

# Study of Dimensional Analysis and Hydraulic Similitude

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## Abstract:

Dimensional analysis is a powerful tool in designing, ordering, and analysing the experiment results and also synthesizing them. One of the important theorems in dimensional analysis is known as the Buckingham  $\Pi$  theorem, so called since it involves non-dimensional groups of the products of the quantities. In this topic, Buckingham  $\Pi$  theorem and its uses are thoroughly discussed. We shall discuss the planning, presentation, and interpretation of experimental data and demonstrate that such data are best presented in dimensionless form. Experiments which might result in tables of output, or even multiple volumes of tables, might be reduced to a single set of curves—or even a single curve—when suitably non-dimensionalized. The technique for doing this is dimensional analysis.

Also, Physical models for hydraulic structures or river courses are usually built to carry out experimental studies under controlled laboratory conditions. The main purposes of physical models are to replicate a small-scale hydraulic structure or flow phenomenon in a river and to investigate the model performance under different flow and sediment conditions. The concept of similitude is commonly used so that the measurements made in a laboratory model study can be used to describe the characteristics of similar systems in the practical field situations.

This report describes hydraulic similitude in terms of geometric, kinematic, and dynamic similitude. The analysis leads to the definition of model-scale ratios. A number of illustrative examples are also presented.

**Key Words:** Dimensions, Dimensional Analysis, Similitude, Model, Model Analysis.

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## 1. Introduction

Although many practical engineering problems involving fluid mechanics can be solved by using the equations and analytical procedures described in the preceding chapters, there remain a large number of problems that rely on experimentally obtained data for their

solution. In fact, it is probably fair to say that very few problems involving real fluids can be solved by analysis alone. The solution to many problems is achieved through the use of a combination of theoretical and numerical analysis and experimental data. Thus, engineers working on fluid mechanics problems should be familiar with the experimental approach to these problems so that they can interpret and make use of data obtained by others, such as might appear in handbooks, or be able to plan and execute the necessary experiments in their own laboratories.

In this paper we consider some techniques and ideas that are important in the planning and execution of experiments, as well as in understanding and correlating data that may have been obtained by other experimenters.

An obvious goal of any experiment is to make the results as widely applicable as possible. To achieve this end, the concept of similitude is often used so that measurements made on one system (for example, in the laboratory) can be used to describe the behaviour of other similar systems (outside the laboratory).

The laboratory systems are usually thought of as models and are used to study the phenomenon of interest under carefully controlled conditions. From these model studies, empirical formulations can be developed, or specific predictions of one or more characteristics of some other similar system can be made.

To do this, it is necessary to establish the relationship between the laboratory model and the “other” system.

## **2. Dimensional Analysis**

Dimensional analysis is the analysis of the relationships between different physical quantities by identifying their base quantities and units of measure and tracking these dimensions as calculations or comparisons are performed. Each physical phenomenon can be expressed by an equation, composed of variable (or physical quantities) which may be dimensional and non-dimensional quantities. Dimensional Analysis helps in determining a systematic arrangement of variables in the physical relationship and combining dimensional variables to form non dimensional parameters. Basically, dimensional analysis is a method for reducing the number and complexity of experimental variables which affect a given physical phenomenon, by using a sort of compacting technique. Dimensional analysis is essential because it keeps the units the same, helping us perform mathematical calculations

smoothly. This mathematical technique is used in research work for design and for conducting model tests.

### 3. Methodology of Dimensional Analysis

The basic principle of dimensional analysis is Dimensional Homogeneity, which means the dimensions of each terms in an equation on both sides are equal.

So, such an equation, in which dimensions of each term on both sides of equation are same, is known as Dimensionally Homogeneous Equation. Such equations are independent of system of units.

#### METHODS OF DIMENSIONAL ANALYSIS

If the number of variables involved in a physical phenomenon are known, then the relation among the variables can be determined by the following two methods;

- Rayleigh's Method
- Buckingham's  $\pi$ -Theorem

##### Rayleigh's Method

Rayleigh's method of dimensional analysis is a conceptual tool used in engineering.

It is used for determining expression for a variable which depends upon maximum three to four variables only.

Let  $X$  is a dependent variable which depends upon  $X_1$ ,  $X_2$ , and  $X_3$  as independent variables.

Then according to Rayleigh's Method

$$X = f(X_1, X_2, X_3)$$

*which can be written as*

$$X = K X_1^a X_2^b X_3^c$$

##### Buckingham's $\pi$ -Theorem

Since Rayleigh's Method becomes laborious if variables are more than fundamental dimensions, so the difficulty is overcome by Buckingham's  $\pi$ -Theorem which states that:

If there are  $n$  variables in a problem and these variables contain  $m$  primary dimensions the equation relating all the variables will have  $(n-m)$  dimensionless groups."

Buckingham referred to these groups as  $\pi$  groups.

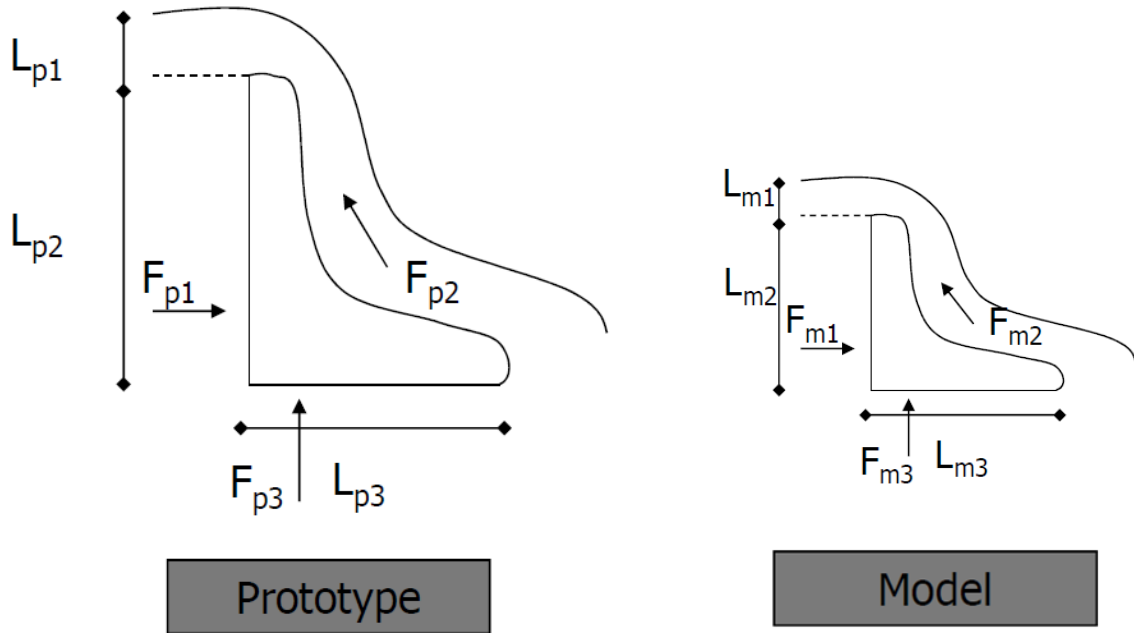
The final equation obtained is in the form of:

$$\pi_1 = f(\pi_2, \pi_3, \dots, \pi_{n-m})$$

#### 4. SIMILITUDE AND MODEL ANALYSIS

Usually, it is impossible to obtain a pure theoretical solution of hydraulic phenomenon. Therefore, experimental investigations are often performed on small scale models, called model analysis.

Model is a small-scale replica of the actual structure while as a prototype is a term used to describe the actual structure or machine.



A few examples, where models may be used are ships in towing basins, air planes in wind tunnel, hydraulic turbines, centrifugal pumps, spillways of dams, river channels etc and to study such phenomenon as the action of waves and tides on beaches, soil erosion, and transportation of sediment etc.

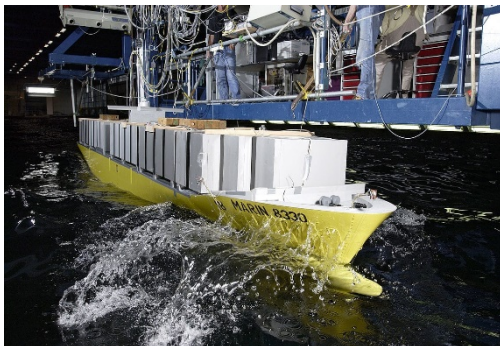


image source: Wikipedia

Ship Model Basin



Wind Tunnel

image source: Nasa.gov

**HYDRAULIC SIMILITUDE:** Hydraulic similitude is an indication of a relationship between a model and a prototype. Or it is a model study of hydraulic structure. It is defined as the similarity between model and prototype in every respect, which means model and prototype have similar properties or model and prototype are completely similar.

The three types of similarities that must exist between model and prototype are:

- Geometric Similarity
- Kinematic Similarity
- Dynamic Similarity

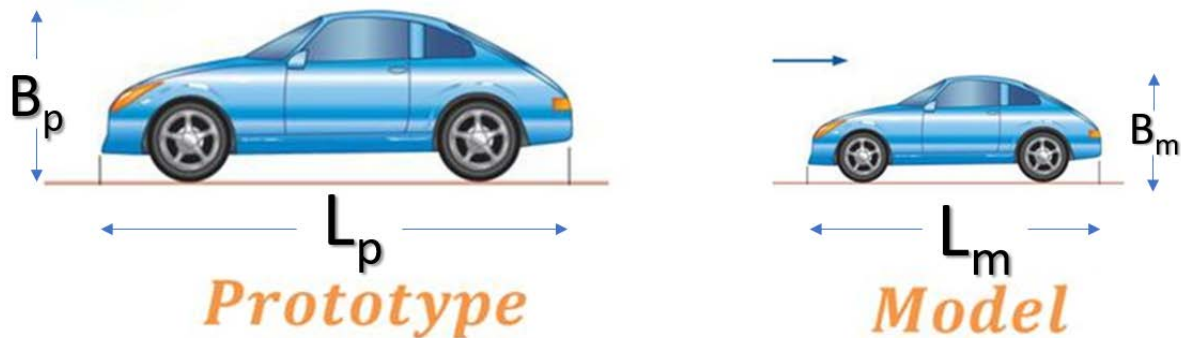
**Geometric Similarity:**

Geometric Similarity is the similarity of shape. It is said to exist between model and prototype if ratio of all the corresponding linear dimensions in the model and prototype are equal. e.g.

$$\frac{L_p}{L_m} = \frac{B_p}{B_m} = \frac{D_p}{D_m} = L_r$$

Where:  $L_p$ ,  $B_p$  and  $D_p$  are Length, Breadth, and diameter of prototype and  $L_m$ ,  $B_m$ ,  $D_m$  are Length, Breadth, and diameter of model.

$L_r$  = Scale ratio



**Kinematic Similarity:**

Kinematic Similarity is the similarity of motion. It is said to exist between model and prototype if ratio of velocities and acceleration at the corresponding points in the model and prototype are equal. e.g.

$$\frac{V_{p1}}{V_{m1}} = \frac{V_{p2}}{V_{m2}} = V_r; \quad \frac{a_{p1}}{a_{m1}} = \frac{a_{p2}}{a_{m2}} = a_r$$

Where  $V_{p1}$  &  $V_{p2}$  and  $a_{p1}$  &  $a_{p2}$  are velocity and accelerations at point 1 & 2 in prototype and  $V_{m1}$  &  $V_{m2}$  and  $a_{m1}$  &  $a_{m2}$  are velocity and accelerations at point 1 & 2 in model.

$V_r$  and  $a_r$  are the velocity ratio and acceleration ratio.

### Dynamic Similarity

Dynamic Similarity is the similarity of forces. It is said to exist between model and prototype if ratio of forces at the corresponding points in the model and prototype are equal. e.g.

Where  $(F_i)_p$ ,  $(F_v)_p$  and  $(F_g)_p$  are inertia, viscous and gravitational forces in prototype and  $(F_i)_m$ ,  $(F_v)_m$  and  $(F_g)_m$  are inertia, viscous and gravitational forces in model.

$F_r$  is the Force ratio

## 5. CONCLUSION

This paper introduces the method: experimentation, as supplemented by the technique of dimensional analysis. Tests and experiments are used both to strengthen existing theories and to provide useful engineering results when theory is inadequate.

The paper begins with a discussion of some familiar physical relations and how they can be recast in dimensionless form because they satisfy the principle of dimensional homogeneity.

A general technique, the pi theorem, is then presented for systematically finding a set of dimensionless parameters by grouping a list of variables which govern any particular physical process. Alternately, direct application of dimensional analysis to the basic equations of fluid mechanics yields the fundamental parameters governing flow patterns.

It is shown that model testing in air and water often leads to scaling difficulties for which compromises must be made. Many model tests do not achieve true dynamic similarity.

The paper ends by pointing out that classic dimensionless charts and data can be manipulated and recast to provide direct solutions to problems that would otherwise be quite cumbersome and laboriously iterative.

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