IJISET - International Journal of Innovative Science, Engineering & Technology, Vol. 8 Issue 5, May 2021
ISSN (Online) 2348 – 7968 | Impact Factor (2020) – 6.72
www.ijiset.com

Vertex Neighborhood Signed Graphs

Dr. Gavirangaiah K
Assistant Professor, Dept. of Mathematics, Government First Grade College, Tumkur
Email: gavirangayya@gmail.com

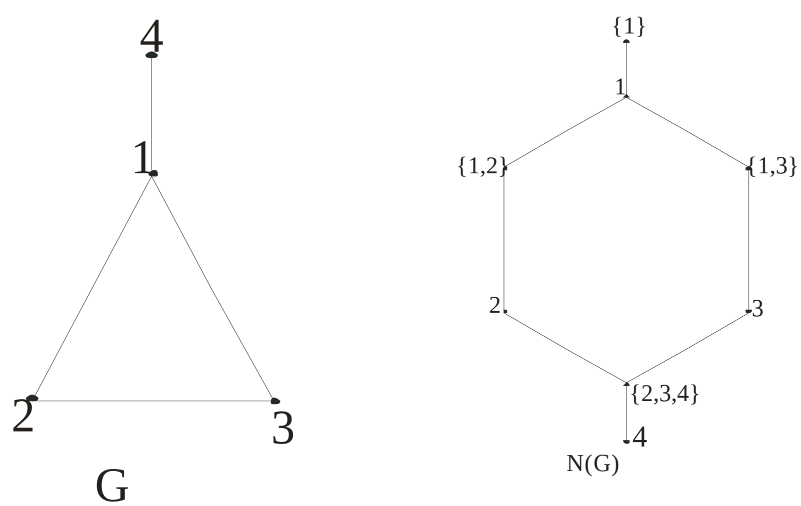
Abstract: We defined the new notion called vertex neighborhood signed graph and studied some properties, also we have discussed the switching equivalence characterizations for vertex neighborhood signed graph. Further we have presented the structural characterization vertex neighborhood signed graph.

Keywords: Neighborhood Graphs; Vertex Neighborhood Signed Graphs

1. INTRODUCTION

Suppose G= (V, E) be a graph with p vertices and q edges. The set $N(u) = \{v \in V : uv \in E\}$ is called the open neighborhood set of a vertex u of G and S be the set of all open neighborhood sets of the vertices of G. Let $D \subseteq V$ and every vertex in VD is adjacent to some vertex in D, and then D is called a dominating set of G. Any vertex $v \in V, if\ d - \{v\}$ is not a dominating set then D is said to be minimal and S_1 be the set of all minimal dominating sets of G. Let $G = (V, E)$ be any graph, $D(G)$ be the dominating graph with $V \cup S_1$ as vertex set and vertices p and q in dominating graph are adjacent if and only if q is a minimal dominating set in G containing p. (See [1])

In [2,3], the author introduced the new notion called neighborhood graph of a graph by the motivation of dominating graph. Let $G = (V, E)$ be any graph, the graph $N(G)$ is called neighborhood graph with vertex set $V \cup S$ and any two vertices p and q in neighborhood graph are adjacent if and only if q is an open neighborhood set containing p. The above figure illustrates the concept of neighborhood graph.



From the definition of neighborhood graph of a graph, we can clearly observe that $N(G)$ is always bipartite. However, $N(G)$ is disconnected, if G is bipartite. In [1], the author remarked that: Let $G = (V, E)$with $|V| =$

$m, |E| = n$ and for each $v \in V, \deg(v) \geq 1$,then neighborhood graph of G contains $V(N(G)) = 2m \text{ and } E(N(G)) = 2n$. Also, the neighborhood graph of any tree T with at least two vertices is 2T. The neighborhood graphs any complete bipartite graph $K_{m,n}$ is $2K_{m,n}$.

Let $G = (V, E)$ be any graph with v is an end vertex, then the corresponding vertices v and $N(v)$ in neighborhood graph are also end vertices. The neighborhood graph of graph is a complete graph is a complete graph if and only if G contains only one vertex. Let $G = (V, E)$ be a one component graph with $|V| \geq 2$, then the neighborhood graph of one such graph will not a complete. Suppose $G = (V, E)$ be any graph with each vertex $v \in V, \deg(v) = k$, then the corresponding neighborhood graph will have the same property (i.e each vertex $p \in V(N(G)), \deg(p) = k$). If G is connected and each vertex of G has even degree, then neighborhood graph of one such graph is not Eulerian. For example, cycle with four vertices is Eulerian but its neighborhood graph is $2C_4$ which is not Eulerian. In [1], the author characteristics the neighborhood graphs which are Eulerian.

2. VERTEX NEIGHBORHOOD SIGNED GRAPHS

By the motivation of neighborhood graph introduced by Kulli [1, 3, 10], in this section we defined the new notion called vertex neighborhood signed graph of signed graph as: the vertex neighborhood signed graph $VND(S) = (N(G), \sigma')$ of a signed graph $S = (G, \sigma)$ is a signed graph, the sign of any edge $pq \in E(VND(S))$ is the product of canonical marking of the vertices p and q. If any signed graph S is isomorphic to vertex neighborhood signed graph of some signed S' ($i.e., VND(S) \cong S'$), then S is called a vertex neighborhood signed graph.

In general, signed graphs can be portioned into groups as: positive signed graphs (i.e., balanced signed graphs) and negative signed graphs (i.e., unbalanced signed graphs). Given signed graph $S = (G, \sigma)$ is either positive or negative, the vertex neighborhood signed graph is always positive.

THEOREM 2.2.1.

$The\ vertex\ neighborhood\ signed\ graph\ VND(S) is\ obtained, for\ any\ signed\ graph$

$\sum = (\tau, \sigma)$.

$Proof$: Let $S = (G, \sigma)$ be any signed graph and S_c is a signed marked graph subsequently employ the canonical marking. Through the Elucidation of vertex neighborhood signed graph $VND(S)$, we examined in order that the sign of any edge uv in $VND(S) is\ \sigma(uv) = \zeta(u)\zeta(v)$. From Theorem 1.1.2, it follows that vertex neighborhood signed graph $VND(S)$ is balanced.

Consider the Z^+ and $k \in Z^+$, the k^{th} iterated vertex neighborhood signed graph $VND(S)$ of S is defined as follows:

$$VND^0(S) = S, VND^k(S) = VND(VND^{k-1}(S)).$$

COROLLARY 2.2.2. The k^{th} iterated vertex neighborhood signed graph $VND^k(S)$ is always positive, for any signed graph $\sum = (\tau, \sigma)$.

THEOREM 2.2.3. *The vertex neighborhood signed graphs of* $S_1 = (G_1, \sigma)$ *and* $S_2 = (G_2, \sigma)$ *are switching equivalent* $\left(i.e., VND(S_1) \sim VND(S_2)\right)$*, if* G_1 *and* G_2 *are isomorphic.*

Proof: Consider two signed graphs S_1 *and* S_2 with their underlying graphs are isomorphic. Thereupon, the corresponding the vertex neighborhood signed graphs $VND(S_1)$ *and* $VND(S_2)$ are positive. From Theorem 1.1.3, it follows that $VND(S_1)$ and $VND(S_2)$ are switching equivalent. In [1, 5], the author proved that: If T is a tree with $V(T) \geq 2$, *then* $N(T) \cong 2T$. In view of this, we have the following:

THEOREM 2.2.4. Let $S = (G, \sigma)$ be any signed graph. Then $VND(G) \sim 2S$ if and only if G is a tree with at least two vertices.

Proof: Suppose $VND(S) \sim 2S$. Then $VND(G) \cong 2G$, form the above observation we have G is a tree with at least two vertices.

Conversely, suppose that G is a tree with at least two vertices. Consider a signed graph with underlying graph as tree with at least two vertices. Then the corresponding vertex neighborhood signed graph is positive. Since G is a tree with at least two vertices, then $N(G) \cong 2G$. From the structural characterization of Harary (Theorem 1.1.1), every graph with underlying graph as 2T is always positive. Hence, from Theorem 1.1.3, it follows that $VND(S)$ and $2S$ are cycle isomorphic.

COROLLARY 2.2.5. *Let* $S = (G, \sigma)$ *be any signed graph. Then* $VND(S) \sim 2S$*, if* G *is any path with at least two vertices.*

COROLLARY 2.2.6. *Let* $S = (G, \sigma)$ *be any signed graph. Then* $VND(S) \sim 2S$*, if* G *is any complete bipartite graph with one vertex,* n *vertices, where* $n \geq 1$.

COROLLARY 2.2.7. *Let* $S = (G, \sigma)$ *be any signed graph. Then* $VND(S) \sim 2S$ $(VND(S) \sim S)$*, if* G *is isomorphic to* $2C_p$*, if* p *is even* $\left(G \text{ is isomorphic to } C_{2p}, \text{if } p \text{ is odd}\right)$.

In [1, 6], Kulli characterize the graphs for which the graphs and its corresponding neighborhood graphs are isomorphic.

THEOREM 2.2.8. *For any graph* $G = (V, E)$*, the neighborhood graph* $N(G)$ *and the graph* G *are isomorphic if and only if the graph* G *is isomorphic to the complememnt of the complete graph with* n *vertices* $(i.e., \overline{k_n})$.

In the context, we now characterize the signed graphs for which the signed graphs and vertex neighborhood signed graphs are cycle isomorphic.

THEOREM 2.2.9. *For any signed graph $S = (G, \sigma)$, the signed graphs and vertex neighborhood signed graphs are cycle isomorphic if and only if S is balanced and G is isomorphic to the complement of the complete graph with n vertices $(i.e., \overline{k_n})$*

Proof: Suppose S is balanced and G is isomorphic to the complement of the complete graph with n vertices $(i.e.\ \overline{k_n})$. Then, G and $N(G)$ are isomorphic. Now the vertex neighborhood signed graph $VND(S)$of a signed graph S with underlying graph is isomorphic to the complement of the complete graph with n vertices, is positive. From the hypothesis, S is positive and just now we have seen that $VND(S)$ is also positive and hence S and $VND(S)$ are cycle isomorphic, from that Theorem 1.1.3.

Conversely suppose that signed graph and its vertex neighborhood signed graph are cycle isomorphic. Then $G \cong N(G)$. Therefore G is isomorphic to the complement of the complete graph with n vertices$(i.e.,\ \overline{k_n})$. Since $VND(S) and$ S are cycle isomorphic. This satisfies only when S is positive.

The concept negation of a signed graph introduced by Harary [3, 7] as follows: Consider a signed graph $S = (G, \sigma)$, the negation of S is denoted by $\eta(S)$ and the underlying graph of S and $\eta(S)$ are isomorphic. Further, the sign of any edge $e = uv$ in $\eta(S)$ is $+(-)$, if the sign of the edge $e = uv$ in S is $-(+)$.

In view of the negation operator introduced by Harary [9, 7], we have the following cycle isomorphic characterizations:

COROLLARY 2.2.10.

The negation of vertex neighborhood signed graphs of $S_1 = (G_1, \sigma)$and $S_2 = (G_2, \sigma)$ are cycle isomorphic $(i.e. \eta(VND(S_1)) \sim \eta(VND(S_2)))$, if G_1 and G_2 are isomorphic.

COROLLARY 2.2.11

For any two signed graphs $S_1 = (G_1, \sigma)$and $S_2 = (G_2, \sigma)$, $(VND\ \eta(S_1))$ and $(VND\ \eta(S_2))$ are cycle isomorphic, if G_1 and G_2 are isomorphic.

COROLLARY 2.2.12.

*For any signed graph $S = (G, \sigma)$,
the signed graph S and vertex neighborhood signed graphs of $\eta(S)$are*

cycle isomorphic if and only if S is balanced and G is isomorphic to the complement of the complete graph with n vertices $(i.e. \overline{k_n})$.

We have observed that, the signed graph is either positive or negative but the vertex neighborhood signed graph of one such signed graph is always positive. Using the concept negation in signed graphs introduced by Harary [7, 8], we have the following result to the vertex neighborhood signed graphs.

THEOREM 2.2.13.

Suppose the vertex neighborhood graph $N(G)$ *is bipartite. Then the negation of vertex neighborhood signed graph* $\eta(VND(S))$ *is positive, where S is any signed graph.*

Proof: Since, by Theorem 2.1.1, vertex neighborhood signed graph $VND(S)$ is positive. Then all the cycles in vertex neighborhood signed graph $VND(S)$ are positive. By the hypothesis, the vertex neighborhood graph $N(G)$ is bipartite. Then each cycle C_n (where n is even) in $VND(S)$ is positive. Therefore, the negation of vertex neighborhood signed graph $VND(S)$ is positive.

2.3 Structural Characterization of Vertex Neighborhood Signed Graphs

We now give the structural characterization of vertex neighborhood signed graphs.

THEOREM 2.3.1.

Suppose $S = (G, \sigma)$ *be any signed graph. Then S is positive and its underlying graph is vertex neighborhood graph (i.e., neighborhood graph)* $N(G)$ *if and only if S is an vertex neighborhood signed graph* $VND(S)$.

Proof: Let us consider that S is a vertex neighborhood signed graph $VND(S)$. Then the signed graph $\sum$ and the vertex neighborhood signed graph of some signed graph S_1 $(i.e., VND(S_1))$ are isomorphic. Since, the vertex neighborhood signed graph of any signed graph is positive and we have $S \cong VND(S_1)$. Consequently, $\sum$ is positive and its underlying graph is a vertex neighborhood graph.

Conversely, suppose that S is positive and its underlying graph is vertex neighborhood graph (i.e., neighborhood graph) $N(G)$. Since, the signed graph S is positive, then establish the S_ζ. With the evidence of Sampathkumar's result (Theorem 1.1.2), every edge pq in S_ζ amuse $\sigma(pq) = \zeta(p)\zeta(q)$. Deliberate, the signed graph $\sum_1 = (G_1, \sigma_1)$ in which each edge $e = (pq)$ in $G_1, \sigma_1(e) = \zeta(p)\zeta(q)$ Therefore, the signed graph and the vertex neighborhood signed graph of S_1 are isomorphic. Hence, S is a vertex neighborhood signed graph $VND(S)$.

REFERENCES

1. Kulli, VR, Janakiram, B., & Niranjan, KM (2004). The dominating graph. *Graph Theory Notes of New York, New York Academy of Sciences*, *46*, 5-8.
2. Konig, D. (2001). *Theorie der endlichen und unendlichen Graphen* (Vol. 72). American Mathematical Soc...
3. Adiga, C., Sampathkumar, E., & Sriraj, M. A. (2014). Color energy of a unitary Cayley graph. *Discussiones Mathematicae - Graph Theory*, *34*(4), 707–721. https://doi.org/10.7151/dmgt.1767
4. Kulli, V. R. (2015a). On Middle Neighborhood Graphs. *International Journal of Mathematics And Its Applications*, *3*(4-D), 79–83.
5. Kulli, V. R. (2015b). The neighborhood graph of a graph. *International Journal of Fuzzy Mathematical Archive*, *8*(October), 93–99.
6. Vasilyev, A. (2014). IJMC Upper and lower bounds of symmetric division deg index. *Iranian Journal of Mathematical Chemistry*, *5*(2), 91–98.
7. F. Harary, Structural duality, *Behavioral Science*, 2 (1957), 255-265.
8. S.B. Bozkurt, A.D. Gungor, I.Gutman, A.S Cevik, *Randic matrix and Randic energy*, *MATCH Commum*. Math. Comput. Chem. 64 (2010) 239250.
9. I. Gutman, B. Furtula, S. B. Bozkurt, On *Randic energy*, *Linear Algebra Appl.*, 442 (2014) 5057.
10. M. R. Rajesh Kanna, B. N. Dharmendra and G. Sridhara, *Minimum dominating energy of a graph*, *International Journal of Pure and Applied Mathematics*, 85 (4) (2013), 707-718.