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Some Applications of Normal Curvature on Smooth Surfaces

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Abstract

Curvatures are important geometric quantities on the surface for different areas of research. many segmentation or mesh smoothing algorithms use normal curvatures to act as the feature to determine region boundaries.There are many uses for curvatures in many fields of chemistry, physics, engineering, and others. We deal with two applications, one is used in the manufacture of car tire tubes, and the other application used to find the normal curvature and ideal level for its manufacture and is modeled via the computer to know its changes and produced in perfect way. Secondly, we took an example in the field of chemistry. The aims of this study is to identify and develop some applications of normal curvature on smooth surfaces . We followed the applied mathematical method using matlab . We found that the normal curvature is a piece of glass that is used for laboratory purposes and for dealing with various liquids that can help researcher to know the liquids nature.

Key Ward : **Applications , Normal Curvature , Smooth Surfaces , Matlab .**

1. Introduction:

Plane geometry is mainly the study of the properties of polygons and circles. Differential geometry is the study of curves that can be locally approximated by straight line segments **[1].** Differential geometry is closely related to differential topology and the geometric aspects of the theory of differential equations. Differential geometry arose and developed as a result of and in connection to the mathematical analysis of curves and surfaces.**[2]** Differential geometry has encountered numerous applications in physics. More and more physical concepts considered as fundamental can be understood as a direct consequence of geometric principles. The mathematical structure of Maxwell's electrodynamics, general theory of relativity, string theory and gauge theories, to name but a few, are of a geometric nature**.** [3**]** The notion of the normal curve as introduced by Whitney [12] is important partly for the following reasons: the normal curves are dense in a natural sense in the regular C" curves; the combinatorial topology embodied in the self-intersection numbers is welldefined. The self-intersection numbers and another minor condition determine a curve up to a sense preserving homeomorphism of the plane onto itself.**[4]**

Gaussian geometry is the study of curves and surfaces in three dimensional Euclidean space. This theory was initiated by the ingenious Carl Friedrich Gauss (1777-1855) in his famous work Disquisitions generals circa superficies curvas from 1828.The work of Gauss, J´anosBolyai (1802-1860) and Nikolai IvanovichLobachevsky (1792-1856) then lead to their independent discovery of non-Euclidean geometry. **[5]** Curvature Some curvature, like normal curvature, has the property such that it depends on how we embed the surface in R^3. Normal curvature is extrinsic; that is, it could not be measured by being on the surface. On the other hand, another measurement of curvature, namely Gauss curvature, does not depend on how we embed the surface in R^3. Gauss curvature is intrinsic; that is, it can be measured from on the surface. **[6]**

2. Normal Curvature:

We now tackle the problem of defining curvature on regular surfaces. We do this by expressing it in terms of the curvatures of regular curves on the surface. We start with the notion of normal curvature, which is defined with respect to a given regular curve on a surface.

Definition(2.1):. Let C be a regular curve on regular surface C passing through point $p \in S$, k be the curvature of C at p, n be the unit normal vector to C at p, N be the unit normal vector to the surface at p, and $cos\ \theta = < n, N >$, Then the normal curvature of C at point p is defined to be the signed quantity$k_n = k \cos\theta$.

At first glance, this definition doesn't seem terribly useful, since we'd like a definition of curvature that only depends on the properties of the surface, independent of the curves we can draw on it. It turns out that the definition of normal curvature is independent of the specific choice of curve C and only depends on the value of its tangent at point p. To do this, we first express the normal curvature in terms of the differential of the Gauss map. $N\ o\ C$denotes the restriction of the Gauss map to the curve C. Since the normal vector $N\ (p)$ is orthogonal to every tangent vector at p, $< N\ (p), C'(0) > = < N\ C(0), C'(0) > = 0$ Differentiating both sides yields $< (N\ oC)'(0), C'(0) > = < N\ (p), C''(0) >$.Thus,

$$k_n = k(0)\cos\theta$$

$$= k(0) < n(0), N\left(C(0)\right) >$$

$$= < C''(0), N\left(C(0)\right) >$$

$$= - < C'(0), (N\ oC)'(0) >$$

$$= - < C'(0), dN_p(C'(0)) >$$

The last line thus shows that the normal curvature only depends on the tangent $C'(0)$ This development is very reassuring, since it gives us a notion of the curvature of a surface in a specific direction in the tangent plane at p, namely $C'(0)$.

A natural next step would be to determine the directions of minimum and maximum normal curvature, and if a minimum and maximum even exist. It turns out that they do, and that they are the negative eigenvalues of the differential of the Gauss map at point p**[7].**

3. The Gaussian curvature:

It would be tempting to try to reduce to a description by means of a single number. One such number is the following measure of curvature, which was introduced by Gauss. Recall, that if $U \subset R^n$ is an m-dimensional linear space and $L: U \to U$ a linear map, the determinant of L, denoted by det L, is defined as the determinant of the $m \times m$ matrix that represents L in some basis for U . It is a theorem of linear algebra that the determinant is independent of the chosen basis (the matrix will be different in another basis, but the determinant will remain the same).

Definition(3.1). The Gaussian curvature (or total curvature) $K(p)\ of\ \sigma\ at\ p$ is the determinant of the map .

$$K(p) = det\left(\begin{pmatrix} E & F \\ F & G \end{pmatrix}^{-1} \begin{pmatrix} L & M \\ M & N \end{pmatrix}\right) = \frac{LN - M^2}{EG - F^2}$$

Notice that the determinant K(p) does not depend on the use of the basis σ'_u, σ'_v for $T_p\sigma$ which is used in the above expression.

It will be seen in the examples below that there exist surfaces with quite different shapes, which have the same Gaussian curvature everywhere. Therefore, the Gaussian curvature does not hold complete information about the shape of the surface.

Example (3.1): For the plane we saw W is the zero operator. Hence its Gaussian curvature is $K = 0$. For the unit sphere we determined W to be the identity operator and we conclude that the Gaussian curvature is $K = 1$.

Example(3.2) : Consider again the cylinder $\sigma(u, v) = (cos\ v, sin\ v, u)$. We will determine the Gauss curvature in the point $\sigma(u, v)$.

We saw that $E = G = 1, F = 0$, and $L = M = 0, N = 1$. It follows that the Gaussian curvature is $K = 0$. Notice that the cylinder and the plane thus have the same Gaussian curvature, although they have different shapes. The sign of the Gaussian curvature has a particular geometric significance, which is explained in the following result. **[8]**

4. The Applications of Normal Curvature in Industry:

1. Tire Tubes Curvature:

Flow patterns in curved tubes have important applications in aerosol science and technology, as well as engineering problems such as heat and mass transfer and analysis of particle movement in curved tubes and aerosol instruments[9]. The curved tube has secondary motion in the plane perpendicular to the axis of the flow, and the axial flow itself is skewed rather than parabolic. These new elements of the flow pattern will affect the transport mechanisms of particles and gases [10].

- **Code in MATLAB:**

```
length_a=5;
length_b=1;
U=linspace(0,2*pi,40);
V=U;
[U,V]=meshgrid(U,V);
x=(length_a+length_b*cos(V)).*cos(U);
y=(length_a+length_b*cos(V)).*sin(U);
z=length_b*sin(V);
surf(x,y,z,...
'FaceColor','interp',...
'EdgeColor','none',...
'FaceLighting','phong')
camlightleft
colormap('default')
axisequal
axisoff
view(160,30)
title('Tire tube')
```

Result:

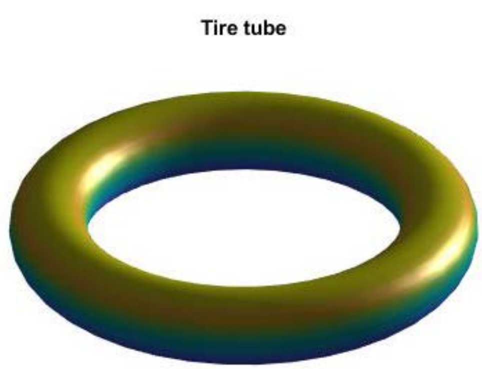

Figure No. (1): Show the Tire Tube Shape

2. A Klein Bottle:

Klein's bottle is formed by joining two sides of an arch into a cylinder (tube), then rolling the ends of a cylinder through itself so that the inside (green) and outside (white) of the cylinder are united. It is obvious that the Klein bottle, like the better-known sphere, is a closed surface: it is finite in the sense that it fits into a finite area of space, but an ant could walk on it forever without ever meeting one[11] . In contrast to the ball, which has an inside and an outside, the small bottle is one-sided: our ant could reach both sides of any point on the surface while walking, so that the bottle does not contain any volume and also reacts to the "why" Question: Klein's bottle is interesting because we don't find many one-sided shapes in nature [12].

MATLAB Code:

```
raduis=1;
U=linspace(0,2*pi,60);
V=linspace(0,2*pi,60);
[U,V]=meshgrid(U,V);
X_Axis=(raduis+cos(U/2).*sin(V)-sin(U/2).*sin(2*V)).*cos(U);
Y_Axis=(raduis+cos(U/2).*sin(V)-sin(U/2).*sin(2*V)).*sin(U);
Z_Axis=sin(U/2).*sin(V)+cos(U/2).*sin(2*V);
surf(X_Axis,Y_Axis,Z_Axis,...
'FaceColor','interp',...
'EdgeColor','none',...
'FaceLighting','phong')
camlightleft
colormap('default')
axisoff
```

Result:

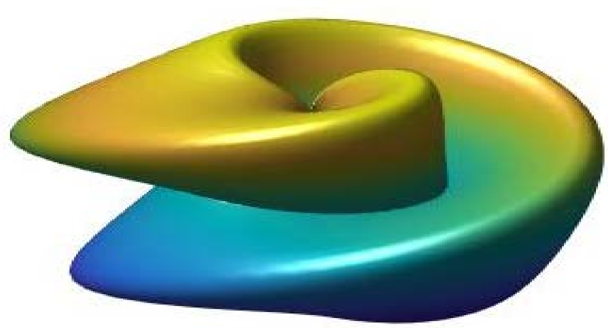

Figure No. (2): A Klein Bottle Shape

Conclusion:

There are many uses for curvatures in many fields of chemistry, physics, engineering, and science. In this paper we handled two applications the first one applied in the manufacture of car tire tubes, and the second one applied the normal curvature and ideal level for its manufacture and is modeled via the computer to know its changes and produced in perfect way. Secondly, we took an example in the field of chemistry It is a piece of glass that is used for laboratory purposes and for dealing with various liquids that can help researcher to know the liquids nature.

References:

[1] Andrew Grossfield, A Two-Dimensional Comparison of Differential Calculus and Differential Geometry, Copyright © 2020, American Society for Engineering Education

[2] Kande Dickson Kinyua ,Kuria Joseph Gikonyo, Differential Geometry: An Introduction to the Theory of Curves, January 10, 2018

[3] Eduardo Nahmad-Achar, Differential Topology and Geometry with Applications to Physics , December 2018

[4] CHARLES J. TITUS, A THEORY OF NORMAL CURVES AND SOME APPLICATIONS, Vol. 10, No. 3 November 1960

[5] SigmundurGudmundsson, An Introduction to Gaussian Geometry, September 2020.

[6] Copyright c Alexander C. R. Belton , MATH329 Geometry of Curves and Surfaces, 2015.

[7] AshwinDevaraj, An Overview of Curvature,May 2020.

[8] HenrikSchlichtkrull, Curves and Surfaces, HenrikSchlichtkrull, March 2011.

[9] Vinberg, E.B. ed., 1993. Geometry II: spaces of constant curvature. Berlin: Springer-Verlag

[10] Hyde, S., Blum, Z., Landh, T., Lidin, S., Ninham, B.W., Andersson, S. and Larsson, K., 1996. The language of shape: the role of curvature in condensed matter: physics, chemistry and biology. Elsevier.

[11] Guoxiang, G.U.I., 2018. The Application of Curvature in Real Life. Management Science and Engineering, 12(3),

[12] Roberts, A., 2001. Curvature attributes and their application to 3 D interpreted horizons. First break, 19(2),