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Symmetrical Component Transformation of a Three-Phase Impedance Coupled to a Single-Phase Impedance in Electric Power Systems

Diego Bellan

Department of Electronics, Information and Bioengineering, Politecnico di Milano
20133 Milan, Italy

Abstract

This paper deals with the use of the Symmetrical Component Transformation (SCT) in the analysis of an electric power system where a three-phase network is interconnected with a single-phase network, and a three-phase impedance is coupled to a single-phase impedance. It is shown that, in this case, specific derivations are required to obtain a consistent equivalent circuit taking into account such impedance coupling. In particular, it is shown that the mutual impedance affects the zero-sequence circuit only, whereas the positive and negative-sequence circuits are not affected. Moreover, beside the ideal transformers taking into account the SCT transformation of the wye connections, two series impedances must be considered in the equivalent circuit to take into account impedance coupling. Further equivalent circuits can be easily obtained by exploiting the well-known properties of ideal transformers. The paper is intended to have a methodological value, providing a further theoretical contribution to the general theory of the SCT.

Keywords: *Symmetrical Component Transformation, Sequence Circuits, Impedance Coupling, Coupled Single-Phase and Three-Phase Networks.*

1. Introduction

The most common mathematical tool for analyzing three-phase electrical networks in the phasor domain is the well-known Symmetrical Component Transformation (SCT). Despite the SCT was originally proposed about one century ago [1], it is still the basic idea underlying most of the modern methodologies for power system analysis [2]-[3]. The main assumption of the SCT approach is that a three-phase system has a symmetrical structure, i.e., the passive part of the three phases is equal, and the mutual interaction between the three phases is equal. Under this basic assumption, the use of the SCT decouples the equations of a given three-phase system. In particular, three decoupled circuits (i.e., the so-called sequence circuits) can be defined and promptly solved without the computational burden due to the originally coupled equations.

In modern power systems, however, the basic SCT assumption concerning the symmetry of the three phases is a critical point to be satisfied. Two main reasons can be identified. First, a pure three-phase power system can show an asymmetrical structure due to untransposed transmission lines and/or large single-phase loads [4]-[9]. Second, three-phase power systems are often interconnected with single-phase networks [10]. Thus, in both cases the conventional SCT must be properly elaborated and/or extended in order to provide a consistent and general methodology.

In this paper, a specific problem related to a single-phase network interconnected to a three-phase network is investigated from a methodological viewpoint. More specifically, the case of an impedance belonging to the single-phase network coupled to a three-phase impedance in the three-phase network is considered. This is not a conventional problem for the SCT which is normally used to decouple the three impedances belonging to the same three-phase impedance. To the Author's knowledge, a general and consistent methodology, within the SCT framework, dealing with the case of a three-phase impedance coupled to a single-phase impedance is not available in the relevant literature. In this paper, thanks to the use of the rational form of the SCT (i.e., the power-invariant form), a general methodology is derived, leading to an equivalent zero-sequence circuit including additive impedances to take into account impedance coupling. The paper is intended to have a methodological value, providing a further theoretical contribution to the general theory of the SCT.

The paper is organized as follows. In Section 2, the theoretical results concerning the use of the SCT to represent the interconnection between three-phase and single-phase networks are recalled. In particular, it is shown that such interconnection can be represented with an ideal transformer in the zero-sequence circuit. In Section 3, the case of a three-phase impedance coupled to an impedance belonging to a single-phase network is investigated. Theoretical derivations are presented to show that impedance coupling can be represented with additive impedances in the zero-sequence equivalent circuit including the ideal transformer. Finally, concluding remarks are provided in Section 4.

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2. SCT of a Wye Connection Interfacing Three-Phase and Single-Phase Networks

The Symmetrical Component Transformation (SCT) is a well-known mathematical tool used to analyze three-phase systems in the phasor domain. The so-called rational form of SCT is defined through the following transformation matrix:

$$\mathbf{S} = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \\ 1 & 1 & 1 \end{bmatrix} \tag{1}$$

where $\alpha = e^{j2\pi/3}$, and $\mathbf{S}$ is a unitary matrix, i.e., $\mathbf{S}^{-1} = \mathbf{S}^*$ (here the asterisk denotes complex conjugate). Notice that the rational form of SCT, consisting in a unitary transformation matrix, leads to the crucial property of power conservation when SCT is used to transform voltages and currents. This property allows the derivation of consistent equivalent circuits in the transformed domain. In particular, let us consider the wye connection depicted in Fig. 1 which is used to connect a three-phase network to a single-phase network. By defining a reference terminal *G*, i.e., a possibly non-physical terminal with zero total current, the voltages represented in Fig. 1 can be defined. The four independent relationships defining the wye connection as a four-port element can be written as:

$$V_a = V_y,\ \ V_b = V_y,\ \ V_c = V_y,\ \ I_a + I_b + I_c = I_y \tag{2}$$

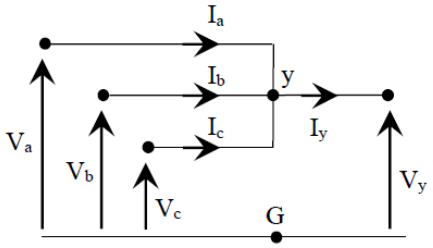

Fig. 1. Interconnection between a three-phase and a single-phase network through the wye connection. The reference terminal G allows the definition of four ports.

By using the SCT (1) to transform the voltages, and by taking into account (2), we obtain:

$$\begin{bmatrix} V_p \\ V_n \\ V_0 \end{bmatrix} = \mathbf{S}\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \sqrt{3}V_y \end{bmatrix} \tag{3}$$

where V_p, V_n, and V_0 are the positive, negative, and zero-sequence voltages, respectively, and where the property $1 + \alpha + \alpha^2 = 0$ has been used. Similarly, the use of SCT for the currents defined in Fig. 1 leads to:

$$\begin{bmatrix} I_p \\ I_n \\ I_0 \end{bmatrix} = \mathbf{S}\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} \frac{I_a+\alpha I_b+\alpha^2 I_c}{\sqrt{3}} \\ \frac{I_a+\alpha^2 I_b+\alpha I_c}{\sqrt{3}} \\ \frac{I_y}{\sqrt{3}} \end{bmatrix} \tag{4}$$

From (3) and (4) we can readily observe that: 1) since $V_p = V_n = 0$, the positive and negative-sequence equivalent circuits of the wye connection in Fig. 1 are given by short circuits; 2) since $V_0 = \sqrt{3}V_y$ and $I_0 = I_y/\sqrt{3}$, the interface between the zero-sequence circuit and the single-phase circuit can be represented by an ideal transformer with ratio $\sqrt{3}$ as shown in Fig. 2. The circuit representation depicted in Fig. 2, showing the interaction between the three-phase and the single-phase networks, is interesting since the well-known properties of ideal transformers can be used (e.g., the possibility to move a network between the primary and the secondary sides of the ideal transformer).

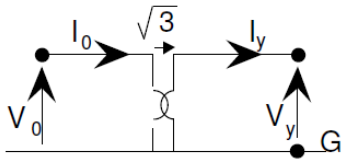

Fig. 2. Zero-sequence equivalent circuit of the wye connection in Fig. 1. The ideal transformer is the circuit representation of coupling between the zero-sequence variables of the three-phase network and the single-phase network.

3. SCT of a Three-Phase Impedance Coupled to a Single-Phase Impedance

Let us consider a three-phase impedance and a simple impedance Z_n in the single-phase network as shown in Fig. 3. In case of single-phase impedance not coupled to the three-phase impedance, the SCT can be used for the three-phase impedance in the conventional way. More specifically, by assuming the three-phase impedance given by the following impedance matrix:

$$\mathbf{Z} = \begin{bmatrix} Z_{ph} & Z_m & Z_m \\ Z_m & Z_{ph} & Z_m \\ Z_m & Z_m & Z_{ph} \end{bmatrix} \tag{5}$$

where Z_{ph} is the phase impedance, and Z_m is the mutual impedance, the voltage/current relationship in the *a*,*b*,*c*, variables:

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \mathbf{Z} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \tag{6}$$

can be readily transformed through the SCT into:

$$\begin{bmatrix} V_p \\ V_n \\ V_o \end{bmatrix} = \mathbf{Z}_s \begin{bmatrix} I_p \\ I_n \\ I_0 \end{bmatrix} \tag{7}$$

where:

$$\mathbf{Z}_s = \mathbf{S}\mathbf{Z}\mathbf{S}^{-1} = \begin{bmatrix} Z & 0 & 0 \\ 0 & Z & 0 \\ 0 & 0 & Z_0 \end{bmatrix} \tag{8}$$

and $Z = Z_{ph} - Z_m$, $Z_0 = Z_{ph} + 2Z_m$. In particular, since in Section 2 it was shown that the single-phase network is connected to the three-phase network through an ideal transformer in the zero-sequence circuit, the relevant equivalent circuit is shown in Fig. 4 where the impedances can be readily reported between the primary and secondary sides of the transformer through the simple factor 3 (i.e., the squared value of the transformer ratio).

The procedure outlined above is valid only under the assumption of single-phase impedance not coupled to the three-phase impedance. In some applications, however, such impedance coupling cannot be neglected. In this case, the conventional procedure outlined above requires further derivations. To this aim, let us assume that the coupling between the single-phase and the three-phase impedances is given by the mutual impedance Z_{st} (Fig. 5). Thus, the voltage/current relationship (6) in terms of the *a*,*b*,*c* variables must be rewritten as:

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \mathbf{Z} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} + Z_{st} \begin{bmatrix} I_y \\ I_y \\ I_y \end{bmatrix} \tag{9}$$

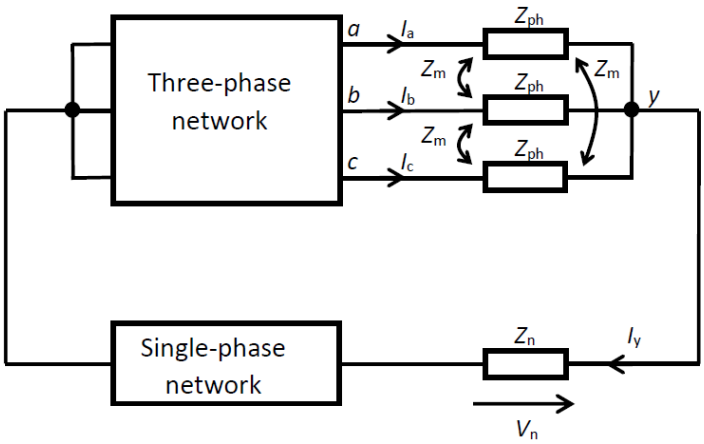

Fig. 3. Three-phase and single-phase networks interconnected through wye connections. In this case the impedance Z_n in the single-phase network is not coupled to the three-phase impedance.

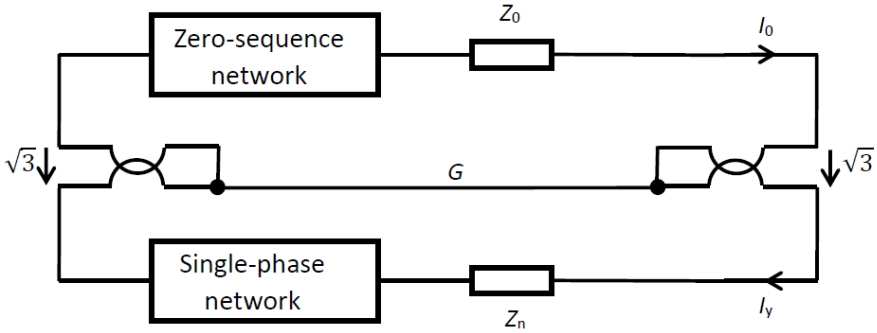

Fig. 4. Circuit coupling between the zero-sequence and the single-phase networks is represented by ideal transformers resulting from the use of the SCT to transform the wye connection.

By taking into account that $I_y = I_a + I_b + I_c$, (9) can be rewritten as:

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \mathbf{Z} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} + Z_{st} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} Z_{ph} + Z_{st} & Z_m + Z_{st} & Z_m + Z_{st} \\ Z_m + Z_{st} & Z_{ph} + Z_{st} & Z_m + Z_{st} \\ Z_m + Z_{st} & Z_m + Z_{st} & Z_{ph} + Z_{st} \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \tag{10}$$

By using the SCT in (10) we obtain:

$$\begin{bmatrix} V_p \\ V_n \\ V_o \end{bmatrix} = \begin{bmatrix} Z_{ph} - Z_m & 0 & 0 \\ 0 & Z_{ph} - Z_m & 0 \\ 0 & 0 & Z_{ph} + 2Z_m + 3Z_{st} \end{bmatrix} \begin{bmatrix} I_p \\ I_n \\ I_0 \end{bmatrix} \tag{11}$$

By comparing (11) and (7) we observe that the mutual impedance Z_{st} has impact only on the zero-sequence impedance with the additive term $3Z_{st}$. Thus, on the three-phase side, the mutual coupling between three-phase and single-phase impedances is taken into account by the impedance $3Z_{st}$ in series with the conventional zero-sequence impedance $Z_0 = Z_{ph} + 2Z_m$. On the single-phase side, however, by considering impedance coupling, for the single-phase impedance we must write:

$$V_n = Z_n I_y + Z_{st} I_a + Z_{st} I_b + Z_{st} I_c \tag{12}$$

By taking into account that $I_a + I_b + I_c = I_y$ we obtain:

$$V_n = (Z_n + Z_{st}) I_y \tag{13}$$

Therefore, on the single-phase side, impedance coupling leads to an equivalent single-phase impedance given by $Z_n + Z_{st}$.
Thus, the original problem consisting in impedance coupling through a mutual impedance Z_{st} between the three-phase and the single-phase impedances, can be treated as an equivalent uncoupled problem where additive series impedances must be added in the zero-sequence circuit, i.e., the impedance $3Z_{st}$ on the three-phase side in series with Z_o, and Z_{st} on the single-phase side in series with Z_n. Such equivalent uncoupled circuit is shown in Fig. 6. Of course, also in this case the impedances can be reported across the ideal transformer with the factor 3. For example, if the whole single-phase impedance $Z_n + Z_{st}$ is reported to the primary side of the ideal transformer, the total impedance on the three-phase side is given by $Z_0 + 6Z_{st} + 3Z_n$.

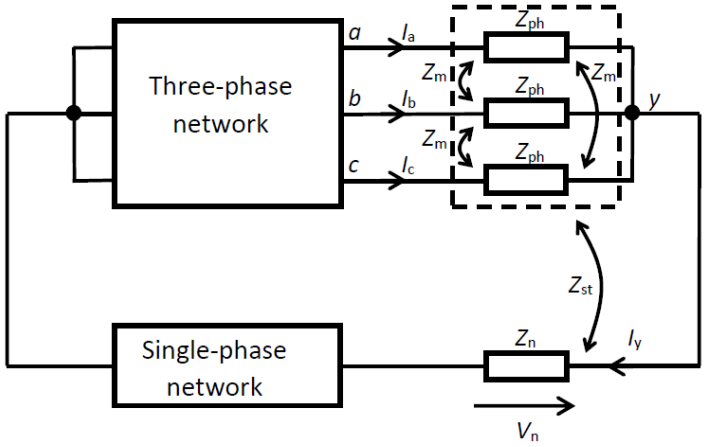

Fig. 5. Three-phase and single-phase networks interconnected through wye connections. In this case the impedance Z_n in the single-phase network is coupled to the three-phase impedance through the mutual impedance Z_{st}.

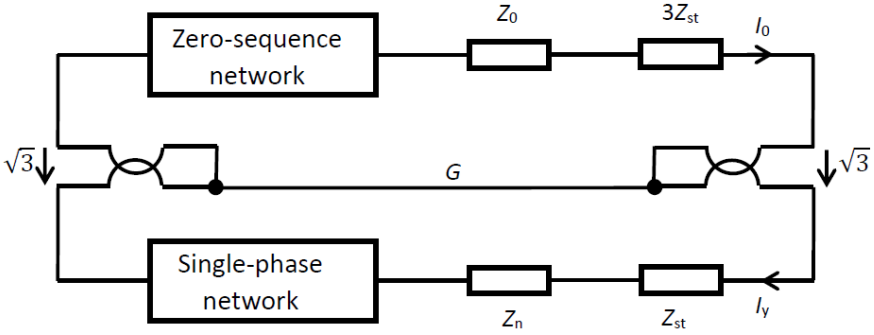

Fig. 6. Coupled zero-sequence and single-phase networks. The ideal transformers take into account the wye connections, whereas the impedance coupling shown in Fig. 5 is taken into account by the series impedances $3Z_{st}$ and Z_{st}.

4. Conclusions

Theoretical aspects related to the use of the SCT in case of a single-phase impedance coupled to a three-phase impedance have been addressed in the paper. A modified zero-sequence equivalent circuit has been derived to take into account impedance coupling. This point was not covered by the conventional SCT theory. Therefore, this paper is intended as a contribution to a more general SCT theory. Future work will be devoted to investigate further issues oriented to extend the use of the SCT under non-conventional assumptions.

References

[1] C. L. Fortescue, “Method of Symmetrical Co-ordinates Applied to the Solution of Poly-Phase Networks (with discussion)”, in the *34th Annual Convention of the AIEE (American Institute of Electrical Engineers)*, Atlantic City, NJ, USA, 28 June 1918, Vol. 37, pp. 1027–1140.
[2] J. C. Das, Understanding Symmetrical Components for Power System Modeling, NJ, USA: Wiley, 2017.
[3] G. Chicco and A. Mazza, “100 Years of Symmetrical Components”, *Energies*, Vol. 12, No. 3, 2019, pp. 1-20.
[4] D. Bellan, “Statistical analysis of voltage unbalance emission due to asymmetrical loads in three-phase power systems,” *International Journal of Circuits, Systems and Signal Processing*, Vol. 16, 2022, pp. 747-753.

[5] D. Bellan, G. Superti-Furga, and S. A. Pignari, Circuit representation of load and line asymmetries in three-phase power systems, *International Journal of Circuits, Systems and Signal Processing*, Vol. 9, 2015, pp. 75-80.
[6] Z. Emin and D. S. Crisford, "Negative phase-sequence voltages on E&W transmission system," *IEEE Trans. on Power Delivery*, Vol. 21, No. 3, 2006, pp. 1607-1612.
[7] P. Paranavithana, S. Perera, R. Koch, and Z. Emin, "Global voltage unbalance in MV networks due to line asymmetries," *IEEE Trans. on Power Delivery*, Vol. 24, No. 4, 2009, pp. 2353-2360.
[8] U. Jayatunga, S. Perera, and P. Ciufo, "Voltage unbalance emission assessment in radial power systems," *IEEE Trans. Power Delivery*, Vol. 27, No. 3, 2012, pp. 1653-1661.
[9] U. Jayatunga, S. Perera, P. Ciufo, and A. P. Agalgaonkar, "Deterministic methodologies for the quantification of voltage unbalance propagation in radial and interconnected networks," *IET Generation, Transmission & Distribution*, Vol. 9, No. 11, 2015, pp. 1069-1076.
[10] D. Bellan, S. A. Pignari, G. Superti-Furga, "Consistent circuit technique for zero-sequence currents evaluation in interconnected single/three-phase power networks," *Journal of Electrical Systems*, Vol. 12, No. 2, 2016, pp. 230-238.