multivalued F-contractions on complete metric spaces*, J. Nonlinear Convex Anal., **16(4)(**2015), 659-666.

[6] Arshad, M., Abbas, M., Hussain, A. and Hussain, N., *Generalized dynamic process for generalized (f, L)-almost F-contraction with Applications*, J. Nonlinear Sci. Appl., **9**(2016), 1702-1715.

[7] Arshad, M., Ameer, E. and Hussain, A., *Hardy-Rogers type fixed point theorems for α −GF-contractions*, Archivum Mathematicum(BRNO) , Tomus **15**(2015), 129-141.

[8] Azam, A. and Shakeel, M., *Weakly contractive maps and common fixed points*, Math. Vasnik, **Vol. 60(2)** (2008), 101-106.

[9] Bae, J. S., *Fixed point theorem for weakly contractive multi-valued maps,* J. Math. Anal . Appl., **Vol. 284** (2003), 690-697.

[10] Banach, B., *Sur les oprations dans les ensembles abstraits et leurs applications auxquations intgrales*, Fund Math., **3**(1922) 133-181.

[11] Batra, R., Vashistha, S. and Kumar, R., *A coincidence point theorem for F-contractions on metric spaces equipped with an altered distance*, J. Math. Comput. Sci., **4(5)(**2014), 826-833.

[12] Batra, R., and Vashistha, S., *Fixed points of an F-contraction on metric spaces with a graph*, Int. J. Comput. Math., **91**(2014), 1-8.

[13] Beg, I., and Azam, A., *Fixed points of asymptotically regular multi-valued mappings*, Bull. Aust. Math. Soc., **53**(1992), 313-326.

[14] Ciric, Lj. B., *A generalization of Banach's contraction principle*, Proc. Amer. Math. Soc., **45**(1974), 267-273.

[15] Cosentino, M. and Vetro, P., *Fixed point results for F-contractive mappings of Hardy-Rogers Type*, Filomat, **28:4**(2014), 715-722.

[16] Dung, N. V. and Hang, V. L., *A fixed point theorem for generalized F-contraction on complete metric spaces*, Vietnam J. Math., **43**(2015), 743-753.

[17] Fisher, B., *Common Fixed point of mappings*, Indian J. pure. Appl. Math, **Vol. 20** (1978), 2-16.

[18] Jleli, M., Kumar, N., Samet, B. and Vetro, C., *On multivalued weakly Picard operators in Partial Hausdorff metric spaces*, Fixed Point Theory Appl., **2015**(2015), 5 pages.

[19] Minak, G., Halvaci, A., and Altun, I., *Ciric type genrealized F-contractions on complete metric spaces and fixed point results*, Filomat, **28(6**)(2014), 1143-1151.

[20] Nadler, S. B., *Multivalued contraction mappings*, Pacific J. Math. **Vol. 30** (1969), 475-488.

[21] Piri, H., and Kumam, P., *Some fixed point theorem concerning F-contraction in complete metric spaces*, Fixed Point Theory and Appl., **2014**, 210(2014).

[22] Secelean, N. A., *Iterated function systems consisting of F-contractions*, Fixed Point Theory Appl., **2013**(2013), 277.

[23] Sgroi, M. and Vetro, C., *Multivalued F-contractions and the solution of certain functional and Integral equations*, Filomat, **27(7**)(2013), 1259-1268.

[24] Suzuki, T., *A generalized Banach contraction principle that characterizes metric completeness*, Proc. Amer. Math. Soc., **Vol. 136 (5)** (2008), 1861-1869.

[25] Wardowski, D., *Fixed points of a new type of contractive mappings in complete metric spaces*, Fixed Point Theory Appl., **2012**, 94(2012).

[26] Wardowski, D., Van Dung N., *Fixed points of F-weak contractions on complete metric space*, Demonstr. Math., **XLVII(1)** (2014), 146-155.