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A Fuzzy Inventory Model With Permissible Delay In Payments

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Abstract:

Our goal in this research is to develop a mathematical model in which payments from the retailer to the supplier can be delayed for a specified amount of time. The trapezoidal fuzzy numbers have been preambled for us. The major goal is to identify optimal solutions for these models by defuzzifying the fuzzy total production and inventory cost using the Graded mean integration representation approach and solving the problem using the Kuhn- Tucker method. The solution procedure is supported by a numerical example.

Keywords:

Graded mean integration representation, Kuhn-Tucker conditions, Permissible delay in payments, Trapezoidal Fuzzy Number.

Introduction:

Scientists define a fuzzy idea as one that can be applied to a circumstance to some extent. That the notion has gradations of relevance or fuzzy (varying) application boundaries. A fuzzy assertion is one that is true to a certain extent, which is commonly expressed by a scaled number. An amber traffic light is the most well-known example of a fuzzy idea in the world, and fuzzy notions are commonly used in traffic management systems.

Inventory is a phrase used in accounting to describe items that are in various stages of being prepared for sale, such as:Finished goods (that are available to be sold)

- This is a work-in-progress

- Supplies of raw materials to be used to produce more finished goods

Inventory is typically a company's largest current asset, consisting of things that are planned to sell within the following year.

Definitions and Methodologies

Fuzzy Set:

A fuzzy set $\tilde{A}$ in a universe of discourse X is defined as the following set of pairs $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$.Here $\mu_{\tilde{A}} : X \to [0,1]$ is a mapping called the membership value of $x \in X$ in a fuzzy set $\tilde{A}$.

Trapezoidal Fuzzy Number:

A Trapezoidal Fuzzy Number $\tilde{A} = (a,b,c,d)$ is represented by membership function

$$\mu_{\tilde{A}}(x) = \begin{cases} L(x) = \dfrac{x-a}{b-a}, a \le x \le b \\ 1, b \le x \le c \\ R(x) = \dfrac{d-x}{d-c}, \mathrm{c} \le x \le d \\ 0, Otherwise \end{cases}$$

Graded Mean Integration Representation Method

The Graded mean integration representation method is a function that maps the set of all trapezoidal fuzzy numbers to the real line R. The real number that corresponds to the trapezoidal fuzzy number using the graded mean representation method is given by

$$\mathrm{R}(\tilde{A}) = \frac{a_1 + 2a_2 + 2a_3 + a_4}{6}$$

Arithmetic Operations under Function Principle

Suppose $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ are two hexagonal fuzzy numbers, then the arithmetic operations are defined as

1. $\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$
2. $\tilde{A} * \tilde{B} = (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4,)$
3. $\tilde{A} - \tilde{B} = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$
4. $\dfrac{\tilde{A}}{\tilde{B}} = \left(\dfrac{a_1}{b_4}, \dfrac{a_2}{b_3}, \dfrac{a_3}{b_2}, \dfrac{a_4}{b_1}\right)$
5. $\alpha \tilde{A} = \begin{cases} \alpha a_1, \alpha a_2, \alpha a_3, \alpha a_4, \alpha \ge 0 \\ \alpha a_4, \alpha a_3, \alpha a_2, \alpha a_{1,} \alpha < 0 \end{cases}$

Notations:

D_r - Demand Rate
P_r - Production Rate
O_c - Ordering Cost
S_c - Setup Cost
H_c - Holding Cost
I_c - Interest earned per cycle
P_d - Permissible delay period
C_t - Cycle Time
Q- Economic Order Quantity

T_C- Total Cost

$\tilde{O}_c$ - Fuzzy ordering Cost

$\tilde{S}_c$ - Fuzzy setup Cost

$\tilde{H}_c$ - Fuzzy holding Cost

$\tilde{I}_c$ - Fuzzy Interest earned per cycle

Q^* - Fuzzy economic order quantity

$\tilde{T}_C$ - Fuzzy total cost

Assumptions:

- The production rate is known and constant Demand.
- The time period is infinite.
- Shortages are not allowed.
- When $C_t \le P_d$, the account is settled at $C_t = P_d$ and
- we do not pay any interest change.

Mathematical model in Crisp Sense:

The total cost of the mathematical model in crisp sense is given by

$$T_C = \frac{Q}{2}\left[H_c + \frac{O_c D_r}{P_r}\right] + \frac{D_r}{Q}\left[O_c + S_c I_c\left(P_d - \frac{C_t}{2}\right)\right] \qquad \text{……………….(1)}$$

By Differentiating with respect to Q and equating to zero, we get

$$Q^* = \sqrt{\frac{2D_r\left[O_c + S_c I_c\left(P_d - \frac{C_t}{2}\right)\right]}{H_c + \frac{O_C D_r}{P_r}}} \qquad \text{……………….(2)}$$

Mathematical model in Fuzzy Sence:

The ordering cost, setup cost, holding cost, and interest received each year are all regarded trapezoidal fuzzy numbers in the fuzzy mathematical model.

Let

$$\tilde{O}_c = (O_{c_1}, O_{c_2}, O_{c_3}, O_{c_4})$$

$$\tilde{S}_c = (S_{c_1}, S_{c_2}, S_{c_3}, S_{c_4})$$

$$\tilde{H}_c = (H_{c_1}, H_{c_2}, H_{c_3}, H_{c_4})$$

$$\tilde{I}_c = (I_{c_1}, I_{c_2}, I_{c_3}, I_{c_4})$$

$$\tilde{Q} = (Q_1, Q_2, Q_3, Q_4)$$

be Trapezoidal fuzzy numbers.

Now, the fuzzy total cost is given by

$$\tilde{T}_C = \frac{\tilde{Q}}{2}\left[\tilde{H}_c \oplus \frac{\tilde{O}_c D_r}{P_r}\right] \oplus \frac{D_r}{\tilde{Q}}\left[\tilde{O}_c \oplus \tilde{S}_c \tilde{I}_c\left(P_d - \frac{C_t}{2}\right)\right] \qquad \text{……………….(3)}$$

$$\tilde{T}_C = \frac{Q_1,Q_2,Q_3,Q_4}{2}\left[\left(H_{c_1},H_{c_2},H_{c_3},H_{c_4}\right)\oplus\frac{\left(O_{c_1},O_{c_2},O_{c_3},O_{c_4}\right)D_r}{P_r}\right]\oplus$$

$$\frac{D_r}{Q_4,Q_3,Q_2,Q_1}\left[\left(O_{c_1},O_{c_2},O_{c_3},O_{c_4}\right)\oplus\left(S_{c_1},S_{c_2},S_{c_3},S_{c_4}\right)\left(I_{c_1},I_{c_2},I_{c_3},I_{c4}\right)\left(P_d-\frac{C_t}{2}\right)\right] \quad \text{................(4)}$$

$$\tilde{T}_C = \begin{Bmatrix} \left[\frac{Q_1}{2}\left[H_{c_1}+\frac{O_{c_1}D_r}{P_r}\right]+\frac{D_r}{Q_4}\left[O_{c_1}+\left(P_d-\frac{C_t}{2}\right)\right]I_{c_1}S_{c_1}\right]+ \\ \left[\frac{Q_2}{2}\left[H_{c_2}+\frac{O_{c_2}D_r}{P_r}\right]+\frac{D_r}{Q_3}\left[O_{c_2}+\left(P_d-\frac{C_t}{2}\right)\right]I_{c_2}S_{c_2}\right]+ \\ \left[\frac{Q_3}{2}\left[H_{c_3}+\frac{O_{c_3}D_r}{P_r}\right]+\frac{D_r}{Q_2}\left[O_{c_3}+\left(P_d-\frac{C_t}{2}\right)\right]I_{c_3}S_{c_3}\right]+ \\ \left[\frac{Q_4}{2}\left[H_{c_4}+\frac{O_{c_4}D_r}{P_r}\right]+\frac{D_r}{Q_1}\left[O_{c_4}+\left(P_d-\frac{C_t}{2}\right)\right]I_{c_4}S_{c_4}\right] \end{Bmatrix} \quad \text{................(5)}$$

We use Graded mean integration function for defuzzification.
By defuzzifying equation (5) using Graded mean integration, we get

$$\frac{1}{6}\left[\frac{Q_1}{2}\left[H_{c_1}+\frac{O_{c_1}D_r}{P_r}\right]+\frac{D_r}{Q_4}\left[O_{c_1}+\left(P_d-\frac{C_t}{2}\right)\right]I_{c_1}S_{c_1}+\lambda_1-\lambda_4\right]+$$
$$\frac{2}{6}\left[\frac{Q_2}{2}\left[H_{c_2}+\frac{O_{c_2}D_r}{P_r}\right]+\frac{D_r}{Q_3}\left[O_{c_2}+\left(P_d-\frac{C_t}{2}\right)\right]I_{c_2}S_{c_2}+\lambda_2-\lambda_1\right]+$$
$$\frac{2}{6}\left[\frac{Q_3}{2}\left[H_{c_3}+\frac{O_{c_3}D_r}{P_r}\right]+\frac{D_r}{Q_2}\left[O_{c_3}+\left(P_d-\frac{C_t}{2}\right)\right]I_{c_3}S_{c_3}-\lambda_2+\lambda_3\right]+ \quad \text{................(6)}$$
$$\frac{1}{6}\left[\frac{Q_4}{2}\left[H_{c_4}+\frac{O_{c_4}D_r}{P_r}\right]+\frac{D_r}{Q_1}\left[O_{c_4}+\left(P_d-\frac{C_t}{2}\right)\right]I_{c_4}S_{c_4}-\lambda_3\right]=0$$

In the following steps, we use Kuhn- Tucker method
to find the solutions of $Q_1,Q_2,Q_3,andQ_4$ to minimize $\tilde{T}_C(Q)$.

(i) $\lambda \le 0$
(ii) $\nabla fp\left(\tilde{T}C\right)$
(iii) $\lambda igi\left(Q\right)=0; i=1,2,3.....m$
(iv) $gi\left(Q\right)\ge 0, i=1,2,3.....m$

From equation 3

$q=\left(q_1,q_2,q_3,q_4\right)$

Apply fuzzy in equation (3)

$$\tilde{T}_C = \begin{Bmatrix} \left[\frac{Q_1}{2}\left[H_{c_1} + \frac{O_{c_1}D_r}{P_r}\right] + \frac{D_r}{Q_4}\left[O_{c_1} + \left(P_d - \frac{C_t}{2}\right)\right] I_{c_1}S_{c_1}\right] + \\ \left[\frac{Q_2}{2}\left[H_{c_2} + \frac{O_{c_2}D_r}{P_r}\right] + \frac{D_r}{Q_3}\left[O_{c_2} + \left(P_d - \frac{C_t}{2}\right)\right] I_{c_2}S_{c_2}\right] + \\ \left[\frac{Q_3}{2}\left[H_{c_3} + \frac{O_{c_3}D_r}{P_r}\right] + \frac{D_r}{Q_2}\left[O_{c_3} + \left(P_d - \frac{C_t}{2}\right)\right] I_{c_3}S_{c_3}\right] + \\ \left[\frac{Q_4}{2}\left[H_{c_4} + \frac{O_{c_4}D_r}{P_r}\right] + \frac{D_r}{Q_1}\left[O_{c_4} + \left(P_d - \frac{C_t}{2}\right)\right] I_{c_4}S_{c_4}\right] \end{Bmatrix}$$

$$\tilde{T}_C = \frac{1}{6}\begin{Bmatrix} \left[\frac{Q_1}{2}\left[H_{c_1} + \frac{O_{c_1}D_r}{P_r}\right] + \frac{D_r}{Q_4}\left[O_{c_1} + \left(P_d - \frac{C_t}{2}\right)\right] I_{c_1}S_{c_1}\right] + \\ 2\left[\frac{Q_2}{2}\left[H_{c_2} + \frac{O_{c_2}D_r}{P_r}\right] + \frac{D_r}{Q_3}\left[O_{c_2} + \left(P_d - \frac{C_t}{2}\right)\right] I_{c_2}S_{c_2}\right] + \\ 2\left[\frac{Q_3}{2}\left[H_{c_3} + \frac{O_{c_3}D_r}{P_r}\right] + \frac{D_r}{Q_2}\left[O_{c_3} + \left(P_d - \frac{C_t}{2}\right)\right] I_{c_3}S_{c_3}\right] + \\ \left[\frac{Q_4}{2}\left[H_{c_4} + \frac{O_{c_4}D_r}{P_r}\right] + \frac{D_r}{Q_1}\left[O_{c_4} + \left(P_d - \frac{C_t}{2}\right)\right] I_{c_4}S_{c_4}\right] \end{Bmatrix}$$

With $0 \le q_1 \le q_2 \le q_3 \le q_4$

It can be written as $q_2 - q_1 \ge 0;\ q_3 - q_2 \ge 0;\ q_4 - q_3 \ge 0;\ q_1 \ge 0$.

Condition :1

$\lambda_1, \lambda_2, \lambda_3, \lambda_4 \le 0$

Condition:2

$$\frac{\partial}{\partial q_1}\left(P\left(Tc\right)\right) - \lambda_1\frac{\partial}{\partial q_1}\left(g_1\left(Q\right)\right) - \lambda_2\frac{\partial}{\partial q_2}\left(g_2\left(Q\right)\right) - \lambda_3\frac{\partial}{\partial q_3}\left(g_3\left(Q\right)\right) - \lambda_4\frac{\partial}{\partial q_4}\left(g_4\left(Q\right)\right) = 0$$

Diff q_1, q_2, q_3, q_4 & equate to zero.

$$\frac{1}{6}\left[\frac{Q_1}{2}\left[H_{c_1} + \frac{O_{c_1}D_r}{P_r}\right] + \frac{D_r}{Q_4}\left[O_{c_1} + \left(P_d - \frac{C_t}{2}\right)\right] I_{c_1}S_{c_1} + \lambda_1 - \lambda_4\right] = 0 ------(7)$$

$$\frac{2}{6}\left[\frac{Q_2}{2}\left[H_{c_2} + \frac{O_{c_2}D_r}{P_r}\right] + \frac{D_r}{Q_3}\left[O_{c_2} + \left(P_d - \frac{C_t}{2}\right)\right] I_{c_2}S_{c_2} + \lambda_2 - \lambda_1\right] = 0 ------(8)$$

$$\frac{2}{6}\left[\frac{Q_3}{2}\left[H_{c_3} + \frac{O_{c_3}D_r}{P_r}\right] + \frac{D_r}{Q_2}\left[O_{c_3} + \left(P_d - \frac{C_t}{2}\right)\right] I_{c_3}S_{c_3} - \lambda_2 + \lambda_3\right] = 0 ------(9)$$

$$\frac{1}{6}\left[\frac{Q_4}{2}\left[H_{c_4} + \frac{O_{c_4}D_r}{P_r}\right] + \frac{D_r}{Q_1}\left[O_{c_4} + \left(P_d - \frac{C_t}{2}\right)\right] I_{c_4}S_{c_4} - \lambda_3\right] = 0 --------(10)$$

Condition :3

$\lambda_1\left(q_2 - q_1\right) = 0; \lambda_2\left(q_3 - q_2\right) = 0; \lambda_3\left(q_4 - q_3\right) = 0; \lambda_4 q_1 = 0$

Condition :4

$q_2 - q_1 \geq 0; q_3 - q_2 \geq 0; q_4 - q_3 \geq 0; q_1 \geq 0$

So we get $\lambda_4 = 0$.

Then replace q_2 by q_1, q_3 by q_2, and q_4 by q_3 then $q_1 = q_2 = q_3 = q_4$.

Then add equations (7),(8),(9),(10)we get,

$$\frac{1}{6}\left[\frac{Q_1}{2}\left[H_{c_1} + \frac{O_{c_1} D_r}{P_r}\right] + \frac{D_r}{Q_4}\left[O_{c_1} + \left(P_d - \frac{C_t}{2}\right)\right] I_{c_1} S_{c_1} + \lambda_1 - \lambda_4\right] +$$

$$\frac{2}{6}\left[\frac{Q_2}{2}\left[H_{c_2} + \frac{O_{c_2} D_r}{P_r}\right] + \frac{D_r}{Q_3}\left[O_{c_2} + \left(P_d - \frac{C_t}{2}\right)\right] I_{c_2} S_{c_2} + \lambda_2 - \lambda_1\right] +$$

$$\frac{2}{6}\left[\frac{Q_3}{2}\left[H_{c_3} + \frac{O_{c_3} D_r}{P_r}\right] + \frac{D_r}{Q_2}\left[O_{c_3} + \left(P_d - \frac{C_t}{2}\right)\right] I_{c_3} S_{c_3} - \lambda_2 + \lambda_3\right] +$$

$$\frac{1}{6}\left[\frac{Q_4}{2}\left[H_{c_4} + \frac{O_{c_4} D_r}{P_r}\right] + \frac{D_r}{Q_1}\left[O_{c_4} + \left(P_d - \frac{C_t}{2}\right)\right] I_{c_4} S_{c_4} - \lambda_3\right] = 0$$

By simplifying

$$Q^* =_c \sqrt{\frac{2D_r\left[1\left(O_{c_1}\left(P_d - \frac{C_t}{2}\right)S_{c_1}I_{c_1}\right) + 2\left(O_{c_2}\left(P_d - \frac{C_t}{2}\right)S_{c_2}I_{c_2}\right) + 2\left(O_{c_3}\left(P_d - \frac{C_t}{2}\right)S_{c_3}I_{c_3}\right) + 1\left(O_{c_4}\left(P_d - \frac{C_t}{2}\right)S_{c_4}I_{c_4}\right)\right]}{\left(H_{c_1} + 2H_{c_2} + 2H_{c_3} + H_{c_4}\right) + \left[\frac{O_{c_1}D_r + 2O_{c_2}D_r + 2O_{c_3}D_r + O_{c_4}D_r}{P_r}\right]}}$$

Numerical Example:

Crisp sense:

D_r =1100, O_c =6, S_c =1600, I_c =0.12, P_d =5, C_t =7, H_c =27, P_r =1300

Q^* =142.72

Tc^* =Rs.4575.41

Fuzzy Sense:

D_r =1100

O_c =(4,5,7,8)

S_c =1300,1500,1700,1900

I_c =(0.8,0.10,0.14,0.16)

P_d =5

C_t =7

H_c =(23,26,29,32)

P_r =1300

Q^* =141.85

$\tilde{T}c$ =Rs.4534.72

Conclusion:

We present a fuzzy inventory model in this work to dispose of wastages that arise throughout the production process. The problem is defined in both a crisp and a fuzzy sense. Defuzzification is accomplished using the Beta distribution approach. Also regarded as trapezoidal fuzzy numbers are parameters such as ordering cost, setup cost, and holding cost. Numerical examples are used to verify the calculated solutions..

References:

[1]. Inventory systems by Eliezer Naddor

[2] Fuzzy sets and logics by Zadeh

[3]. Harris, F., Operations and cost, AW Shaw Co. Chicago, (1915).

[4]. Wilson, R., A scientific routine for stock control. Harvard Business review, 13, 1934, 116.128

[5]. Hadley, G., Whitin T. M., Analysis of inventory systems, Prentice – Hall, Englewood dipps, NJ, 1963.

[6]. Zimmerman, H. J., Using fuzzy sets in operational research, European journal of operational research

[7]. Urgeletti Tinarelli, G., Inventory control models and problems, European Journal of operations research, 14, 1983, 1 – 12.

[8]. Chan, Wang, Backorder fuzzy inventory model under function principle, Information science, 95, 1996,1-2, 71-79.

[9]. Vujosevic, M., Petrovic, D., Petrovic, R., EOQ formula when inventory cost is Fuzzy, International Journal of production Economics, 45, 1996, 499-504.

[10]. D. Dutta, Pavan kumar, Fuzzy inventory model without shortages using Trapezoidal fuzzy number with sensitivity analysis.