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Derivations of Intuitionistic Fuzzy Sub-Implicative Ideals of BCK – Algebras

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Abstract: The aim of this paper is to introduce the notion of derivation of intuitionistic fuzzy sub-implicative ideals (DIFSII) in BCK – algebras and to investigate some of their related properties.

Keywords and Phrases: *BCK-algebras*, Derivations, Intuitionistic Fuzzy Set, implicative ideals, Intuitionistic Fuzzy Positive Implicative Ideal, Derivations of Intuitionistic Fuzzy Ideal and Derivations of Intuitionistic Fuzzy Positive Implicative Ideals.

Mathematics subject classification: 06F35, 03B52

1. Introduction:

In 1966 by means of K. Iseki and Y. Imai [6], introduced a new perception known as a BCK-algebras. So many Researchers investigated assorted houses of this algebra. Such Algebra generalized the thought of units with the set subtraction as the most effective nonnulary operation. In [5], Iseki and Tanaka introduce the idea of Sub-Algebra, Ideals and Positive Implicative Ideals in BCK-algebras. Meng [9], introduce the notion of Implicative ideals in BCK-algebras and investigate the affiliation of it with the concept of Positive Implicative ideals and Commutative Ideals in BCK-Algebras. In 1991, Xi [17] applied the concept of Fuzzy Set to BCK-algebras and had given some houses of it. Jun and Xin [8] applied the notion of derivation in rings and near-rings. Recently Aboujabal and Alshehri [1] discussed the derivation on BCK-algebra. After the work of Jun and Xin [8], many research articles have been appeared on the derivations of *BCK*-algebras and a greater interest has been devoted to the study of derivation in *BCK*-algebras on various aspects. Motivated by the notions of left derivation and generalized derivation in rings and near rings theory, Satyanarayana et.al. ([13], [14, [16]) applied the concept of derivation and introduced Derivation of intuitionistic fuzzy commutative ideals, Derivation of intuitionistic fuzzy positive implicative ideals and Derivation of intuitionistic fuzzy implicative ideals in BCK-algebras.

The aim of this paper is to introduce the notion of derivation of intuitionistic fuzzy sub-implicative ideals and derivation of intuitionistic fuzzy positive implicative ideals in BCK – algebras and to investigate some of their related properties.

2. *Preliminaries:*

Definition 2.1. The following are the elementary definitions. A BCK – algebra is an algebra of type (2, 0), if it satisfies the following axioms for all $p,q,r \in K$.

(BCK-1) $((p\otimes q)\otimes(p\otimes r))\otimes(r\otimes q)=0$

(BCK-2) $(p\otimes(p\otimes q))\otimes q=0$

(BCK-3) $p\otimes p=0$

(BCK-4) $0\otimes p=0$

(BCK-5) $p\otimes q=0$ and $q\otimes p=0$ implies $p=q$.

We can define a binary relation $\le$ on K by assuming $p\le q$ if and only if $p\otimes q=0$. In that case $(K,\le)$ is a partial ordered set with least element 0 and $(K,\otimes,0)$ is a BCK – algebra if and only if, it satisfies the following axioms: For all $p,q,r\in K$

(i) $((p\otimes q)\otimes(p\otimes r))\le(r\otimes q)$ (ii) $(p\otimes(p\otimes q))\le q$ (iii) $p\le p$ (iv) $0\le p$

(v) $p\le q$ and $q\le p$ implies $p=q$, for all $p,q,r\in K$.

The following are the properties in a BCK – algebra:

(P_1) $p\otimes 0=p$, (P_2) $p\otimes q\le p$, (P_3) $(p\otimes q)\otimes r=(p\otimes r)\otimes q$, ($P_4$) $(p\otimes r)\otimes(q\otimes r)\le(p\otimes q)$

(P_5) $p\otimes(p\otimes(p\otimes q))=p\otimes q$, ($P_6$) $p\le q\Rightarrow p\otimes r\le q\otimes r$ and $r\otimes q\le r\otimes p$

(P_7) $p\otimes q\le r\Rightarrow p\otimes r\le q$ for all $p,q,r\in K$.

A BCK-Algebra K is said to be implicative if $p=p\otimes(q\otimes p)$, for all $p,q,r\in K$.

A non-empty subset I of K is said to be an ideal of K if (I_1) $0\in I$ and (I_2) $p\otimes q$ and $q\in I$ imply that $p\in I$ for all $p,q\in K$,

A non-empty subset I of K is said to be an implicative ideal if (I_1) $0\in I$ and (I_3) $(p\otimes(q\otimes p))\otimes r\in I$ and $r\in I$ imply $p\in I$ for all $p,q,r\in K$.

A non-empty subset I of K is said to be a positive implicative ideal if (I_1) $0\in I$ and (I_4) $(p\otimes q)\otimes r\in I$ and $q\otimes r\in I$ imply $p\otimes r\in I$ for all $p,q,r\in K$.

A non-empty subset I of K is said to be a sub- implicative ideal if $0\in I$ and $((p\otimes(p\otimes q))\otimes(q\otimes p))\otimes r\in I$ *and* $r\in I$ *imply* $q\otimes(q\otimes p)\in I$ *forall* $p,q,r\in K$.

Once we recollect the contents of fuzzy set and intuitionistic fuzzy set.

A fuzzy set in a set K is a function $M:K\to[0,1]$ and the complement of M denoted by $\bar{M}$ the fuzzy set on K given by $\bar{M}(p)=1-M(p)$ for all $p\in K$. Let M and N be the fuzzy sets on K. For $s,t\in[0,1]$ the set $U(M,s)=\{p\in K/M_A(p)\ge s\}$ is called upper s-level cut of M and the set $L(N,t)=\{p\in K/N_A(p)\le t\}$ is called lower t-level cut of N and can be used to characterize of M and N.

A fuzzy set M in K is called a fuzzy sub-implicative ideal (briefly FSI) of K if (1) $M(0)\ge M(p)$ and (2) $M(q^2\otimes p)\ge \min\{M(((p^2\otimes q)\otimes(q\otimes p))\otimes r)),\ M(r)\}$ for all $p,q,r\in K$.

An intuitionistic fuzzy set (briefly IFS) A in a non-empty set K is an object having the form $A=\{p,M_A(p),N_A(p)/p\in K\}$ where the functions $M_A:K\to[0,1]$ and $N_A:K\to[0,1]$ denoted the degree of membership of each element $p\in K$ to the set A respectively and $0\le M_A(p)+N_A(p)\le 1$ for all $p\in K$. Let K denotes a BCK – algebra.

A fuzzy set M in K is called an intuitionistic fuzzy sub-implicative ideal (briefly IFSII) of K if (1) $M_A(0)\ge M_A(p)$ and $N_A(0)\le N_A(p)$

(2) $M_A(q^2\otimes p)\ge \min\{M_A(((p^2\otimes q)\otimes(q\otimes p))\otimes r)),\ M_A(r)\}$

(3) $N_A(q^2\otimes p)\le \max\{N_A(((p^2\otimes q)\otimes(q\otimes p))\otimes r)),\ N_A(r)\}$ for all $p,q,r\in K$.

Example 1. Let K = {0, 1, 2} with the following Cayley table by a BCK-algebra.

$\otimes$	0	1	2
0	0	0	2
1	1	0	2
2	2	2	0

Let $A=(K,M_A,N_A)$ be an IFS in K define by $M_A(0)=M_A(1)=0.6,\ M_A(2)=0.2$ and $N_A(0)=N_A(1)=0.2,\ N_A(2)=0.6$. Then A is an IFSII of K.

A map $\Delta:K\to K$ is known as a left-right derivation (briefly (L, R)-derivation of K if:

$$\Delta(p\otimes q)=(\Delta(p)\otimes q)\wedge(p\otimes\Delta(q)),\ \text{for all } p,q\in K$$

A map $\Delta:K\to K$ is known as a right derivation (briefly (R, L)-derivation of K if:

$$\Delta(p\otimes q)=(p\otimes\Delta(q))\wedge(\Delta(p)\otimes q),\ \text{for all } p,q\in K$$

A map $\Delta:K\to K$ is known as a derivation of K if Δ is both a (L, R)-derivation and (R, L)-derivation of K. Let $(K,\otimes,0)$ be a BCK-algebra, $\Delta:K\to K$ be a self map. A non-empty subset A of BCK-algebra K and $p,q,r\in K$ is called DMI (derivation implicative ideal) of BCK-algebra K if it satisfies: (DMI-1) $0\in A$ (DMI-2) $\Delta((p\otimes(q\otimes p))\otimes r)\in A$ and $\Delta(r)\in A$ imply $\Delta(p)\in A$.

Definition 2.2. A self mapping of BCK-algebra is called regular if $\Delta(0)=0$

3. DIFSII in BCK-ALGEBRA: (Derivations of intuitionistic fuzzy sub-implicative ideal in BCK – algebra)

In this part, we apply the concept of Derivation to Intuitionistic fuzzy sub-implicative ideal (DIFSII) and related properties are investigated.

Definition 3.1. A derivation $\Delta: K \to K$ is a mapping of BCK-algebra. Let $A = (K, M_A, N_A)$ be a non-empty IFS of K for all $p, q, r \in K$ is called derivation intuitionistic fuzzy sub- implicative ideal (briefly DIFSII) of K if it satisfies:

(DIFSII-1) $M_A(0) \geq M_A(\Delta(p))$ and $N_A(0) \leq N_A(\Delta(p))$

(DIFSII -2) $M_A(\Delta(q^2 \otimes p)) \geq \min\{M_A(\Delta(((p^2 \otimes q) \otimes (q \otimes p)) \otimes r)),\ M_A(\Delta(r))\}$

(DIFSII -3) $N_A(\Delta(q^2 \otimes p)) \leq \max\{N_A(\Delta(((p^2 \otimes q) \otimes (q \otimes p)) \otimes r)),\ N_A(\Delta(r))\}$

Example 2. Let K = {0, a, b} be a BCK-algebra with the following Cayley table:

⊗	0	a	b
0	0	0	0
a	a	0	0
b	b	b	0

Let $A = (K, M_A, N_A)$ be an IFS in K define by $M_A(0) = 0.4, M_A(a) = 0.6,\ M_A(b) = 0.8$ and $N_A(0) = 0.5, N_A(a) = 0.3,\ N_A(2) = 0.19.$ Then A is a DIFSII of K.

Theorem 3.2. Let $A = (K, M_A, N_A)$ be an IFS in K satisfying $M_A(0) \geq M_A(p)$ and $N_A(0) \leq N_A(p)$. If A is DIFSII of K, then satisfies the following inequality

$M_A(\Delta(q^2 \otimes p)) \geq M_A((p^2 \otimes q) \otimes (q \otimes p))$ and $N_A(\Delta(q^2 \otimes p)) \leq N_A((p^2 \otimes q) \otimes (q \otimes p))$

for all $p, q \in K.$

Proof: Let A be a DIFSII of K.

Then $M_A(\Delta(q^2 \otimes p)) \geq \min\{M_A(\Delta(((p^2 \otimes q) \otimes (q \otimes p)) \otimes r)),\ M_A(\Delta(r))\}$ and

$N_A(\Delta(q^2 \otimes p)) \leq \max\{N_A(\Delta(((p^2 \otimes q) \otimes (q \otimes p)) \otimes r)),\ N_A(\Delta(r))\}$

Taking r = 0, then $M_A(\Delta(q^2 \otimes p)) \geq \min\{M_A(\Delta(((p^2 \otimes q) \otimes (q \otimes p)) \otimes 0)),\ M_A(\Delta(0))\}$

Since derivation on BCK – algebra is regular (ie) $\Delta(0) = 0.$

$$\therefore M_A(\Delta(q^2 \otimes p)) \geq \min\{M_A(\Delta((p^2 \otimes q) \otimes (q \otimes p))),\ M_A(0)\}$$

$$= M_A(\Delta((p^2 \otimes q) \otimes (q \otimes p))$$

and $N_A(\Delta(q^2 \otimes p)) \leq \max\{N_A(\Delta(((p^2 \otimes q) \otimes (q \otimes p)) \otimes 0)),\ N_A(\Delta(0))\}$

$$N_A(\Delta(q^2 \otimes p) \leq \max\{N_A(\Delta((p^2 \otimes q) \otimes (q \otimes p))),\ N_A(0)\}$$

$= N_A(\Delta((p^2 \otimes q) \otimes (q \otimes p))$, Hence proved.

Theorem 3.3. Every DIFSII of K is a DIFI.

Proof: Assume $A = (K, M_A, N_A)$ is a DIFSII of K. Then

(DIFSII-1) $M_A(0) \geq M_A(\Delta(p))$ and $N_A(0) \leq N_A(\Delta(p))$

(DIFSII -2) $M_A(\Delta(q^2 \otimes p)) \geq \min\{M_A(\Delta(((p^2 \otimes q) \otimes (q \otimes p)) \otimes r)), M_A(\Delta(r))\}$

(DIFSII -3) $N_A(\Delta(q^2 \otimes p)) \leq \max\{N_A(\Delta(((p^2 \otimes q) \otimes (q \otimes p)) \otimes r)), N_A(\Delta(r))\}$

Put q = p in DIFSII-2 and DIFSII-3, we get

$M_A(\Delta(p^2 \otimes p)) \geq \min\{M_A(\Delta(((p^2 \otimes p) \otimes (p \otimes p)) \otimes r)), M_A(\Delta(r))\}$

$M_A(\Delta(p)) \geq \min\{M_A(\Delta((p \otimes 0)) \otimes r), M_A(\Delta(r))\}$
$(Since\ \ \Delta(p^2 \otimes p) = \Delta(p \otimes (p \otimes p)) = \Delta(p \otimes 0) = \Delta(p))$

$M_A(\Delta(p)) \geq \min\{M_A(\Delta(p \otimes r)), M_A(\Delta(r))\}$

Similarly $N_A(\Delta(p^2 \otimes p)) \leq \max\{N_A(\Delta(((p^2 \otimes p) \otimes (p \otimes p)) \otimes r)), N_A(\Delta(r))\}$

$N_A(\Delta(p)) \leq \max\{N_A(\Delta((p \otimes 0)) \otimes r), N_A(\Delta(r))\}$

$N_A(\Delta(p)) \leq \max\{N_A(\Delta(p \otimes r)), N_A(\Delta(r))\}$ for all $p, q \in K$.

Hence A is DIFI.

But the converse of the above theorem need not be true. It can be explained by the following example.

Let K = {0, a, b, c} be a BCK-algebra with the following Cayley table:

⊗	0	a	b	C
0	0	0	0	C
a	a	0	0	C
b	b	b	0	C
c	c	c	c	0

Let $A = (K, M_A, N_A)$ be an IFS in K define by $M_A(0) = 0.7, M_A(x) = 0.2\ \ for\ all\ \ x \neq 0$ and $N_A(0) = 0.2, N_A(x) = 0.7\ \ for\ all\ \ x \neq 0$.

Define $\Delta : K \to K$ as $\Delta(x) = 0,\ x = 0$

$= c,\ \ x = a, b, c$

and if you define derivation on IFS by $M_A : K \to K$ and $N_A : K \to K$ such that

$M_A(\Delta(0)) = 0.7, M_A(\Delta(a)) = M_A(\Delta(b)) = M_A(\Delta(c)) = 0.2$ and

$N_A(\Delta(0)) = 0.2, N_A(\Delta(a)) = N_A(\Delta(b)) = N_A(\Delta(c)) = 0.7$

Then it is easily to check that A is DIFI of K. But it is not DIFSII of K because

$M_A(\Delta(a^2 \otimes b)) < \min\{M_A(\Delta(((b^2 \otimes a) \otimes (a \otimes b)) \otimes 0)), M_A(\Delta(0))\}$

$N_A(\Delta(a^2 \otimes b)) > \max\{N_A(\Delta(((b^2 \otimes a) \otimes (a \otimes b)) \otimes 0)), N_A(\Delta(0))\}$

Theorem 3.4. Every DIFI satisfying the condition

$M_A(\Delta(q^2 \otimes p)) \geq M_A(\Delta((p^2 \otimes q) \otimes (q \otimes p)))$ and

$N_A(\Delta(q^2 \otimes p)) \leq N_A(\Delta((p^2 \otimes q) \otimes (q \otimes p)))$ is an DIFSII of K for all $p, q \in K$.

Proof: Let $A = (K, M_A, N_A)$ be a DIFI of K satisfying

$M_A(\Delta(q^2 \otimes p)) \geq M_A(\Delta((p^2 \otimes q) \otimes (q \otimes p)))$ and

$N_A(\Delta(q^2 \otimes p)) \leq N_A(\Delta((p^2 \otimes q) \otimes (q \otimes p)))$

$M_A(\Delta(q^2 \otimes p)) \geq M_A(\Delta((p^2 \otimes q) \otimes (q \otimes p)))$

$\geq \min\{M_A(\Delta(((p^2 \otimes q) \otimes (q \otimes p)) \otimes r)), M_A(\Delta(r))\}$

$N_A(\Delta(q^2 \otimes p)) \leq N_A(\Delta((p^2 \otimes q) \otimes (q \otimes p)))$

$\leq \max\{N_A(\Delta(((p^2 \otimes q) \otimes (q \otimes p)) \otimes r)), N_A(\Delta(r))\}$

Hence the proof.

4. Derivation of Intuitionistic fuzzy positive implicative ideal of BCK-algebras.

Definition 4.1. An IFS A in K is called DIFPII of K if

(DIFPII-1) $M_A(0) \geq M_A(\Delta(p))$ and $N_A(0) \leq N_A(\Delta(p))$

(DIFPII -2) $M_A(\Delta(p \otimes r)) \geq \min\{M_A(\Delta(((p \otimes r) \otimes r) \otimes (q \otimes r))), M_A(\Delta(q))\}$

(DIFPII -3) $N_A(\Delta(p \otimes r)) \leq \max\{N_A(\Delta(((p \otimes r) \otimes r) \otimes (q \otimes r))), N_A(\Delta(q))\}$

Note:

$((p \otimes r) \otimes r) \otimes (q \otimes r) = ((p \otimes r) \otimes (q \otimes r)) \otimes r \leq (p \otimes q) \otimes r$

$(p \otimes q) \otimes r \geq ((p \otimes r) \otimes r) \otimes (q \otimes r)$

We know that $q \otimes r \leq q \Rightarrow q \geq q \otimes r$ instead of $(q \otimes r)$ we can write q.

Hence the definition of DIFPII is modified accordingly to the definition of DIFSII.

Example 3. Let K = {0, 1, 2, 3} be a BCK-algebra with the following Cayley table:

$\otimes$	0	1	2	3
0	0	0	0	3
1	1	0	0	3
2	2	2	0	3
3	3	3	3	0

Let $A=(K, M_A, N_A)$ be an IFS in K define by $M_A(0)=0.5, M_A(x)=0.1 \ for\ all\ x \neq 0$ and $N_A(0)=0.1, N_A(x)=0.5 \ for\ all\ x \neq 0$.

Define $\Delta: K \to K$ as $\Delta(x)=0,\ x=0$

$$=3,\ \ x=1,2,3$$

and if you define derivation on IFS by $M_A: K \to K$ and $N_A: K \to K$ such that

$M_A(\Delta(0))=0.5, M_A(\Delta(1))=M_A(\Delta(2))=M_A(\Delta(3))=0.1$ and

$N_A(\Delta(0))=0.1, N_A(\Delta(1))=N_A(\Delta(2))=N_A(\Delta(3))=0.5$

Then it is easily to check that A is DIFPII of K.

Theorem 4.2. Every DIFSII is a DIFPII.

Proof: Let A be a DIFSII of K. Then A is a DIFI of K.

From theorem 3.4, we have $M_A(\Delta(q^2 \otimes p)) \geq M_A(\Delta((p^2 \otimes q) \otimes (q \otimes p)))$ and

$N_A(\Delta(q^2 \otimes p)) \leq N_A(\Delta((p^2 \otimes q) \otimes (q \otimes p)))$ for all $p, q \in K$.

Substituting $x \otimes y$ for p and x for q we have,

$$\begin{aligned} M_A(\Delta(x \otimes y)) = M_A(\Delta((x \otimes (x \otimes (x \otimes y))))) &= M_A(\Delta((q^2 \otimes p)) \\ &\geq M_A(\Delta((p^2 \otimes q) \otimes (q \otimes p))) \\ &= M_A(\Delta((((x \otimes y) \otimes ((x \otimes y) \otimes x))) \otimes (x \otimes (x \otimes y)))) \\ &= M_A(\Delta((((x \otimes y) \otimes (x \otimes (x \otimes y))) \otimes ((x \otimes y) \otimes x))) \\ &= M_A(\Delta((((x \otimes (x \otimes (x \otimes y))) \otimes y) \otimes ((x \otimes x) \otimes y))) \\ &= M_A(\Delta(((x \otimes y) \otimes y) \otimes (0 \otimes y))) \end{aligned}$$

$$\begin{aligned} N_A(\Delta(x \otimes y)) = N_A(\Delta((x \otimes (x \otimes (x \otimes y))))) &= N_A(\Delta((q^2 \otimes p)) \\ &\leq N_A(\Delta((p^2 \otimes q) \otimes (q \otimes p))) \\ &= N_A(\Delta((((x \otimes y) \otimes ((x \otimes y) \otimes x))) \otimes (x \otimes (x \otimes y)))) \\ &= N_A(\Delta((((x \otimes y) \otimes (x \otimes (x \otimes y))) \otimes ((x \otimes y) \otimes x))) \\ &= N_A(\Delta((((x \otimes (x \otimes (x \otimes y))) \otimes y) \otimes ((x \otimes x) \otimes y))) \\ &= N_A(\Delta(((x \otimes y) \otimes y) \otimes (0 \otimes y))) \end{aligned}$$

Hence A is DIFPII of K.

Theorem 4.3. For any DIFSII A of K, the set

$K_A = \{p \in K; M_A(\Delta(p) = M_A(0)\ \ and\ \ N_A(\Delta(p)) = N_A(0)\}$ is a Sub-implicative Ideal (SII).

Proof: Clearly $0 \in K_A$.

Let $p, q, r \in K$ be such that $((p^2 \otimes q) \otimes (q \otimes p)) \otimes r \in K_A\ \ and\ \ r \in K_A$.

Then $M_A(\Delta(q^2 \otimes p)) \geq \min\{M_A(\Delta(((p^2 \otimes q) \otimes (q \otimes p)) \otimes r)),\ M_A(\Delta(r))\} = M_A(0)$.

$M_A(\Delta(q^2 \otimes p)) = M_A(0)$.

and $N_A(\Delta(q^2 \otimes p)) \leq \max\{ N_A(\Delta(((p^2 \otimes q) \otimes (q \otimes p)) \otimes r)),\ N_A(\Delta(r))\} = N_A(0)$.

$N_A(\Delta(q^2 \otimes p)) = N_A(0)$ for all $p, q, r \in K$.

(ie)., $q^2 \otimes p \in K_A$.

Therefore, K_A is SII of K.

Definition 4.4. Let $A = (K, M_A, N_A)$ be an IFS in BCK-algebra K. For each pair $\langle t, s\rangle \in [0, 1]$, the set $A_{\langle t, s\rangle} = \{p \in K : M_A(\Delta(p) \geq t \;\; and \;\; N_A(\Delta(p)) \leq s\}$ is called the level subset of A.

Definition 4.5. Let $A = (K, M_A, N_A)$ be an IFS in BCK-algebra K and let $t \in [0, 1]$. Then the sets $U(M_A : t) = \{p \in K : M_A(\Delta(p) \geq t\}$ and $L(N_A : t) = \{p \in K : N_A(\Delta(p) \leq t\}$ are called M-level t-cut and N-level t-cut of A respectively.

Theorem 4.6. Let $A = (M_A, N_A)$ be a DIFSII of K. Then $A_{\langle t, s\rangle}$ is a SII of K for every $t, s \in I_m(M_A) \times I_m(N_A)$ with $t + s \leq 1$.

Proof: Let A be a DIFSII of K. Obviously $0 \in A_{\langle t, s\rangle}$.

Let $p, q, r \in K$ be such that $((p^2 \otimes q) \otimes (q \otimes p)) \otimes r \in A_{\langle t, s\rangle}$ *and* $r \in A_{\langle t, s\rangle}$.

$M_A(\Delta(((p^2 \otimes q) \otimes (q \otimes p)) \otimes r)) \geq t$, $N_A(\Delta(((p^2 \otimes q) \otimes (q \otimes p)) \otimes r)) \leq s$ and $M_A(\Delta(r)) \geq t$, $N_A(\Delta(r)) \leq s$.

It follows that $M_A(\Delta(q^2 \otimes p)) \geq \min\{M_A(\Delta(((p^2 \otimes q) \otimes (q \otimes p)) \otimes r)),\; M_A(\Delta(r))\} \geq t$

$N_A(\Delta(q^2 \otimes p)) \leq \max\{N_A(\Delta(((p^2 \otimes q) \otimes (q \otimes p)) \otimes r)),\; N_A(\Delta(r))\} \leq s$

So that, $q^2 \otimes p \in A_{\langle t, s\rangle}$. Hence $A_{\langle t, s\rangle}$ is a SII.

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